2018_19 AP Calculus AB – Summer Packet

NOTE - Please mark July 30 on your calendars. We will have a boot camp in my room from 12pm-3pm on this day. We will work together on the summer packet. Time permitting, we will also tackle additional topics.

IF you participate fully in this boot camp, you will get full credit for the summer packet.

| Mr. Lynch | 1-055 | Calculus BC | 7/30 | 8am-11am |
| Mr. Lynch | 1-055 | Calculus AB | 7/30 | 12pm-3pm |

Welcome to AP Calculus AB at Boca Ciega High School. I'm Mr Lynch - many of you know me already. You should feel free to reach me at lynchr@pcsb.org. I will check my email most days.

There are certain math skills necessary to be successful in AP Calculus and ultimately on the AP exam. This packet is designed to review those topics so that you will be "math-ready" in August. AP Calculus is a college level class, requiring the ability to read a college text (which we will do) and the ability to do internet research (which you will do). To this end, I have included websites that you will need to finish the packet and the course. You can also email me if you have specific questions.   I STRONGLY recommend you work together if you have that option.

If you do not fully understand the topics in this packet, it is possible that you will get calculus problems wrong in the future not because you do not understand the calculus concept but because you do not understand the algebra or trigonometry behind it.   Don't fake your way through any of these problems because you will need to understand everything in this to be successful. There are ten assignments – each should take an hour or so. You should aim to complete one assignment per week. Most of the assignments are review, but there are one or two that may be new to you. Use the internet !

Do NOT depend on in-class review to learn the content in these assignments. Also, do not rely on a calculator to do the work (more than half of the AP exam does not allow any calculator at all).

The ten PHYSICAL assignments are:
1) Some algebra basics (proportionality) and common algebra mistakes
2) polynomial zeros, multiplicity, and end behavior
3) slope
4) composite functions
5) the unit circle and all 6 trig functions
6) Pascal's triangle and the binomial expansion
7) position, velocity, and acceleration
8) rational functions and asymptotes
9) the shape of functions
10) basic limits (review for most of you)

I want to finish with a quick word of encouragement. I know many of you, and, from what I've seen of your past work, I know you can master AP Calculus. The content is tougher than any math you've taken so far, but with focus and hard work throughout the year, you should do fine in class and on the AP exam.

Important websites that you must be familiar with:

1) Patrickjmt.com – perhaps the best calculus contributor online.

2) Khanacademy.org – excellent material

3) AP Central   http://apcentral.collegeboard.com/apc/members/exam/exam_information/1997.html
    -    Important AP exam information (regarding exam format, calculutator use, etc)
    -    AP free response questions for the last 10-15 years. We will do many of these together.

4) desmos.com – great free online graphing calculator

5) Paul's Online Notes for algebra and calculus   http://tutorial.math.lamar.edu/Classes/Alg/Alg.aspx
    -    excellent material for both algebra and calculus

6) Hippocampus for algebra and AP Calc AB   http://www.hippocampus.org/Calculus
    -    lessons and related problems

7) Miscellaneous other websites – find them and let me know. I'll include the best of them.
         YouTube – http://www.youtube.com/results?search_query=calculus&aq=f
    -    many good tutorials covering all math topics. Search by keyword.

PHYSICAL Assignment 1 - Some algebra basics (proportionality) and common algebra mistakes

REVIEW – There are two types of proportionality that often show up in word problems:

1) y is directly proportional to x. Sometimes this is written as y is proportional to x (skipping "directly")

    In equation form, this means $y = kx$ where k is the constant of proportionality

2) y is inversely proportional to x

    In equation form, this means $y = k/x$ where, again, k is the constant of proportionality

    Note that this one can also be written as $xy = k$

PROBLEMS –

1) y is proportional to x. If y is 10 when x is 24, find x when y is 15.
2) P is inversely proportional to V. If P is 16 when V is 12, find P when V is 4.
3) The number of bacteria P in a Petri dish is proportional to the square root of time t (in hours). Write an equation representing this problem. If the population is 200 bacteria after 1 hour, how many bacteria are there after 1 day (24 hours) ?
4) The amount of time t that it takes a painting company to paint a house is inversely proportional to the number of workers. If it takes 3 workers 14 hours to complete the house, how long will it take 7 workers ?
5) The force (F) of gravity between two objects varies inversely with the distance (r) between the two objects squared. If the force between two objects is 100 lbs when the objects are 4000 miles apart, what is the force between the objects when the objects are 8000 miles apart ?

COMMON ALGEBRA MISTAKES – Nine of the ten problems below have a commonly made mistake. Identify the one problem that is done correctly, then fix the remaining nine problems. Note that – for one or two problems – the correct answer is to leave it "as-is" rather than trying to rewrite it. You may want to work with others on this section!

1)   If $y = 3(2)^x$, then when $x = 3, y = 216$

2) $(x + 3)^2 = x^2 + 9$

3) $\frac{3+2x}{3} = 1 + 2x$

4) $\sqrt{x^2 - 8x + 16} = x - 4$

5)   If $y = \log_{16} x$, then when $x = 2, y = -4$

6) $\frac{\pi}{2} + \frac{2\pi}{3} = \frac{3\pi}{5}$

7) $\sqrt{9x^2 - 16} = 3x - 4$

8) $\frac{3x-2}{x^2+5} = 3x - 2/x^2 + 5$

9) $\frac{2}{3} / \frac{3}{4} = \frac{1}{2}$

10) $\frac{1}{x^2+x^3} = x^{-2} + x^{-3}$

PHYSICAL Assignment 2 – Polynomial Zeroes, Multiplicity, and End Behavior

REVIEW –   Remember...   zeroes = roots = x-intercepts

You find zeroes by determining where a polynomial hits the x-axis (set $y = 0$)

Generally – to find zeroes – you must factor the polynomial if it is not already factored.

Multiplicity refers to the exponent on each factored term in the polynomial.

If multiplicity is: odd, the curve crosses the x-axis at that zero.

even, the curve touches (but does not cross) the x-axis at that zero.

End behavior (checked when a polynomial is in standard form) depends on the sign of the leading coefficient (a_n) and whether the degree (n) of the polynomial is even or odd, as shown below.

| n is even | n is even | n is odd | n is odd |
| $a_n > 0$ | $a_n < 0$ | $a_N > 0$ | $a_n < 0$ |
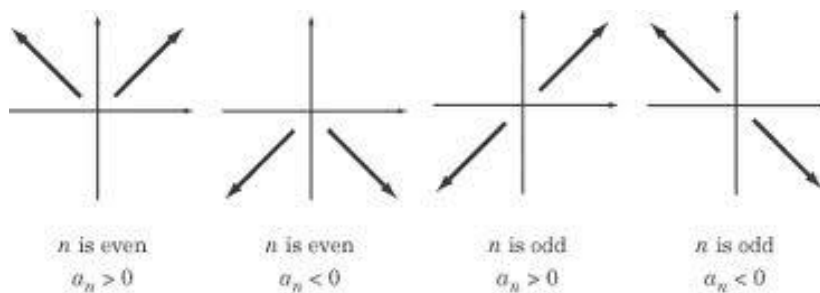
EXAMPLE –   Given $y = x^3 - 5x^2 + 3x + 9 = (x + 1)(x - 3)^2$:   1) Find the zeroes and multiplicity.

2) Discuss end behavior.

Solution: zero at $x = -1$ with odd multiplicity (exponent is 1) → curve crosses x-axis

zero at $x = 3$ with even multiplicity (exponent is 2) → curve touches x-axis

Because $a_n = +1$ and degree is odd ($n = 3$), end behavior is down/up

PROBLEMS – For each of the following polynomials, determine zeroes and multiplicity, then sketch end behavior.

