ACCELERATED MATHEMATICS
GRADE 7
1205050 or 1205055 (Cambridge) or 1205100 (IB MYP)


### Course Pacing

<table>
<thead>
<tr>
<th>Unit of Instruction</th>
<th># of Days</th>
<th>Dates of Instruction</th>
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<td>Review Grade 7 Standards from 6th Advanced</td>
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<td>8.27-9.10</td>
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<td>9.11-9.28</td>
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<td>Unit 2: Geometric Figures</td>
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<td>9.29-10.12</td>
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<tr>
<td><strong>Cycle 1 Assessment (Review and Units 1 &amp; 2)</strong></td>
<td>1</td>
<td><strong>10.13</strong> (10.7 – 10.23)</td>
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<td>Unit 3: Circumference, Area, Surface Area, and Volume of Compound Figures</td>
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<td>Unit 4: Probability</td>
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<td>Unit 5: Statistics</td>
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<td>Thanksgiving Break 11/21 – 11/29</td>
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<td><strong>Midterm Exam (Units 1-6)</strong></td>
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<td><strong>12.14 – 12.18</strong></td>
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<tr>
<td>Unit 7: Linear Equations in One Variable</td>
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<td>Semester 1 Ends on 1.15.21</td>
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<td>Unit 8: Linear Equations in Two Variables</td>
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<td>Unit 9: Functions</td>
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<td>Unit 10: Triangles and Pythagorean Theorem</td>
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<td>Unit 11: Transformations, Congruence and Similarity</td>
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<td>Unit 12: Volume</td>
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<td>4.19-4.23</td>
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<td>Unit 13: Scatter Plots and Data Analysis</td>
<td>7</td>
<td>4.26-5.4</td>
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<tr>
<td><strong>FSA Grade 7 Math</strong></td>
<td>2</td>
<td><strong>5/3 – 5/28</strong></td>
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**ACCELERATED MATHEMATICS GRADE 7**

**Pinellas County Schools**

**Unit 1: Multi-Step Equations & Inequalities**
- **Cycle 1 Assessment**
  - Review, Units 1 & 2
  - Window: Oct. 12 - Oct. 23

**Unit 2: Geometric Figures**
- **Unit 3: Circumference, Area, Surface Area, and Volume of Compound Figures**

**Unit 4: Probability**

**Unit 5: Statistics**

**Unit 6: Real Numbers**

**Midterm Review**

**Midterm Exam (Units 1-7)**
- Window: Dec. 14 - Dec. 18

**Unit 7: Linear Equations in One Variable**

*Unit 7 will likely be zeroed out on the Midterm Exam*

**Progression of Units**

- **Unit 8: Linear Equations in Two Variables**
- **Unit 9: Functions**
- **Unit 10: Triangles & Pythagorean Theorem**
- **Unit 11: Transformations, Congruence, & Similarity**
- **Unit 12: Volume**
- **Unit 13: Scatter Plots and Data Analysis**

**Grade 7 FSA**
- Window: May 3-May 28

**Non-Teacher Day**

**Non-Student Day**

**Month** | **Dates** | **Unit** | **Standards** | **Dates**
--- | --- | --- | --- | ---
**August 2020** | **10 11 12 13 14** | **Review Grade 7 Standards from Accelerated Math 6** | **MAFS.7.RP.1.1** | **MAFS.7.EE.1.1**
**17 18 19 20 21** | **MAFS.7.RP.1.3** | **MAFS.7.EE.1.2**
**24 25 26 27 28** | **MAFS.7.NS.1.1** | **MAFS.7.EE.2.3**
**31** | **MAFS.7.NS.1.2** | **MAFS.7.NS.1.3**

**September 2020** | **1 2 3 4** | **Unit 1: Multi-Step Equations & Inequalities** | **MAFS.7.EE.2.3** | **MAFS.7.EE.2.4**
**7 8 9 10 11** | **MAFS.7.G.1.1** | **MAFS.7.G.1.3**
**14 15 16 17 18** | **MAFS.7.G.1.2** | **MAFS.7.G.2.5**
**21 22 23 24 25** | **MAFS.7.G.2.4** | **MAFS.7.G.2.6**
**28 29 30** | **MAFS.7.SP.3.5** | **MAFS.7.SP.3.7**

**October 2020** | **1 2** | **Cycle 1 Assessment** | **Review, Units 1 & 2**
 | **Window: Oct. 12 - Oct. 23**
**5 6 7 8 9** | **MAFS.7.SP.3.5** | **MAFS.7.SP.3.7**
**12 13 14 15 16** | **MAFS.7.SP.3.6** | **MAFS.7.SP.3.8**
**19 20 21 22 23** | **MAFS.7.SP.1.1** | **MAFS.7.SP.2.3**
**26 27 28 29 30** | **MAFS.7.SP.1.2** | **MAFS.7.SP.2.4**

