2017_18 AP Calculus BC – Summer Packet

Welcome to AP Calculus BC at Boca Ciega High School. I'm Mr Lynch - all of you know me already. You should feel free to reach me at lynchr@pcsb.org. I will check my email most days.

Whether or not you passed the AP Calculus AB exam in May, you can be successful in AP Calculus BC.

We will spend a fair amount of time reviewing the math we learned this last year in AB. The following pages include a review of most of the key concepts we learned. You should spend 1-2 hours on each of these 5 pages. For the FRQs on the last two pages, you can obviously find the answers posted online. However, you should seriously attempt these problems PRIOR to checking your answers.

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Focus 1: Log and Exponential Functions

Brief Notes: If $y = b^x$ then $x = \log_b y$

rewrite as logs to solve exp equations

rewrite as exp to solve log equations

1) Rewrite $y = \ln(2x)$ as an exponential equation
2) Rewrite $P = e^t$ as a logarithmic equation
3) Rewrite $Q = 2^m$ as a logarithmic equation
4) Without a calculator, solve for the unknown: $3 \cdot 2^x + 4 = 28$
5) Use a calculator to solve for the unknown: $5^r - 3 = 27$
6) Without a calculator, solve for the unknown: $5 + \log_2(x - 3) = 21$
7) Use a calculator to solve for the unknown: $3 + \ln(4x) = 16$
8) Rewrite using properties of logarithms: $\ln\left(x^2 \cdot \sqrt[3]{y}\right)$
9) Rewrite using properties of logarithms: $\log\left(\frac{3x^5}{\sin(x)}\right)$
10) Rewrite using properties of logarithms: $\ln\left(\frac{x^5\tan(x)}{\sqrt{x^2+1}}\right)$
11) Rewrite using properties of logarithms: $\ln(5e^x/x^2)$
12) Rewrite using properties of logarithms: $\ln\left(\sqrt[3]{e^x}/5x^7\right)$
13) Condense using properties of logarithms: $\ln(x^2) - \ln(\sqrt[3]{x+1})$
14) Condense using properties of logarithms: $\ln(x) - \ln(y) - \ln(z)$
15) Condense using properties of logarithms: $\ln(x) - \ln(y) + \ln(z)$
16) Condense using properties of logarithms: $3\ln(x) - 0.25\ln(x + 1)$
17) Find the derivative $f'(x)$: $f(x) = ln\left(\frac{3x^2}{\cos(x)}\right)$
18) Find the derivative $f'(x)$: $f(x) = ln\left(\frac{\tan(x)}{\sqrt{x}}\right)$
19) Find the derivative $f'(x)$: $f(x) = \log_2(5x^3 + 2x)$

Focus 2: Derivatives

Some rules: constant, power, product, quotient

trig fns, e^x, ln(x), implicit

1) Find $\frac{dy}{dx}$ given $y = 3x^2 + \frac{2}{x} - \cos(x)$

2) Find y' given $y = e^{(x^2+3x)}$

3) Find $\frac{dh}{dx}$ given $h = \frac{x^2-3x}{\sqrt[3]{x}}$

4) Given functions u and v, with $u(2) = 4$, $u'(2) = -1$, $v(2) = -2$, and $v'(2) = 5$, find at $x = 2$:

   a. $\frac{d}{dx}(uv)$

   b. $\frac{d}{dx}\left(\frac{u}{v}\right)$

   c. $\frac{d}{dx}\left(\frac{v}{2u}\right)$

   d. $\frac{d}{dx}(3u - 2uv)$

5) Given $h = xe^{2x}$, find $h''(x)$

6) Given $3xy - y^2 - cosx = 2y$, find $\frac{dy}{dx}$

7) Find the slope of the "Witch of Agnesi", given by $y = \frac{8}{4+x^2}$ at the point (2,1)

8) Given $y = \sqrt{\sin(x^3 - 2)}$, find $y'(x)$

9) Given $\ln(xy^2) + x^4y + y = 2$, find the slope at point (1,1)

10) Given functions u and v and values shown in the table, determine the following quantities :

t	u	v	u'	v'
2	-1	5	-4	-3
5	3	-6	7	11
8	6	3	4	2

   a. Average RoC of u on (2,8)

   b. $\frac{d}{dt}(u(v(2)))$

   c. Instantaneous RoC of v at $t = 5$

   d. $\frac{d}{dt}(u(2) \cdot u(v(2)))$

11) Find any horizontal tangents to the curve $y = x^3 - 9x^2 - 21x + 5$

12) Determine the point at which the function $y = \sqrt{x}$ is perpendicular to the line $2y + 8x = 11$

13) Find $\frac{dm}{dx}$ given $m = \frac{e^x}{x^2+1}$

14) Given $f(x) = \tan(x)$, find $f''(x)$

Focus 3: Integrals

Brief Notes: Do you need u-substitution ? (probably)