1) $y = (x + 3)^2(x - 7)^3$

2) $y = (x - 2)^2(1 - x)$

3) $y = x^2 - 8x + 16$

4) $y = (x - 3)^2(1 - x)^2$

5) $y = x^3 - 125$ (note special factoring required)

6) $y = (x - 1)^2(x + 2)^3(x - 5)$

7) $y = x^4 - 16$

8) $y = x^3 + x^2 - 9x - 9$

9) $y = 3 + 2x - x^2$

PHYSICAL Assignment 3 – Slope !!!

Slope is so incredibly important for a study of calculus. For the first half of the course, we will study <u>the derivative,</u> <u>which is simply the slope of a curve at a point</u>. You must have mastered the slope of a line prior to mastering the slope of a curve.

<u>PROBLEMS</u> –

1) Determine – in <u>slope-intercept form</u> – the equation for each of the following lines:
   a. slope of 3 and y-intercept of 7
   b. slope of -2 going through (2,5)
   c. going through (3,5) and $(-2,8)$
   d. going through $(-3,-3)$ and parallel to line $x - 2y = 7$

2) Determine – <u>in point-slope form</u> – the equation for each of the following lines:
   a. slope of -1 going through $(3,-2)$
   b. going through (2,1) and (8,4)
   c. going through (10,3) and parallel to line $3x + 2y = 7$
   d. going through (2,0) and perpendicular to line $2x - 5y = 1$

3) On the blank graph to the right, sketch the following lines and label each one (either a, b, or c):

   a. going through (2,2) with slope of -1

   b. going through (5,2) with slope of 4

   c. going through (1,4) with slope of 0
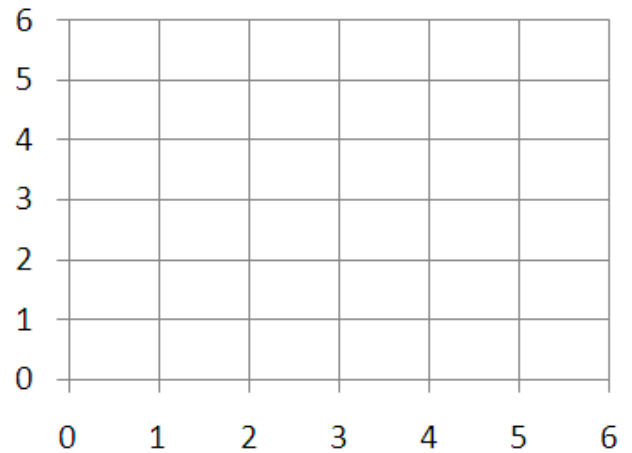
4) On the graph below is the (solid) function $y = f(x)$. At three different points – labelled a, b, and c – along the function - a (dotted) tangent line is drawn. <u>The slope of each tangent line represents the derivative, that</u> <u>is, the slope of the function $y = f(x)$ at that point</u>. Estimate the slope of each of the three tangent lines, then write an equation – in point-slope form – for each of these three tangent lines.
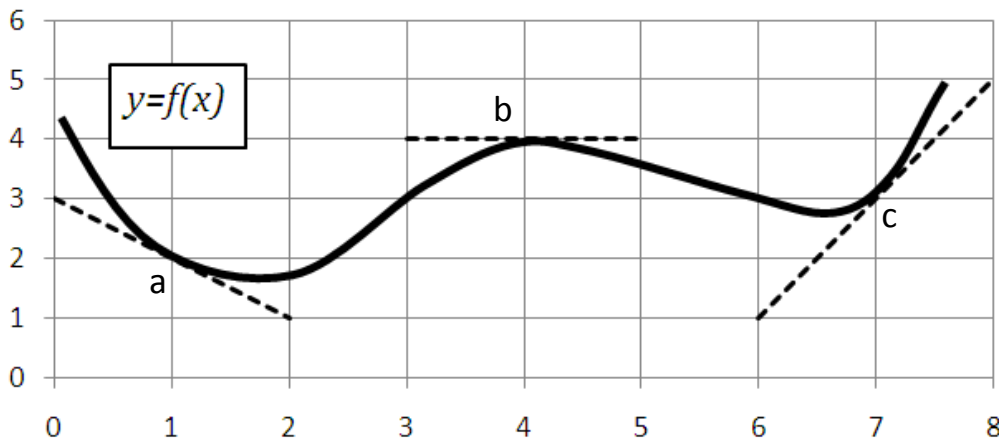
PHYSICAL Assignment 4 – Composite Functions

REVIEW – We can add, subtract, multiply, and divide two functions. We can also take the <u>composite</u> of two functions. In simple terms, the composite of two functions uses the output of one function as the input of the second function. If we have two functions $f(x)$ and $g(x)$, the composite of $f(x)$ of $g(x)$ is written as $f(g(x))$ or sometimes as $(f \circ g)(x)$. In calculus, we use composite functions all the time, and we have to recognize that a single function can often be broken down as a composite of two simpler functions.

WATCH: PatrickJMT on YouTube - http://www.youtube.com/watch?v=S4AEZElTPDo

<u>EXAMPLE</u> – Given $f(x) = 3x^2 - 2x$ and $g(x) = \sqrt{x}$, find the following (answers provided in purple):

a) $f(x) + g(x)$ $= 3x^2 - 2x + \sqrt{x}$

b) $f(x) \bullet g(x)$ (multiplication) $= (3x^2 - 2x) \bullet (\sqrt{x})$ $= 3x^2\sqrt{x} - 2x\sqrt{x}$

c) $f(g(x))$ $= 3(g(x))^2 - 2(g(x))$ $= 3(\sqrt{x})^2 - 2(\sqrt{x})$ $= 3x - 2\sqrt{x}$

d) $g(f(x))$ $= \sqrt{(f(x)}$ $= \sqrt{3x^2 - 2x}$

e) $f(g(4))$ $= 3(4) - 2\sqrt{(4)} = 12 - 4 = 8$

f) $g(f(4))$ $= \sqrt{3(4)^2 - 2(4)}$ $= \sqrt{40}$

<u>PROBLEMS</u> – For each of the following sets of two functions, find 1) $f(g(x))$, 2) $g(f(x))$, and 3) $g(f(2))$

1) $f(x) = x^3$ $g(x) = x + 1$

2) $f(x) = \sqrt{x + 7}$ $g(x) = 3x - 2$

3) $f(x) = \log(50x)$ $g(x) = 5x - 3$

4) $f(x) = \sqrt[3]{x + 6}$ $g(x) = 2x$

5) $f(x) = 3^x$ $g(x) = \sqrt{x + 7}$

6) $f(x) = \sin\left(\frac{\pi x}{6}\right)$ $g(x) = 5x - 2$

7) $f(x) = \csc\left(\frac{\pi x}{12}\right)$ $g(x) = \sqrt{5x - 1}$

8) $f(x) = 3x + 1$ $g(x) = \sin\left(\frac{\pi x}{6}\right)$

<u>PROBLEMS</u> – Express each of the following functions as the composite of two simpler functions. These problems are similar to the previous section, except they are done IN REVERSE. Problem 9 is done as an example.