**November 2020** | **2 3 4 5 6** | **Unit 4: Probability**
 | **MAFS.7.SP.3.5** | **MAFS.7.SP.3.7**
**9 10 11 12 13** | **MAFS.7.SP.3.6** | **MAFS.7.SP.3.8**
**16 17 18 19 20** | **MAFS.8.NS.1.1** | **MAFS.8.EE.1.2**
**23 24 25 26 27** | **MAFS.8.NS.1.2** | **MAFS.8.EE.1.3**
**30** | **MAFS.8.EE.1.1** | **MAFS.8.EE.1.4**

**December 2020** | **1 2 3 4** | **Unit 6: Real Numbers**
 | **MAFS.8.NS.1.1** | **MAFS.8.EE.1.2**
**7 8 9 10 11** | **MAFS.8.NS.1.2** | **MAFS.8.EE.1.3**
**14 15 16 17 18** | **MAFS.8.EE.1.1** | **MAFS.8.EE.1.4**

**January 2021** | **1** | **Midterm Review**
**4 5 6 7 8** | **Midterm Exam (Units 1-7)**
 | **Window: Dec. 14 - Dec. 18**
**11 12 13 14 15** | **MAFS.8.EE.3.7**

**February 2021** | **18 19 20 21 22**
**25 26 27 28 29**
**March 2021** | **1 2 3 4 5**
**8 9 10 11 12**
**April 2021** | **15 16 17 18 19**

**May 2021** | **3 4 5 6 7**
**10 11 12 13 14**
**17 18 19 20 21**
**24 25 26 27 28**
**June 2021** | **31**
**July 2021** | **1 2 3 4 5 6 7 8 9 10 11**
<table>
<thead>
<tr>
<th>Standards/Learning Goals</th>
<th>Content Limits, Assessment Types, Calculator</th>
</tr>
</thead>
</table>
| **MAFS.7.RP.1.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, area and other quantities measured in like or different units. | - The item stem must include at least one fraction.  
- Ratios may be expressed as fractions, with “/” or with words.  
- Units may be the same or different across the two quantities.  
**Calculator:** YES  
**Context:** ALLOWABLE |
| **MAFS.7.RP.1.2** Recognize and represent proportional relationships between quantities. | - Ratios should be expressed as fractions, with “/” or with words.  
- Units may be the same or different across the two quantities.  
**Calculator:** NEUTRAL  
**Context:** ALLOWABLE |
| a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.  
| **b.** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.  
| **c.** Represent proportional relationships by equations. For example, if total cost \( t \) is proportional to the number \( n \) of items purchased at a constant price \( p \), the relationship between the total cost and the number of items can be expressed as \( t = pn \).  
| **d.** Explain what a point \((x, y)\) on the graph of a proportional relationship means in terms of the situation, with special attention to the points \((0, 0)\) and \((1, r)\) were \( r \) is the unit rate. |  
| **MAFS.7.RP.1.3** Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. | - Units may be the same or different across the two quantities.  
**Calculator:** YES  
**Context:** ALLOWABLE |
| **MAFS.7.EE.1.1** Apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients. | - Expressions must be linear and contain a variable.  
**Calculator:** NEUTRAL  
**Context:** ALLOWABLE |
| **MAFS.7.EE.1.2** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, \( a + 0.05a = 1.05a \) means that “increase by 5%” is the same as “multiplying by 1.05”. | - Expressions must be linear.  
**Calculator:** NEUTRAL  
**Context:** ALLOWABLE |
| **MAFS.7.EE.2.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making \$25 an hours gets a 10% raise, she will make an additional \( \frac{1}{10} \) of her salary an hour, or \#2.50, for a new salary of \$27.50. If you want to place a towel bar 9 \( \frac{3}{4} \) inches long in the center of a door that is 27 \( \frac{1}{2} \) inches wide, you will need to | - Item should not use variables.  
- Items should require two or more steps.  
**Calculator:** YES  
**Context:** REQUIRED |
**MAFS.7.NS.1.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal and vertical number line diagram.

b. Describe situations in which opposite quantities combine to make 0.

c. Understand \( p+q \) as the number located a distance \(|q|\) from \( p \), in the positive or negative direction depending on whether \( q \) is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

d. Understand subtraction of rational numbers as adding the additive inverse, \( p-q=p+(-q) \). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

e. Apply properties of operations as strategies to add and subtract rational numbers.

**MAFS.7.NS.1.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as \((-1)(-1)=1\) and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If \( p \) and \( q \) are integers, the \(-\frac{p}{q}=\frac{-p}{q}=\frac{p}{(-q)}\). Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

**MAFS.7.NS.1.3** Solve real-world and mathematical problems involving the four operations with rational numbers.

(Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

- Complex fractions may be used, but should contain fractions with single-digit numerators and denominators.