PATTERN RECOGNITION

1) Find $\int (3\sin(x) + 4x)\ dx$

2) Without a calculator, determine $\int_0^3 e^{2t} dt$

3) Find $\int \frac{4x+6}{x^2+3x}\ dx$

4) Without a calculator, determine $\int_0^{\pi/4} \tan(t)\sec(t)\ dt$

5) Without a calculator, determine $\int_2^5 \left(\frac{x^4-2x+1}{x^2}\right) dx$

6) Given $\int_2^9 f(x)dx = -7$ and $\int_5^9 f(x)dx = 3$, determine $\int_5^2 f(x)dx$

7) Given $f(x) = \int_3^x (e^t - \sqrt{t})\ dt$, find $f'(x)$

8) Without a calculator, determine $\int_0^3 3xe^{(x^2+1)}\ dx$

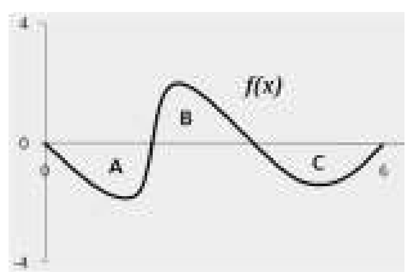
9) Given $g(x) = \int_3^{x^2} (\sqrt[7]{x})\ dx$, find $g''(x)$

10) Determine the average value of $y = \sqrt[3]{x+2}$ on the interval (6,25)

11) Without a calculator, determine $\int_{-4}^8 |x|\ dx$

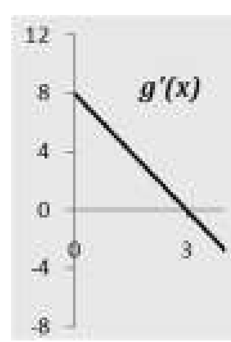
12) Without a calculator, determine $\int_0^1 (e^{-3x}) dx$

13) Determine $\int \frac{cos\left(\frac{1}{x}\right)}{x^2}\ dx$



14) Each labeled region (A, B, and C) in the graph of $f(x)$ (shown to the right) has an area of 3 square units. Determine $\int_0^6 (f(x) + 2)dx$.

15) Use a calculator to determine $\int_2^9 \frac{e^x}{x^2+21} dx$ accurate to 3 decimal places.



16) Use a calculator to determine $\int_{\pi/6}^{3\pi/4} 5\sin\left(\frac{x^4+2}{\sqrt{x+1}}\right) dx$ accurate to 3 decimal places.

17) The graph of $g'(x)$ is shown to the right. If $g(3) = 6$, determine $g(0)$.

Focus 4: Net Accumulation $\int_{t=a}^{t=b} (rate\ in - rate\ out)dt = Final\ Amt - Initial\ Amt$

1) 2009 B - Calculator Required

At a certain height, a tree trunk has a circular cross section. The radius $R(t)$ of that cross section grows at a rate modeled by the function

$$\frac{dR}{dt} = \frac{1}{16}\left(3 + \sin\left(t^2\right)\right) \text{ centimeters per year}$$

for $0 \le t \le 3$, where time t is measured in years. At time $t = 0$, the radius is 6 centimeters. The area of the cross section at time t is denoted by $A(t)$.

(a) Write an expression, involving an integral, for the radius $R(t)$ for $0 \le t \le 3$. Use your expression to find $R(3)$.

(b) Find the rate at which the cross-sectional area $A(t)$ is increasing at time $t = 3$ years. Indicate units of measure.

(c) Evaluate $\int_0^3 A'(t)\,dt$. Using appropriate units, interpret the meaning of that integral in terms of cross-sectional area.

2) 2008 A – Calculator Required

Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

(b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

(c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

3) 2004 A – Calculator Required

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

Focus 5: Volume $$Vol = \int_{x=a}^{x=b} (PerpendicularCrossSectionalArea)\, dx$$

1) 2008 B – Calculator Required

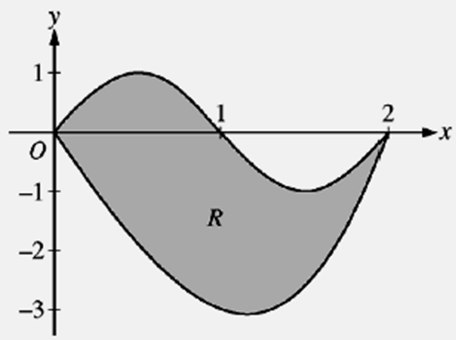
Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

2) 2008 A – Calculator Required



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

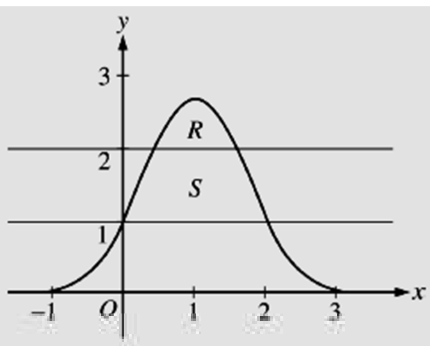
(a) Find the area of R.

(b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

(d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

3) 2007 – B – Calculator Required



Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.