9) $y = \sin(3x + 1)$ $y = f(g(x))$ if $f(x) = \sin(x)$ and $g(x) = 3x + 1$

10) $y = \sqrt{2x + 1}$

11) $y = \log(x^2 + 5)$

12) $y = \sqrt[3]{3x + 4}$

13) $y = 3\sin^2 x + 2\sin x$

PHYSICAL Assignment 5 – The Unit Circle and *all* Six Trig Functions

NOTES - In AP Calculus, we measure all angles using radians. We will put our graphing calculators in radian mode and leave them there for the full year. You must be fluent with the unit circle, knowing all the angles (in radians!) and coordinates, as well as the six trig functions for any of the standard angles. You will be tested on these without access to a calculator. You must also know how to set up and solve problems that relate the sides and angles of a right triangle.

PROBLEMS – Determine exact answers for each of the following, **WITHOUT** the use of a calculator.

1) $\sin\left(\frac{\pi}{4}\right)$

2) $\cos(3\pi)$

3) $\tan\left(\frac{5\pi}{3}\right)$

4) $\csc\left(\frac{\pi}{6}\right)$

5) $\sec\left(\frac{21\pi}{4}\right)$

6) $\cot\left(\frac{3\pi}{4}\right)$

7) $2\sin\left(\frac{-11\pi}{6}\right)$

8) $12 - 7\csc\left(\frac{5\pi}{6}\right)$

9) $\tan\left(\frac{3\pi}{2}\right)$

10) $\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)$

11) $\sin\left(\frac{1807\pi}{6}\right)$

12) $\cos\left(-\frac{\pi}{2}\right)$

13) $\cot\left(\frac{\pi}{3}\right)$

14) $\csc\left(\frac{5\pi}{3}\right)$

15) $7 - 12\cos\left(\frac{-11\pi}{4}\right)$

16) $\cos\left(\frac{5\pi}{6}\right)$

PROBLEMS – Solve each problem using right triangle trigonometry. Calculators are allowed.

17) The distance from you to a building is 85 feet along level ground. How tall is the building if the angle of elevation from you to the top of the building is 0.45 radians ?

18) A cell phone tower is supported by 4 cables, each going from the top of the tower to the ground. If the tower is 170 feet tall and the angle each cable makes with the ground is 1.21 radians, what is the total length of cable required ?

19) You are watching a rocket launch at a distance of 12000 feet (just over 2 miles) from the launch site. Exactly 10 seconds after the launch, the angle of elevation from you to the rocket is 0.89 radians. How far has the rocket travelled in this time ? What was the average speed of the rocket (in units of ft/sec) for the first 10 seconds ?

20) Two boats leave a harbor at the same time, one travelling directly west at 14 miles/hr and one travelling directly south at 9 miles/hr. What is the distance between the two boats after 2.5 hours ?

21) Along a valley floor, the distance to the base of a cliff is 250 feet. If the cliff is 1800 feet high, what is the angle of elevation (in radians) to the top of the cliff.

22) The hypotenuse of a right triangle has a length of 12.1 cm. If one side of the triangle has a length of 7.4 cm, what are the measure (in radians) of each angle in the triangle ? Also, what is the length (in cm) of the second side ?

PHYSICAL Assignment 6 –
Pascal's Triangle and the
Binomial Expansion

<u>NOTES</u> - For this assignment, you
may need to do independent online
research. I've included a few links
below that may be useful. Note
that the math in many of these
links starts out easy, but quickly
becomes complicated. While you
are not expected to know all the
math discussed in these links, you
are expected to understand enough
of the math to properly complete
this assignment.

<u>PROBLEM 1</u> - Fill out rows 0
through 8 in the blank Pascal's
Triangle shown to the right.
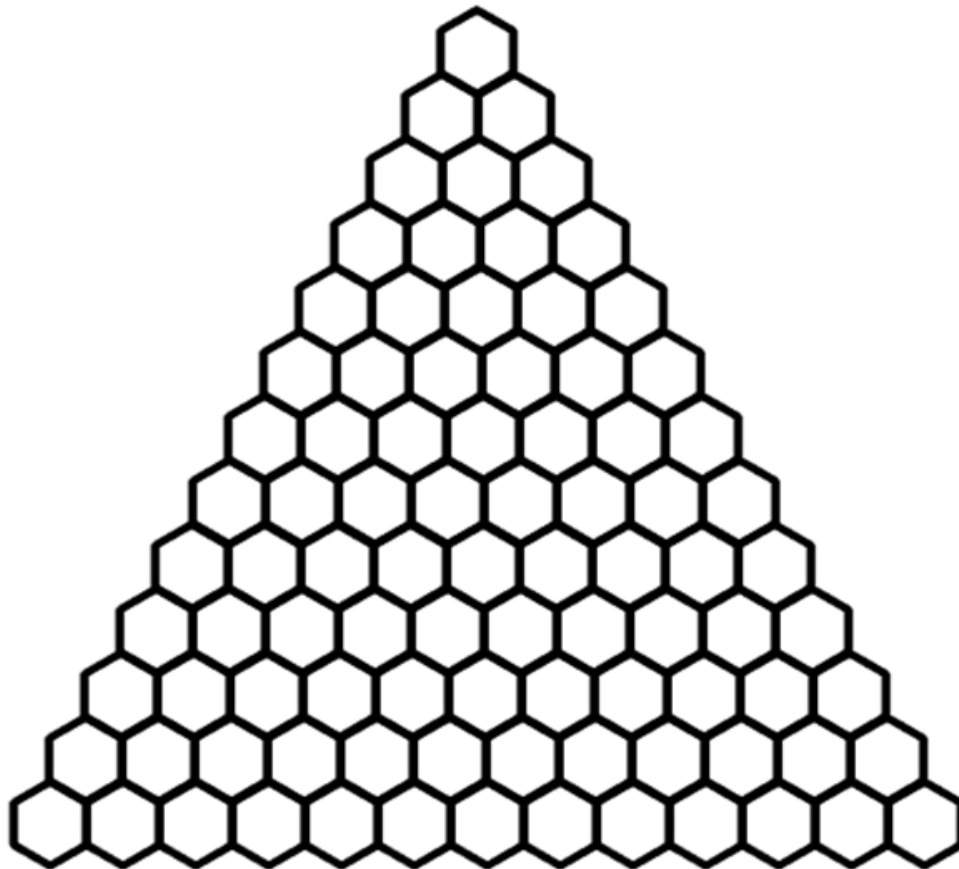
<u>ADDITIONAL PROBLEMS</u> - Use Pascal's Triangle and the Binomial Expansion to expand the following binomials.

1. $(x + 1)^4 =$

2. $(x + y)^6 =$

3. $(x + 2)^3 =$

4. $(b - 1)^5 =$

5. $(2x + y)^3 =$

6. $(x - y)^7 =$

7. $(x - 4)^3 =$

8. $(2x + 3y)^4 =$

<u>HELPFUL LINKS</u> -

http://www.youtube.com/watch?v=OMr9ZF1jgNc

http://www.youtube.com/watch?v=1pSD8cYyqUo&NR=1&feature=fvwp

http://en.wikipedia.org/wiki/Pascal%27s_triangle

PHYSICAL Assignment 7 – Position, Velocity, and Acceleration

NOTES –The topic of one dimensional kinematics – the study of position, distance, displacement, speed, velocity, and acceleration - serves as an excellent study of how math is relevant in the real world.   As such, this topic is very heavily emphasized on AP Calculus exams (usually 4-5 questions per each exam). Also note that this topic is typically the first subject covered in an introductory physics course.

In this assignment, you will navigate an online tutorial to understand the basics of kinematics. You are expected to understand this online tutorial on your own, pulling in additional resources (collaboration with friends, email to me, other online sites, library books) as needed.