<table>
<thead>
<tr>
<th>Place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.</th>
<th>N/A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculator: NEUTRAL</td>
<td>7.NS.1.2a,b,c require the incorporation of a negative value.</td>
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<tr>
<td>Context: ALLOWABLE</td>
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<tr>
<td>Calculator: NO</td>
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<tr>
<td>Context: ALLOWABLE</td>
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</table>
### Standards/Learning Goals:

<table>
<thead>
<tr>
<th>Standards/Learning Goals</th>
<th>Content Limits, Assessment Types, Calculator</th>
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<tbody>
<tr>
<td>MAFS.7.EE.2.3</td>
<td>- Items should not use variables.</td>
</tr>
<tr>
<td></td>
<td>- Items should require two or more steps.</td>
</tr>
<tr>
<td></td>
<td>- Calculator: YES</td>
</tr>
<tr>
<td></td>
<td>- Context: REQUIRED</td>
</tr>
</tbody>
</table>

### Decoded Standard

MAFS.7.EE.2.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hours gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

### MAFS.7.EE.2.4

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

#### a.

Solve word problems leading to equations of the form $px+q=r$ and $p(x+q)=r$, where $p$, $q$, and $r$ are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is the width?

#### b.

Solve word problems leading to inequalities of the form $px+q>r$ or $px+q<r$, where $p$, $q$, and $r$ are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.

### Open Up Resources Lessons (be selective, you can't use all in the time provided)

- Grade 7, Unit 6: Expressions, Equations, and Inequalities
- Grade 7, Unit 5: Rational Number Arithmetic
- Using benchmark numbers that are easy to work with such as using 2 for $1\frac{7}{8}$ to make an estimate.

### Instructional Resources

#### Mathematics Formative Assessments (MFAS)
- **Reeling in Expressions** Solve a multi-step problem involving rational numbers.
- **Discount and Tax** Solve a multi-step problem involving percent.

#### Illustrative Mathematics Assessment Tasks
- **Anna in D.C.** Solve a multi-step percentage problem that can be approached in many ways.
- **Discounted Books** Determine two different ways to look at percentages both as a decrease and an increase of an original amount and turn a verbal description of several operations into mathematical symbols.
- **Shrinking** Calculating and explaining percent decrease within context.
- **Who is the better batter?** Given a natural real-world context for comparing fractions, convert the fractions to decimals or describe the situation in terms of percents.
- **Gotham City Taxis** Solve a multi-step ratio problem that can be approached in many ways.

#### EngageNY
- **Module 3, Topic B, Lesson 8** Use properties of equality to solve word problems.

#### Lesson Resources
- **Travel Troubles** This activity engages the students into time scheduling, budgeting, and decision making to maximize time efficiency.
- **It’s All About Properties of Equality** Complete a gallery walk as formative assessment, to determine students’ understanding of properties of operations and equality when applied to equations.
- **Steps to Solving Equations** Work collaboratively in pairs or threes, matching equations to stories and then ordering the steps used to solve these equations and explain their reasoning to their peers.
- **Scaffold Lesson to Increase Deeper Understanding in Solving Problems Involving Discount, Tax, and Tip** Understand terminology needed to solve problems involving discount, tax, and tip. SWBAT apply the deeper understanding to solve real-world problems.

#### Better Lesson
- Problem-Solving Investigation: Work Backward

### Decoded Standard

#### MAFS.7.EE.2.4

**A.** Students will become fluent in solving equations. Students use the arithmetic from the problem to generalize an algebraic solution.

Use word problems that lend themselves to equations in the forms of $px + q = r$ and $p(x + q) = r$. Two examples are as follows:

1. Three consecutive even numbers add up to 48. What is the lowest number of the three? $x + x + 2 + x + 4 = 3x + 6 = 48$  ($px + q = r$)
2. Ms. Thomas had $25 to spend on party favors. She had $10.40 left after buying 10 balloons. How much did she spend on each balloon? $0.1(25 - 10.40) = r$  ($p(x + q) = r$)

Students should develop fluency solving word problems that can be modeled by linear equations in the form $px + q = r$. Integers, fractions, and decimals should be included as values in the word problems. ([Common Core Mathematics Companion, Pg. 109](#))

**B.** In this standard, students move from solving word problems with equations to word problems with inequalities. Inequalities follow a similar form to those of the equations, $px + q > r$ and $px + q < r$. Students graph the solution set of the inequality on a number line and describe what it means of the context of the word problem. Be aware that sometimes the solution set to the inequality contains values that do not make sense as solutions for the word problems. For example, in the word problem, “Donna has at most $60 to spend on a shopping spree. She wants to buy a dress for $22 dollars and spend the rest on bracelets. Each Bracelet costs $8. How many bracelets can she purchase?” we see a solution of

$$
\begin{align*}
60 - 22 &= 38 \\
8x &\leq 38 \\
\frac{8x}{8} &\leq \frac{38}{8}
\end{align*}
$$
The number of bracelets is less than or equal to 4.75. However, Donna cannot buy .75 of a bracelet, so when we graph the inequality as below:

see image on page 110 of the Common Core Mathematics Companion

we see that the only viable solutions to the word problems are 4, 3, 2, 1, or no bracelets.

(Common Core Mathematics Companion, Pg. 110)

*Emphasis should be placed on two-step equations. Teachers will need to supplement the text to fully address the standard.

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### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**
- **Solve Equations** Solve two multistep equations involving rational numbers.
- **Squares** Write and solve an equation of the form \( p(x + q) = r \) in the context of a problem about the perimeter of a square.
- **Write and Solve an Equation** Write and solve a two-step equation to model the relationship among variables in a given scenario.
- **Algebra or Arithmetic?** Compare an arithmetic solution to an algebraic solution of a word problem.