The tutorial is "Lesson 1" from "the Physics Classroom" at http://www.physicsclassroom.com/Class/1DKin/. Read all five sections of Lesson 1, watching the various animations and testing yourself on the questions at the end of each section. **Write down key concepts in a notebook as a means of retaining the information.** Also, keep a list of items that you are confused about so that we can focus on these in class.

Once you are done reading all five sections of Lesson 1, go to http://www.physicsclassroom.com/reviews/#1Dkin and click on "Printable Version" to create a hardcopy of 50 questions.   Complete problems 8 – 42 to be handed in with your packet. Optional – you may wish to attempt problems 1-7 (true/false) and 43-50 (computation) to further assess your understanding of this material. While the answers to all 50 questions are available online, you should only use these answers to check your work. Highlight any questions that you do not understand so that we can discuss these in class.

Please note that this assignment is significantly longer than the other assignments. You should expect to spend at least 4-5 hours on this assignment, perhaps closer to twice that time. Proceed slowly, focussing on understanding, and do it right !

Note that I have included the full set of the 50 questions at the end of this packet. You may choose to use this set rather than the printable set from the internet.

PHYSICAL Assignment 8 – Rational Functions and Asymptotes

<u>NOTES</u> - Recall that rational functions are of the form $R(x) = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are themselves polynomial functions. Rational functions arise often in calculus when investigating limits (which you will study in assignment 10). Some key features of rational functions include x and y intercepts, holes, and vertical and horizontal asymptotes. You are expected to know how to identify all of these features. A brief summary is below, followed by excellent websites.

      x-intercept(s) - occur when $y = 0$ y-intercept - occurs when $x = 0$

      holes - occur when both $P(x) = 0$ (numerator) AND $Q(x) = 0$ (denominator).

      vertical asymptotes - occur when $Q(x) = 0$, but $P(x) \neq 0$

      horizontal asymptotes - compare degree of $P(x)$ and $Q(x)$

<u>ONLINE RESOURCES</u> - providing more detail and examples:

      http://people.richland.edu/james/lecture/m116/polynomials/rational.html

      http://www.youtube.com/watch?v=c-yK2hUnSB0

<u>PROBLEMS</u> - For each of the following, identify 1) x-intercepts, 2) y-intercept, 3) holes, 4) VAs, and 5) HAs. Answers to an example problem are provided.

Example: $y = \frac{x^2-3x+2}{x^2-1}$ which factors to $y = \frac{(x-1)(x-2)}{(x-1)(x+1)}$

      hole @ $x = 1$ when $x = 1$, both num and den equal 0

      VA @ $x = -1$ when $x = -1$, den equals zero but num does not

      HA @ $y = 1$ degree of both num and den is 2, so HA occurs when y equals ratio of leading
                                          coefficients (which is simply 1/1)

      x-int @ $x = 2$ set $y = 0$, true when num equals 0, but den does not

      y-int @ $y = -2$ set $x = 0$

1. $y = \frac{x^2-3x+2}{x^2-1}$

2. $y = \frac{x^2+4x+3}{x^2+2x+1}$

3. $y = \frac{x^2-7x-8}{x-8}$

4. $y = \frac{3x+9}{x^2-9}$

5. $y = \frac{x^2+3x+2}{3x^2-12}$

PHYSICAL Assignment 9 – The Shape of Functions

<u>NOTES</u> - At this point in your mathematical careers, you should recognize many types of functions, including linear, polynomial, square root, rational, logarithmic, exponential, and trigonometric (sine, cosine, and tangent). In the context of calculus, we will study all these types of function. In this exercise, you will need graph paper (found at http://mathbits.com/mathbits/studentresources/graphpaper/graphpaper.htm or elsewhere) to sketch the different functions.

For each problem, you should first sketch the "root" function. Then you should complete any required translations or transformations to sketch the "final" function. If you are confused about any of these functions, I recommend making a simple table of x and y coordinates prior to making a sketch. As always, use online resources in necessary.

<u>EXAMPLE</u> - Sketch $y = 2^{(x-3)} + 5$. In this case, the "root" function is $y = 2^x$, which is an exponential function with a base of 2. The "final" function is found by translating the "root" function <u>3 units to the right</u> and <u>5 units up</u>. A table of values for the "root" function might look like:

x	-3	-2	-1	0	1	2	3
y	1/8	1/4	1/2	1	2	4	8

On graph paper, you would sketch these coordinates for the "root" function, then perform the indicated translations to sketch the "final" function (perhaps using two different colored pencils for clarity).

<u>PROBLEMS</u> - Sketch the following functions on graph paper.

1. $y = 3x - 2$

2. $y = x^2 - 9$

3. $y = \sqrt{x - 5} + 1$

4. $y = \log_3(x - 2)$

5. $y = 2^x - 4$

6. $y = \cos(x - \pi) - 1$

7. $y = -\log_2 x + 3$

8. $y = e^{-x}$

9. $y = 2\sin\left(x + \frac{\pi}{2}\right) + 3$

PHYSICAL Assignment 10 – Basic Limits (and some continuity)

NOTES - Limits are the first topic addressed in typical calculus courses, as limits are used in the definition of the two key calculus concepts - the derivative and the integral. Most of you spent 5-6 week at the end of this last school year learning about limits. If you did not, you must go online (youtube "calculus limits patrickjmt" as well as other resources) to research this topic.
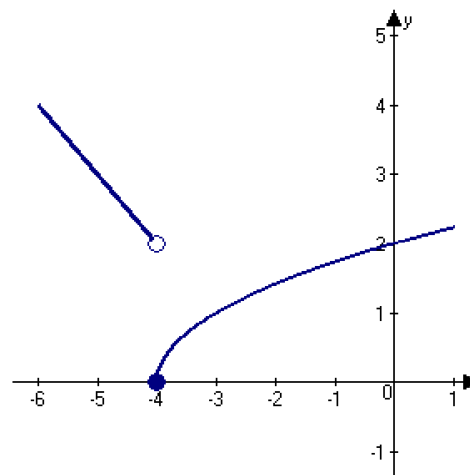
PROBLEMS - Complete the following.   If a limit does not exist, write DNE.

1) Use the graph at the right to determine the following limits.
   Then discuss the continuity of the function at $x = -4$.

   (i) $\lim\limits_{x \to -4^+} f(x)$

   (ii) $\lim\limits_{x \to -4^-} f(x)$

   (iii) $\lim\limits_{x \to -4} f(x)$

2) Find $\lim\limits_{x \to 5} \cos\left(\frac{x\pi}{6}\right)$

3) Find $\lim\limits_{x \to 11} \frac{x-11}{x^2-121}$

4) Find $\lim\limits_{x \to 4} \frac{\sqrt{x+96}}{x-2}$

5) Find $\lim\limits_{x \to \infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)}$

6) Let $f(x) = \begin{cases} x^2 + 7 & x \neq 3 \\ 1 & x = 3 \end{cases}$ Determine $\lim\limits_{x \to 3} f(x)$

7) Find $\lim\limits_{x \to 4} \frac{x}{x^2+4}$

8) Find $\lim\limits_{x \to \infty} \frac{x^3-2x^2+3x-5}{3x^3+x^2-2x+1}$

9) Find $\lim\limits_{x \to \infty} \frac{20x^2-13x+4}{5-4x^3}$

10) Find $\lim\limits_{x \to \pi} \tan\left(\frac{2x}{3}\right)$

# Questions for Physical Assignment 7 (1-D Kinematics)

Part A: Multiple TRUE/FALSE:

1. Which of the following statements about vectors and scalars are TRUE? List all that apply.

    a.   A vector is a large quantity and a scalar is a small quantity.

    b.   A scalar quantity has a magnitude and a vector quantity does not.

    c.   A vector quantity is described with a direction and a scalar is not.

    d.   Scalar quantities are path dependent quantities and vector quantities are not.

    e.   A scalar quantity depends only upon the initial and final values of the quantity; this is not the case for vector quantities.

    f.   The quantity 20 m/s, north is a speed and as such is a scalar quantity.

    g.   The quantity 9.8 m/s/s is an acceleration value and as such is a vector quantity.