**Illustrative Mathematics Assessment Tasks**
- **Fishing Adventures 2** Write and solve inequalities, and represent the solutions graphically.
- **Bookstore Account** Use algebra and the number line to understand why we sometimes represent debt using negative numbers.
- **Gotham City Taxis** Solve a multi-step ratio problem that can be approached in many ways.
- **Sports Equipment Set** An instructional task with context that can naturally be represented with an inequality; explore the relationship between the context and the mathematical representation of that context.

### Lesson Resources

**EngageNY**
- **Module 2, Topic C, Lesson 17** Students use tape diagrams to solve equations of the form \( px + q = r \) and \( p(x + q) = r \) (where \( p, q, \) and \( r \) are small positive integers), and identify the sequence of operations used to find the solution.
- **Module 3, Topic B, Lesson 8** and **Module 3, Topic B, Lesson 9** Students understand and use the addition, subtraction, multiplication, division, and substitution properties of equality to solve word problems leading to equations of the form \( px + q = r \) and \( p(x + q) = r \) where \( p, q, \) and \( r \) are specific rational numbers.
- **Module 3, Topic B, Lesson 13** Students understand that an inequality is a statement that one expression is less than (or equal to) or greater than (or equal to) another expression, such as \( 2x + 5 < 3 \) or \( 3x + 50 \geq 100 \). Students interpret a solution to an inequality as a number that makes the inequality true when substituted for the variable.
- **Module 3, Topic B, Lesson 14** Students solve word problems leading to inequalities that compare \( px + q \) and \( r \), where \( p, q, \) and \( r \) are specific rational numbers. Students interpret the solutions in the context of the problem.
- **Module 3, Topic B, Lesson 15** Students graph solutions to inequalities taking care to interpret the solutions in the context of the problem.

**CPalms**
- **Understanding Equations Using Perimeter** Introductory lesson in writing and solving equations in the form \( p(x + q) = r \) using the perimeter of rectangles.
- **Translating Word Problems into Equations** Shows students how to translate word problems into equations in seven steps.
- **Inequal-tile-ies** Work with Algebra Tiles to solve inequalities.
- **Guess My Number** Represent a sequence of operations using an expression and then to write and solve simple equations. The problem is posed as a game and allows the students to visualize mathematical operations.

**MARS/Shell**
- **Steps to Solving Equations** Work collaboratively in pairs or threes, matching equations to stories and then ordering the steps used to solve these equations and explain their reasoning to their peers.

**Better Lesson**
- **Word Problems with Equations** Students will be able to set up and solve equations for real world problems.
- **Equations with Distributive Property** In the activity, students will be working with a partner, and will be charged with the role of analyzing their partner's work if mistakes exist.
- **Inequalities** Students will be able to solve and graph inequalities with one variable.
- **Inequalities - Negative Rule** Students will be able to solve and graph inequalities with a negative coefficient for $x$.

**McGraw-Hill**

Course 2, Chapter 6

- Lessons 1, 2, and 3: Focus on the problems with signed rational numbers; students were taught one step equations in 6th grade. The only new content in these sections is solving equations with rational numbers (integers, signed fractions and decimals).
- Inquiry Lab: Solve Two-Step Equations
  - Lesson 4
- Inquiry Lab: More Two-Step Equations
  - Lesson 5
- Inquiry Lab: Solve Inequalities
  - Lessons 6, 7, and 8
**Pinellas County Schools**

**ACCELERATED MATH GRADE 7**

**2020-2021**

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<tr>
<th>Semester 1</th>
<th>Unit 2: Geometric Figures</th>
<th>10 days: 9/29 – 10/12</th>
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<tr>
<td><strong>Standards/Learning Goals:</strong></td>
<td><strong>Content Limits, Assessment Types, Calculator</strong></td>
<td></td>
</tr>
<tr>
<td><strong>MAFS.7.G.1.1</strong> Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.</td>
<td>• Geometric figures must be two-dimensional polygons.</td>
<td></td>
</tr>
<tr>
<td><strong>MAFS.7.G.1.1</strong></td>
<td>Calculator: <strong>YES</strong></td>
<td></td>
</tr>
<tr>
<td><strong>MAFS.7.G.1.2</strong> Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, notice when the conditions determine a unique triangle, more than one triangle, or no triangle.</td>
<td>• Given conditions should not focus on similarity or congruence or that the sum of angles in a triangle is 180 degrees.</td>
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<tr>
<td><strong>MAFS.7.G.1.2</strong></td>
<td>• Be aware of the scoring capabilities for the GRID tool when designing these items.</td>
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</tr>
<tr>
<td><strong>MAFS.7.G.1.2</strong></td>
<td>• To distinguish from other grades, conditions should include factors other than parallel/perpendicular lines and angle measure, such as symmetry and side length.</td>
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<tr>
<td><strong>MAFS.7.G.1.3</strong> Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.</td>
<td>• Spheres, cones, and cylinders are allowed.</td>
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<tr>
<td><strong>MAFS.7.G.1.3</strong></td>
<td>• Slicing is limited to horizontal or vertical slices.</td>
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<tr>
<td><strong>MAFS.7.G.1.3</strong></td>
<td>• Bases of prisms and pyramids can be a triangle (any type); a square; a rectangle; or a regular pentagon or hexagon.</td>
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<tr>
<td><strong>MAFS.7.G.1.3</strong></td>
<td>• Items should not use composite figures.</td>
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<tr>
<td><strong>MAFS.7.G.2.5</strong> Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.</td>
<td>• Items should use angles measured in degrees only.</td>
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<tr>
<td><strong>MAFS.7.G.2.5</strong></td>
<td>Calculator: <strong>YES</strong></td>
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<td><strong>MAFS.7.G.2.5</strong></td>
<td>Context: <strong>ALLOWABLE</strong></td>
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</tr>
</tbody>
</table>

**Open Up Resources Lessons** (be selective, you can’t use all in the time provided)

- Grade 7, **Unit 1: Scale Drawings**
- Grade 7, **Unit 7: Angles, Triangles, and Prisms**

**Decoded Standard**

**MAFS.7.G.1.1**

Students work with scale drawings. They learn how to read them, calculate the scale, compute the actual lengths from the scale in the drawings, and reproduce a scale drawing using another scale. Scale drawings are proportional to one another. Problems should center on experiences in the students’ own lives. Examples include but are not limited to scale drawings of student rooms at home, the classroom, and comic book strips. The term scale factor should be used when students are asked to reproduce a scale drawing at a different scale. A scale factor is a number that multiplies some quantity. For example, doubling the length of a window that is 3 ft long corresponds to a scale factor of 2 ($2 \times 3 = 6$).

*(Common Core Mathematics Companion, Pg. 165)*

*Special Note: This would be an excellent place to introduce the conversion tables on the reference sheet.*

**Formative Tasks**

**Mathematics Formative Assessments (MFAS)**

- **Flying Scale** find the length and area of an object when given a scale drawing of the object.
- **Space Station Scale** Find the ratio of the area of an object in a scale drawing to its actual area and then relate this ratio to the scale factor in the drawing.
- **Garden Design** Reproduce a scale drawing using a different scale.

**Lesson Resources**

**Engage NY**

- **Grade 7 Module 1 Topic D Lesson 16** Students understand scale drawings.
- **Grade 7 Module 1 Topic D Lesson 18** Students compute the lengths of pictures using a scale drawing.
- **Grade 7 Module 1 Topic D Lesson 19** Given a scale drawing students compute the area of the actual picture.
### Illustrative Mathematics Assessment Tasks

- **Floor Plan**  
  Translate between measurements given in a scale drawing and the corresponding measurements of the object represented by the scale drawing. If used in an instructional setting, it would be good for students to have an opportunity to see other solution methods, perhaps by having students with different approaches explain their strategies to the class.

- **Map distance**  
  Translate between information provided on a map that is drawn to scale and the distance between two cities represented on the map.

- **Rescaling Washington Park**  
  Think critically about the effect that changing from one scaling to another has on an image, and then to physically produce the desired image.

- **Grade 7 Module 1 Topic D Lesson 20**  
  Students create their own scale drawings of a room or building.

- **Grade 7 Module 1 Topic D Lesson 21**  
  Students produce scale drawings at a different scale.

### Illuminations

- **Off the Scale**  
  Examine maps of their home states and calculate distances between cities.

- **Planning a Playground**  
  Design a playground using manipulatives and multiple representations and scaling the City using SimCity Objects.

### CPalms

- **Designing a Geo-World: A Geometry Based Theme Park**  
  Investigate a contextual problem in which they will need to construct a 2-dimensional polygon, compute actual lengths and areas and then reproduce a scale drawing of the figure at a different scale.

- **Making a Scale Drawing**  
  Create a detailed scale drawing.

### MARS/Shell

- **Drawing to Scale: Designing a Garden**  
  Interpret and use scale drawings to plan a garden layout.

### McGraw-Hill

- **Course 2, Chapter 7 Inquiry Lab: Scale Drawing; Lesson 4**

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### Decoded Standard

**MAFS.7.G.1.2**

Students practice drawing geometric shapes using technology (computer programs both commercial and free on the Internet), rulers and protractors, and free hand. While giving practice with multiple shapes, focus on triangles and constructing them from three given angles or sides. Students should determine, by looking at the given measures, whether one, more than one, or no triangles can be created. Angles need to add up to 180° to make a triangle. The sum of two side lengths of a triangle is always greater than the third side. If this is true for all three combinations of added side lengths, then you will have a triangle. (*Common Core Mathematics Companion, Pg. 166*)

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**

- **Drawing Triangles AAA**  
  Draw a triangle with given angle measures, and explain if these conditions determine a unique triangle.

- **Drawing Triangles AAS**  
  Draw a triangle given the measures of two angles and a non-included side and to explain if these conditions determine a unique triangle.

- **Drawing Triangles ASA**  
  Draw a triangle given the measures of two angles and their included side and to explain if these conditions determine a unique triangle.