2. Which of the following statements about distance and/or displacement are TRUE? List all that apply.

    a.   Distance is a vector quantity and displacement is a scalar quantity.

    b.   A person makes a round-trip journey, finishing where she started. The displacement for the trip is 0 and the distance is some nonzero value.

    c.   A person starts at position A and finishes at position B. The distance for the trip is the length of the segment measured from A to B.

    d.   If a person walks in a straight line and never changes direction, then the distance and the displacement will have exactly the same magnitude.

    e.   The phrase "20 mi, northwest" likely describes the distance for a motion.

    f.   The phrase "20 m, west" likely describes the displacement for a motion.

    g.   The diagram below depicts the path of a person walking to and fro from position A to B to C to D. The distance for this motion is 100 yds.

    h.   For the same diagram below, the displacement is 50 yds.
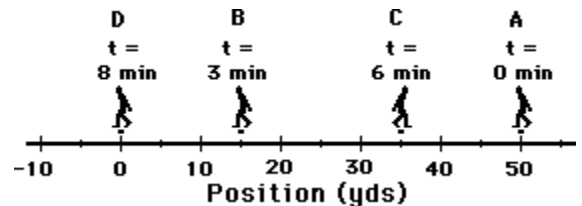
3. Which of the following statements about velocity and/or speed are TRUE? List all that apply.

    a.   Velocity is a vector quantity and speed is a scalar quantity.

    b.   Both speed and velocity refer to how fast an object is moving.

    c.   Person X moves from location A to location B in 5 seconds. Person Y moves between the same two locations in 10 seconds. Person Y is moving with twice the speed as person X.

    d.   The velocity of an object refers to the rate at which the object's position changes.

    e.   For any given motion, it is possible that an object could move very fast yet have an abnormally small velocity.

    f.   The phrase "30 mi/hr, west" likely refers to a scalar quantity.

    g.   The average velocity of an object on a round-trip journey would be 0.

    h.   The direction of the velocity vector is dependent upon two factors: the direction the object is moving and whether the object is speeding up or slowing down.

    i.   The diagram below depicts the path of a person walking to and fro from position A to B to C to D. The entire motion takes 8 minutes. The average speed for this motion is approximately 11.3 yds/min.

    j.   For the same diagram below, the average velocity for this motion is 0 yds/min.
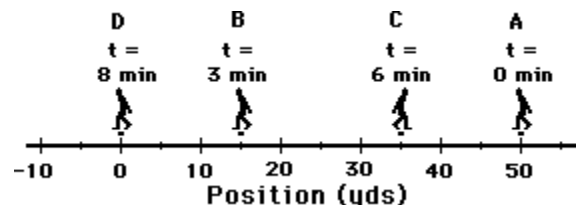
4. Which of the following statements about acceleration are TRUE? List all that apply.

   a.   Acceleration is a vector quantity.
   b.   Accelerating objects MUST be changing their speed.
   c.   Accelerating objects MUST be changing their velocity.
   d.   Acceleration units include the following; m/s^2, mi/hr/sec, cm/s^2, km/hr/m.
   e.   The direction of the acceleration vector is dependent upon two factors: the direction the object is moving and whether the object is speeding up or slowing down.
   f.   An object which is slowing down has an acceleration.
   g.   An object which is moving at constant speed in a circle has an acceleration.
   h.   Acceleration is the rate at which the velocity changes.
   i.   An object that is accelerating is moving fast.
   j.   An object that is accelerating will eventually (if given enough time) be moving fast.
   k.   An object that is moving rightward has a rightward acceleration.
   l.   An object that is moving rightward and speeding up has a rightward acceleration.
   m.   An object that is moving upwards and slowing down has an upwards acceleration.

5. Which of the following statements about position-time graphs are TRUE? List all that apply.

   a.   Position-time graphs cannot be used to represent the motion of objects with accelerated motion.
   b.   The slope on a position-time graph is representative of the acceleration of the object.
   c.   A straight, diagonal line on a position-time graph is representative of an object with a constant velocity.
   d.   If an object is at rest, then the position-time graph will be a horizontal line located on the time-axis.
   e.   Accelerated objects are represented on position-time graphs by curved lines.
   f.   An object with a positive velocity will be represented on a position-time graph by a line with a positive slope.
   g.   An object with a negative velocity will be represented on a position-time graph by a line with a negative slope.
   h.   An object with a positive acceleration will be represented on a position-time graph by a line which curves upwards.
   i.   An object with a negative acceleration will be represented on a position-time graph by a line which curves downwards.

6. Which of the following statements about velocity-time graphs are TRUE? List all that apply.

   a.   The slope on a velocity-time graph is representative of the acceleration of the object.
   b.   The area on a velocity -time graph is representative of the change in position of the object.
   c.   An accelerated object's motion will be represented by a curved line on a velocity-time graph.
   d.   Objects with positive acceleration will be represented by upwardly-curved lines on a velocity-time graph.
   e.   If an object is at rest, then the velocity-time graph will be a line with zero slope.
   f.   A line with zero slope on a velocity-time graph will be representative of an object which is at rest.
   g.   A line with a negative slope on a velocity-time graph is representative of an object with negative velocity.
   h.   If an object changes its direction, then the line on the velocity-time graph will have a changing slope.
   i.   An object which is slowing down is represented by a line on a velocity-time graph which is moving in the downward direction.

7. Which of the following statements about free fall and the acceleration of gravity are TRUE? List all that apply.

   a.   An object that is free-falling is acted upon by the force of gravity alone.
   b.   A falling skydiver which has reached terminal velocity is considered to be in a *state of* free fall.
   c.   A ball is thrown upwards and is rising towards its peak. As it rises upwards, it is NOT considered to be in a*state of* free fall.
   d.   An object in free fall experiences an acceleration which is independent of the mass of the object.
   e.   A ball is thrown upwards, rises to its peak and eventually falls back to the original height. As the ball rises, its acceleration is upwards; as it falls, its acceleration is downwards.
   f.   A ball is thrown upwards, rises to its peak and eventually falls back to the original height. The speed at which it is launched equals the speed at which it *lands*. (Assume negligible air resistance.)
   g.   A very massive object will free fall at the same rate of acceleration as a less massive object.
   h.   The value of **g** on Earth is approximately 9.8 m/s^2.
   i.   The symbol **g** stands for the force of gravity.

Part B: Multiple Choice:

8. If an object has an acceleration of 0 m/s^2, then one can be sure that the object is not _____.

a. moving   b. changing position   c. changing velocity

9. If car A passes car B, then car A must be _____.

   a.   accelerating.
   b.   accelerating at a greater rate than car B.
   c.   moving faster than car B and accelerating more than car B.
   d.   moving faster than car B, but not necessarily accelerating.

10. Which one of the following is NOT consistent with a car which is accelerating?

   a.   A car is moving with an increasing speed.
   b.   A car is moving with a decreasing speed.
   c.   A car is moving with a high speed.
   d.   A car is changing direction.