- **Drawing Triangles SAS**  
  Draw a triangle given the measures of two sides and their included angle and to explain if these conditions determine a unique triangle.

- **Drawing Triangles SSA**  
  Draw a triangle given the lengths of two of its sides and the measure of a non-included angle and to decide if these conditions determine a unique triangle.

### Lesson Resources

**Engage NY**

- **Grade 7 Module 6 Topic B Lesson 6**

- **Grade 7 Module 6 Topic B Lesson 7 Lesson 6 &7 - Students use tools to draw geometric shapes based on given conditions.**

- **Grade 7 Module 6 Topic B Lesson 8 Students draw triangles under different conditions to explore if it forms many, few or one triangle**

- **Grade 7 Module 6 Topic B Lesson 9**

- **Grade 7 Module 6 Topic B Lesson 10 Lesson 9 & 10- Students explore conditions of triangles.**

- **Grade 7 Module 6 Topic B Lesson 11 Students understand that three given lengths determine a triangle, provided the largest length is less than the sum of the other two lengths; otherwise, no triangle can be formed**
• **Drawing Triangles SSS** Draw a triangle with given side lengths, and explain if these conditions determine a unique triangle.
• **Sides of Triangles** Determine if given lengths will create a triangle.

• **Grade 7 Module 6 Topic B Lesson 12** Students explore unique triangles
• **Grade 7 Module 6 Topic B Lesson 13** Students use conditions to determine a unique triangle to determine when two triangles are identical.

**CPalms**
• **Triangle Inequality Investigation** Use hands-on materials to understand that only certain combinations of lengths will create closed triangles.
• **Congruent Triangles** Construct triangles with the parts provided.

**MARS/Shell**
• **Possible Triangle Constructions** Recall, sketch, construct and apply triangle properties and to determine whether given conditions describe a unique triangle, more than one possible triangle or does not describe a possible triangle.

**McGraw-Hill**

Course 2, Chapter

Inquiry Lab: Investigate Online Maps and Scale Drawings; Inquiry Lab: Create Triangles; Lesson 3

---

**Decoded Standard**

MAFS.7.G.1.3

Students relate the two-dimensional shape that results from slicing a three-dimensional figure. Three-dimensional shapes will include right rectangular prisms and right rectangular pyramids. *(Common Core Mathematics Companion, Pg. 167)*

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**Formative Tasks**

Mathematics Formative Assessments (MFAS)

• **Square Pyramid Slices** Sketch and describe the two-dimensional figures that result from slicing a square pyramid.
• **Rectangular Prism Slices** Sketch and describe two-dimensional figures that result from slicing a rectangular prism.
• **Cylinder Slices** Sketch and describe the two-dimensional figures that result from slicing a cylinder.
• **Cone Slices** Sketch and describe the two-dimensional figures that result from slicing a cylinder.

Illustrative Mathematics Assessment Tasks

• **Cube Ninjas!** Explore various cross sections of a cube and use precise language to describe the shape of the resulting faces.

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**Engage NY**

• **Grade 7 Module 6 Topic C Lesson 16** Students describe rectangular regions that result from slicing a right rectangular prism by a plane perpendicular to one of the faces.
• **Grade 7 Module 6 Topic C Lesson 17** Students describe polygonal regions that result from slicing a right rectangular pyramid by a plane perpendicular to the base and by another plane parallel to the base.
• **Grade 7 Module 6 Topic C Lesson 18** Students describe polygonal regions that result from slicing a right rectangular prism or pyramid by a plane.
• **Grade 7 Module 6 Topic C Lesson 19** Students describe three-dimensional figures built from cubes by looking at horizontal slicing planes

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**CPalms**

• **Can You Cut It? Slicing Three Dimensional Figures** Sketch, model, and describe cross-sections formed by a plane passing through a three-dimensional figures.

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**Virtual Manipulative**

• **Shodor: Cross Section Flyer** Explore cross sections of various cones, cylinders, prisms, and pyramids.

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**McGraw-Hill**
Decoded Standard

MAFS.7.G.2.5
Explore supplementary, complementary, vertical, and adjacent angles and their relationships to one another. These facts are used in multi-step problems.

see images on page 170 of the Common Core Mathematics Companion

(Common Core Mathematics Companion, Pg. 170)

Instructional Resources

<table>
<thead>
<tr>
<th>Formative Tasks</th>
<th>Lesson Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematics Formative Assessments (MFAS)</strong></td>
<td><strong>Engage NY</strong></td>
</tr>
<tr>
<td>• Solve for the Angle Write and solve equations to determine unknown angle measures in supplementary and complementary angle pairs.</td>
<td>• Grade 7 Module 6 Topic A Lesson 1 Students solve for unknown angles in word problems and in diagrams involving complementary and supplementary angles.</td>
</tr>
<tr>
<td>• Find the Angle Measure Use knowledge of angle relationships to write and solve equations to determine unknown angle measures.</td>
<td>• Grade 7 Module 6 Topic A Lesson 2 Students solve for unknown angles in word problems and in diagrams involving complementary, supplementary, vertical, and adjacent angles.</td>
</tr>
<tr>
<td>• Straight Angles Write and solve equations to determine unknown angle measures in supplementary angle relationships.</td>
<td>• Grade 7 Module 6 Topic A Lesson 3 Students solve for unknown angles in word problems and in diagrams involving all learned angle facts.</td>
</tr>
<tr>
<td>• What Is Your Angle? Use knowledge of angle relationships to write and solve equations to determine unknown angle measures.</td>
<td>• Grade 7 Module 6 Topic A Lesson 4 Students solve for unknown angles in word problems and in diagrams involving all learned angle facts</td>
</tr>
</tbody>
</table>

**CPalms**

• Angles, Angles Everywhere Discover complementary and supplementary angles by measuring the degrees for sets of angles.

**MARS/Shell**

• Applying Angle Theorems Use geometric properties to solve problems using the measures of the interior and exterior angles of polygons.

**McGraw-Hill**

Course 2, Chapter 7

Lessons 1 & 2 with an emphasis on supplementary, complementary, vertical, and adjacent
Standards/Learning Goals:

**MAFS.7.G.2.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

- Circles are limited to whole circles and semicircles.
- Calculator: **YES**
- Context: **ALLOWABLE**

**MAFS.7.G.2.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

- Three-dimensional shapes may include right prisms and right pyramids.
- When the base of a figure has more than four sides, the area of the base must be given.
- Calculator: **YES**
- Context: **ALLOWABLE**

### Open Up Resources Lessons (be selective, you can’t use all in the time provided)

- Grade 7, **Unit 3: Measuring Circles**
- Grade 7, **Unit 7: Angles, Triangles, and Prisms**

### Decoded Standard

**MAFS.7.G.2.4**

Students learn formulas for area \((A = \pi r^2)\) and circumference \((C = 2\pi r)\) of circles and then solve problems (mathematical and real-world) using these formulas. Students participate in discovering the relationship between the two formulas. *(Common Core Mathematics Companion, Pg. 169)*

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**

- **Circumference Formula** Students are asked to write the formula for the circumference of a circle, explain what each symbol represents, and label the variables on a diagram.
- **Circle Area Formula** Students are asked to write the formula for the area of a circle, explain what each symbol represents, and label the radius on a diagram.
- **Eye on Circumference** Students are asked to write the formula for the area of a circle, explain what each symbol represents, and label the radius on a diagram.
- **Center Circle Area** Students are asked to solve a problem involving the area of a circle.
- **Broken Circles** Students are asked to complete and explain an informal derivation of the relationship between the circumference and area of a circle.

#### Illustrative Mathematics Assessment Tasks

- **The Circumference of a Circle and the Area of the Region it Encloses** The purpose of this task is to help students differentiate between a circle and the region inside of the circle so that they understand what is being measured when the circumference and area are being found. This task is best used as a lead-in to the formulas for circumference and area of a circle.
- **Approximating the area of a circle** Use formulas for the area of squares and triangles to estimate.
- **Circumference of a Circle** The goal of this task is to study the circumferences of different sized circles, both using manipulatives and from the point of view of scaling.
- **Eight Circles** The purpose of this task is to strengthen students' understanding of area.

#### Lesson Resources

**Engage NY**

- **Module 3, Topic C, Lesson 16** Students know the formula for circumference \(C\) of a circle of diameter \(d\) and radius \(r\). Students discover that the ratio of the circumference to the diameter of a circle is called \(\pi\), written \(\pi\).
- **Module 3, Topic C, Lesson 17** Students know the formula for the area of a circle and use it to solve problems.

**Illuminations**

- **Tree Talk** Students will measure circumference of trees in order to find diameter and calculate age of local trees using a growth rate table.
- **The Ratio of Circumference to Diameter** Students measure the circumference and diameter of circular objects.
- **Geometry of Circles** Using a MIRA™ geometry tool, students determine the relationships between radius, diameter, circumference and area of a circle.

**CPalms**

- **The Circle** This interactive lesson introduces students to the circle, its attributes, and the formulas for finding its circumference and area.

**Videos**

- **Math Antics - Circles, Circumference And Area** Learn the difference and similarities between Circumference and Area formulas while relating them to real-life.

**McGraw Hill**

*Course 2, Chapter 8*
• **Measuring the area of a circle** This goal of this task is to give students familiarity using the formula for the area of a circle while also addressing measurement error while looking at the cross-section of a pipe.

Inquiry Lab: Circumference; Inquiry Lab: Area of Circles; Lessons 1, 2, and 3

### Decoded Standard

MAFS.7.G.2.6
This standard pulls together much of what the students know and can do in geometry through problems solving of both mathematical and real-world problems. Students will work with two- and three-dimensional objects and apply what they know about area, volume and surface area. *(Common Core Mathematics Companion, Pg. 171)*

**Special Note:** Questions should not include cylinders, spheres or cones for this standard.

### Instructional Resources

#### Formative Tasks

- **Mathematics Formative Assessments (MFAS)**
  - **Composite Polygon Area** Students are asked to find the area of a composite figure.
  - **Octagon Area** Students are asked to find the area of a composite figure.
  - **Cube Volume and Surface Area** Students are asked to calculate the volume and surface area of a cube.
  - **Chilling Volumes** Students are asked to solve a problem involving the volume of a composite figure.
  - **Composite Surface Area** Students are asked to find the surface area of a composite figure.
  - **Prismatic Surface Area** Students are asked to determine the surface area of a right triangular prism and explain the procedure.