11. A fullback is running down the football field in a straight line. He starts at the 0-yard line at 0 seconds. At 1 second, he is on the 10-yard line; at 2 seconds, he is on the 20-yard line; at 3 seconds, he is on the 30-yard line; and at 4 seconds, he is on the 40-yard line. This is evidence that

   a.   he is accelerating
   b.   he is covering a greater distance in each consecutive second.
   c.   he is moving with a constant speed (on average).

12. A fullback is running down the football field in a straight line. He starts at the 0-yard line at 0 seconds. At 1 second, he is on the 10-yard line; at 2 seconds, he is on the 20-yard line; at 3 seconds, he is on the 30-yard line; and at 4 seconds, he is on the 40-yard line. What is the player's acceleration?

13. Olympic gold medalist Michael Johnson runs one time around the track - 400 meters - in 38 seconds. What is his displacement? _____ What is his average velocity? _____

14. If an object is moving eastward and slowing down, then the direction of its velocity vector is _____.

a. eastward   b. westward   c. neither   d. not enough info to tell

15. If an object is moving eastward and slowing down, then the direction of its acceleration vector is _____.

a. eastward   b. westward   c. neither   d. not enough info to tell

16. Which one of the following quantities is NOT a vector?

a. 10 mi/hr, east   b. 10 mi/hr/sec, west   c. 35 m/s, north   d. 20 m/s

17. Which one of the following quantities is NOT a speed?

a. 10 mi/hr   b. 10 mi/hr/sec   c. 35 m/s   d. 20 m/s

18. Which one of the following statements is NOT true of a free-falling object? An object in a state of free fall _____.

    a.   falls with a constant speed of -10 m/s.
    b.   falls with a acceleration of -10 m/s/s.
    c.   falls under the sole influence of gravity.
    d.   falls with downward acceleration which has a constant magnitude.

19. The average speed of an object which moves 10 kilometers (km) in 30 minutes is _____.

a. 10 km/hr   b. 20 km/hr   c. 30 km/hr   d. more than 30 km/hr

20. What is the acceleration of a car that maintains a constant velocity of 55 mi/hr for 10.0 seconds?

a. 0   b. 5.5 mi /hr/s   c. 5.5 mi /s/s   d. 550 mi/hr/s

21. As an object freely falls, its _____.

a. speed increases   b. acceleration increases

c. both of these   d. none of these

22. A speedometer is placed upon a free-falling object in order to measure its instantaneous speed during the course of its fall. Its speed reading (neglecting air resistance) would increase each second by _____.

a. about 5 m/s   b. about 10 m/s   c. about 15 m/s

d. a variable amount   e. depends on its initial speed.

23. Ten seconds after being dropped from rest, a free-falling object will be moving with a speed of _____.

a. about 10 m/s.   b. about 50 m/s.   c. about 100 m/s.   d. more than 100 m/s.

24. A baseball pitcher delivers a fast ball. During the throw, the speed of the ball increases from 0 to 30.0 m/s over a time of 0.100 seconds. The average acceleration of the baseball is _____ m/s^2.

a. 3.00   b. 30.0   c. 300.   d. 3000   e. none of these

25. On takeoff, a rocket accelerates from rest at a rate of 50.0 m/s^2 for exactly 1 minute. The rocket's speed after this minute of steady acceleration will be _____ m/s.

a. 50.0   b. 500.   c. 3.00 x 10^3 d. 3.60 x 10^3 e. none of these

26. When a rock is dropped, it will accelerate downward at a rate of 9.8 m/s^2. If the same rock is thrown downward (instead of being dropped from rest), it acceleration will be _____. (Ignore air resistance effects.)

a. less than 9.8 m/s^2 b. 9.8 m/s^2 c. more than 9.8 m/s^2

27. Consider drops of water that leak from a dripping faucet at a constant rate. As the drops fall they _____.

a. get closer together   b. get farther apart

c. remain at a relatively fixed distance from one another

28. Renatta Oyle is again found driving her '86 Yugo down Lake Avenue, leaving the following trail of oil drops on the pavement.

If her car is moving from right to left, then ...

    a.   her velocity has a rightward direction and her acceleration has a rightward direction.
    b.   her velocity has a rightward direction and her acceleration has a leftward direction.
    c.   her velocity has a leftward direction and her acceleration has a rightward direction.
    d.   her velocity has a leftward direction and her acceleration has a leftward direction.

Part C: Diagramming:

29. On the diagrams below, construct a dot diagram representing the motion of an object with a ... .

   a.   constant rightward velocity
   b.   rightward velocity and a rightward acceleration
   c.   rightward velocity and a leftward acceleration
   d.   rightward velocity, first slow and constant, and then accelerating to a high speed
   e.   rightward velocity, first decelerating from a high speed to a rest position, then maintaining the rest position, and finally accelerating at a lower rate than the initial deceleration.

30. On a dot diagram, how does the motion of an object moving to the right and slowing down differ from an object moving to the left and speeding up? Explain.

Part D: Kinematic Graphing:

31. On the position-time graph below, sketch a plot representing the motion of an object which is ... . Label each line with the corresponding letter (e.g., "a", "b", "c", etc.)

    a.   at rest.
    b.   moving in the positive direction with constant speed
    c.   moving in the negative direction and speeding up
    d.   moving in the positive direction and slowing down
    e.   moving in the positive direction at a constant speed (slow) and then later fast at constant speed
    f.   moving with a negative velocity and a negative acceleration
    g.   moving with a negative velocity and a positive acceleration
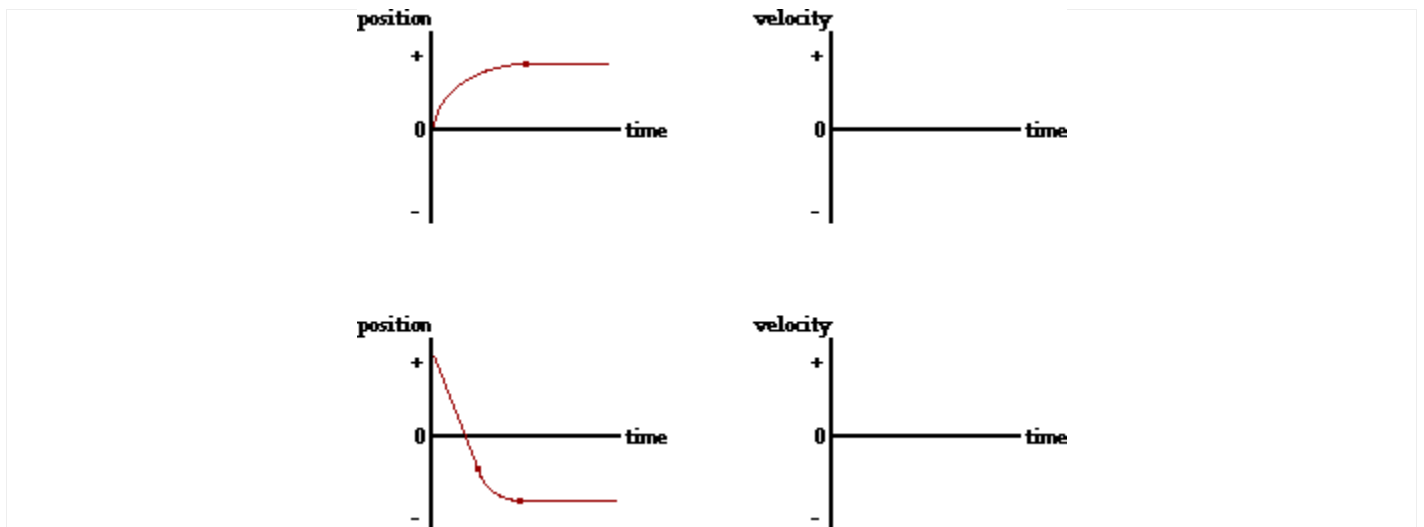
32. On the velocity-time graph below, sketch a plot representing the motion of an object which is ... . Label each line with the corresponding letter (e.g., "a", "b", "c", etc.)

    a.   at rest
    b.   moving in the positive direction at constant speed
    c.   moving in the negative direction from slow to fast
    d.   moving in the negative direction from fast to slow
    e.   moving with a positive velocity and a positive acceleration
    f.   moving with a positive velocity and a negative acceleration
    g.   moving with a positive velocity at constant speed and then decelerating to a rest position
    h.   moving in the positive direction while slowing down, changing directions and moving in the negative directions while speeding up
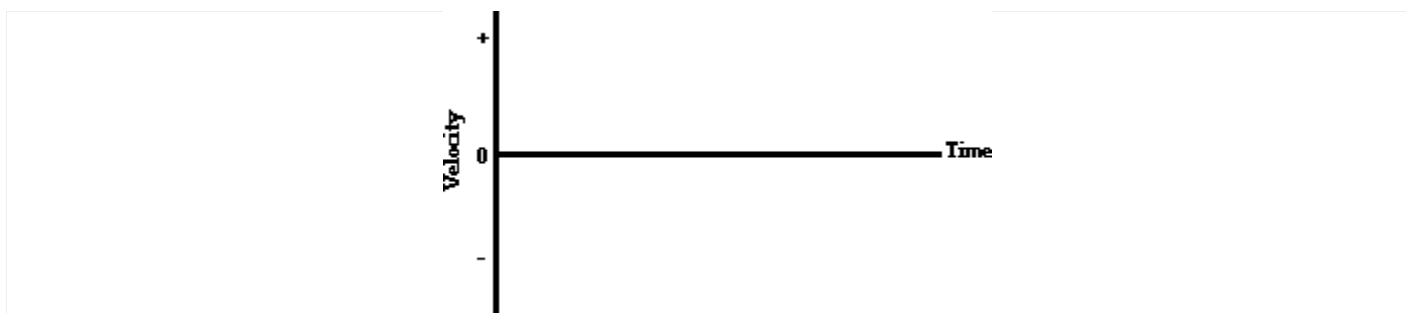
33. Consider the position-time plots below. Sketch the shape of the corresponding velocity-time graphs.

position	velocity
position	velocity

34. Consider the velocity-time plots below. Sketch the shape of the corresponding position-time graphs.

velocity	position
velocity	position
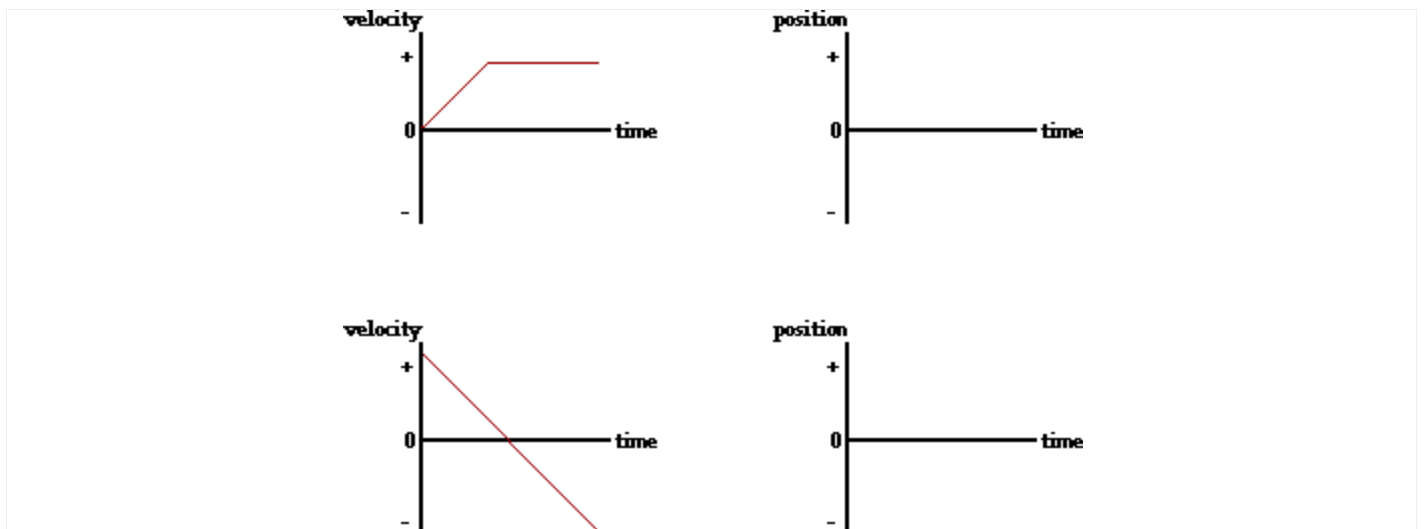
The velocity-time graph below depicts the motion of an automobile as it moves through Glenview during rush hour traffic. Use the graph to answer questions #35 - #39.
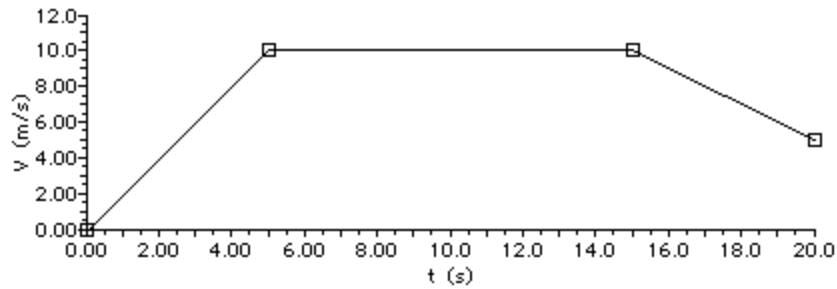
35. Determine the displacement of the automobile during the following intervals of time. **PSYW**

t = 0.0 s - 5.0 s	t = 5.0 s - 15.0 s	t = 15.0 s - 20.0 s

36. Determine the velocity of the automobile at the following instant(s) in time.

t = 3.0 s	t = 8.0 s	t = 17.0 s

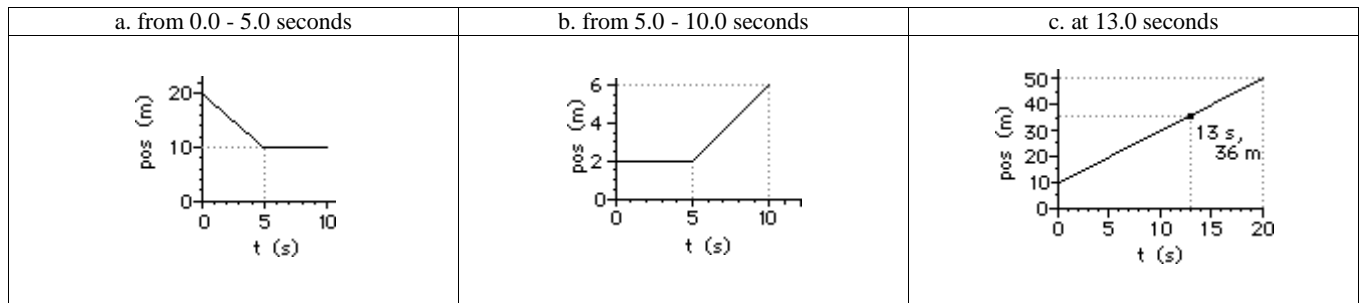37. Determine the acceleration of the automobile during the following intervals of time.

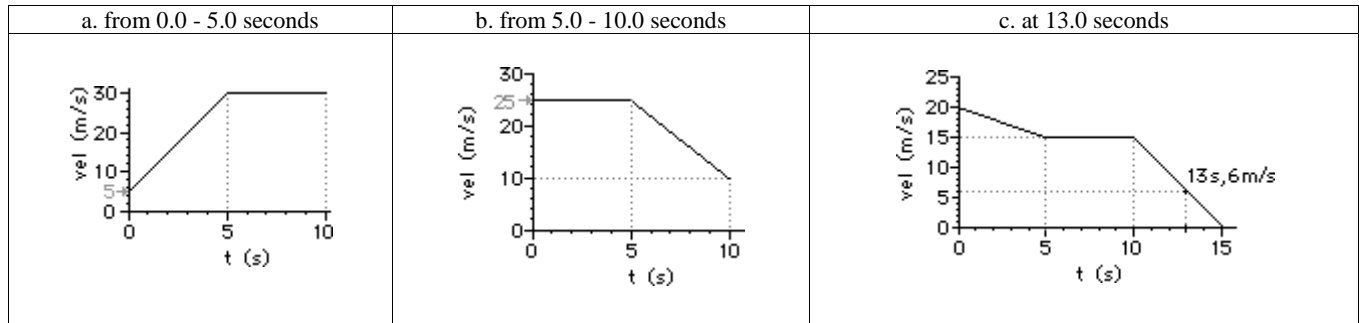| t = 0.0 s - 5.0 s	t = 5.0 s - 15.0 s	t = 15.0 s - 20.0 s

38. Using complete sentences and the language of physics, describe the motion of the automobile during the entire 20.0 seconds. Explicitly describe any changes in speed or direction which might occur; identify intervals of time for which the automobile is at rest, the automobile is moving with constant speed, or the automobile is accelerating.

39. Supposing the automobile has an oil leak, demonstrate your understanding of its motion by drawing an oil drop diagram for the 20.0 seconds of motion. Divide the diagram into three distinct time intervals (0.0 - 5.0 seconds, 5.0 - 15.0 seconds, 15.0 - 20.0 seconds).
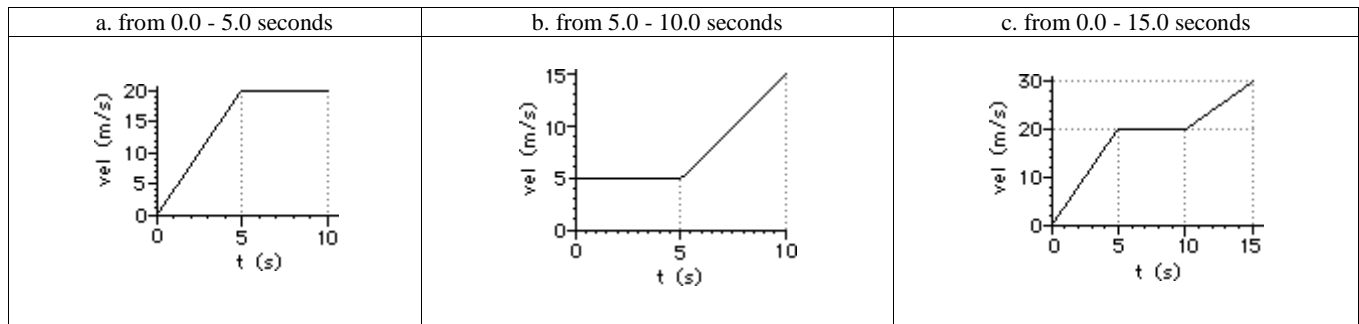
40. For the plots below, determine the velocity of the object ... .

a. from 0.0 - 5.0 seconds	b. from 5.0 - 10.0 seconds	c. at 13.0 seconds

41. For the plots below, determine the acceleration of the object ... .

a. from 0.0 - 5.0 seconds	b. from 5.0 - 10.0 seconds	c. at 13.0 seconds

42. For the plots below, determine the displacement of the object ... .

a. from 0.0 - 5.0 seconds	b. from 5.0 - 10.0 seconds	c. from 0.0 - 15.0 seconds

Part E: Computational Problems:

43. Determine the acceleration (in m/s^2) of an object which ... .

a. moves in a straight line with a constant speed of 20.0 m/s for 12.0 seconds
b. changes its velocity from 12.1 m/s to 23.5 m/s in 7.81 seconds
c. changes its velocity from 0.0 mi/hr to 60.0 mi/hr in 4.20 seconds
d. accelerates from 33.4 m/s to 18.9 m/s over a distance of 109 m

44. Determine the magnitude of the displacement (in meters) of an object which ... .

a. moves from Hither to Yon (with an average speed of 28.0 m/s) and then back to Hither (with an average speed of 28.0 m/s) if both the forward and the return trip take 46 minutes each.
b. moves at a constant speed of 8.30 m/s in a straight line for 15.0 seconds.
c. decelerates at a rate of -4.35 m/s/s from a speed of 38.1 m/s to a speed of 17.6 m/s
d. accelerates from rest at a rate of 3.67 m/s^2 for 12.1 seconds
e. is moving at 12.2 m/s and then accelerates at a rate of +1.88 m/s^2 for 17.0 seconds

45. The hare is sleeping at a location that is 1200 m from the finish line. The tortoise passes him at a steady speed of 5.0 cm/s. If the hare finally wakes up 6.5 hours later, then what minimum acceleration (assumed constant) must he have in order to pass the tortoise before the finish line.

46. A **Gold Car** moving at 12.0 m/s passes a **Green Car** while the **Green Car** is at rest at a stoplight. The **Green Car** immediately accelerates at a rate of +1.80 m/s/s for 11.0 seconds seconds and then maintains a constant speed. After how much time (relative to the initial starting time) must the **Green Car** drive before catching up with the **Gold Car**.

47. Ima Rilla Saari is cruising at 28.0 m/s down Lake Avenue and through the forest preserve. She notices a deer jump into the road at a location 62.0 m in front of her. Ima first reacts to the event, then slams on her brakes and decelerates at -8.10 m/s^2, and ultimately stops a picometer in front of the *frozen* deer. What is Ima's **reaction time**? (i.e., how long did it take Ima to react to the event prior to decelerating?)

48. A two-stage rocket accelerates from rest at +3.57 m/s/s for 6.82 seconds. It then accelerates at +2.98 m/s/s for another 5.90 seconds. After the second stage, it enters into a state of free fall. Determine:

     a.   the maximum speed
     b.   the maximum altitude
     c.   the height of the rocket after 20.0 seconds
     d.   the total time the rocket is in the air (assuming it is launched from the ground)

49. In a 200.0-m relay race (each leg of the race is 50.0 m long), one swimmer has a 0.450 second lead and is swimming at a constant speed of 3.90 m/s towards the opposite end of the pool. What minimum speed must the second swimmer have in order to catch up with the first swimmer by the end of the pool?

50. A drag racer accelerates from rest at an average rate of +13.2 m/s^2 for a distance of 100. m. The driver coasts for 0.500 seconds and then uses the brakes and parachute to decelerate until the end of the track. If the total length of the track is 180. m, what minimum deceleration rate must the racer have in order to stop prior to the end of the track?