#### Illustrative Mathematics Assessment Tasks

- **Drinking the Lake** The purpose of this task is for students to solve a volume problem in a modeling context. This task asks students to work with volumes that do not have a well-defined shape, and so is more abstract than it appears.
- **Designs** The purpose of this task is for students to find the area and perimeter of figures composed of squares and fractions of circles.
- **Stained Glass** The purpose of this task is for students to find the area and perimeter of geometric figures whose boundaries are segments and fractions of circles and to combine that information to calculate the cost of a project.

### Engage NY

- **Module 3, Topic C, Lesson 19** Students find the areas of triangles and simple polygonal regions in the coordinate plane with vertices at grid points by composing into rectangles and decomposing into triangles and quadrilaterals.
- **Module 3, Topic C, Lesson 20** Students find the area of regions in the plane with polygonal boundaries by decomposing the plane into triangles and quadrilaterals, including regions with polygonal holes.
- **Module 3, Topic C, Lesson 21** Students find the surface area of three-dimensional objects whose surface area is composed of triangles and quadrilaterals. They use polyhedron nets to understand that surface area is simply the sum of the area of the lateral faces and the area of the base(s).
- **Module 3, Topic C, Lesson 22** Students find the surface area of three-dimensional objects whose surface area is composed of triangles and quadrilaterals, specifically focusing on pyramids.
- **Module 3, Topic C, Lesson 23** Students use the known formula for the volume of a right rectangular prism (length x width x height).
- **Module 3, Topic C, Lesson 24** Students use the formula for the volume of a right rectangular prism to answer questions about the capacity of tanks. Students compute volumes of right prisms involving fractional values for length.
- **Module 3, Topic C, Lesson 25** Students solve real-world and mathematical problems involving volume and surface areas of three-dimensional objects composed of cubes and right prisms.

### Illuminations

- **Patterns and Functions** Students investigate properties of perimeter, area, and volume related to various geometric two- and three-dimensions shapes.

### MARS/Shell

- **Maximizing Area: Gold Rush** Students will explore the effects on a rectangle’s area of systematically varying the dimensions whilst keeping the perimeter constant. Interpret and evaluate the data generated, identifying the optimum case.

### McGraw-Hill Textbook

**Course 2, Chapter 8**

<table>
<thead>
<tr>
<th>Semester 1</th>
<th>Unit 4: Probability</th>
<th>7 days: 11/2 – 11/10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standards/Learning Goals:</strong></td>
<td><strong>Content Limits, Assessment Types, Calculator</strong></td>
<td></td>
</tr>
<tr>
<td><strong>MAFS.7.SP.3.5</strong> Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>MAFS.7.SP.3.6</strong> Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. <em>For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but the probably not exactly 200 times.</em></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| **MAFS.7.SP.3.7** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.  
  a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*  
  b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* |  | |
| **MAFS.7.SP.3.8** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.  
  a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.  
  b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.  
  c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?* |  | **ASSESSED with MAFS.7.SP.3.7** |
Decoded Standard

MAFS.7.SP.3.5
This standard introduces students to the concept of chance with events that are likely, unlikely, or neither likely nor unlikely. Students learn to use a scale from 0-1 representing probabilities that range from impossible to certain as in the scale from 0-1 below:

See image on page 224 of the Common Core Mathematics Companion

Numerical probabilities are numbers from 0-1, and the larger the number (the closer to 1), the more likely the event is to occur. A number near 0 (i.e., \(\frac{1}{50}\)) indicates an unlikely event and a number in the middle (\(\approx 0.5\)) is neither likely nor unlikely. A 0 probability is an impossible event, and a 1 is a certainty. Probabilities are expressed as ratios of the number of times an event occurs to the total number of trials performed. Probabilities can be represented as fractions, decimals, and percents. (Common Core Mathematics Companion, Pg. 224)

Instructional Resources

Formative Tasks
Mathematics Formative Assessments (MFAS)
- **Probability or Not?** Students are asked to determine whether or not a given number could represent the probability of an event.
- **Likely or Unlikely?** Students are asked to determine the likelihood of an event given a probability.
- **Likelihood of an Event** Students are asked to determine the likelihood of an event given a probability.

Lesson Resources
Engage NY
- **Module 5, Topic A, Lesson 1** A probability is a number between 0 and 1 that represents the likelihood that an event will occur; interpret a probability as the proportion of the time that an event occurs when a chance experiment is repeated many times.

MARS/Shell
- **Probability Games** In this lesson students confront and overcome common probability misconceptions. The will count equally likely outcomes using diagrams, discuss relationships between theoretical probabilities, observe outcomes and samples sizes and calculate probabilities of independent events.

Illuminations
- **What Are My Chances?** Students will conduct five experiments through stations to compare theoretical and experimental probability. The class data will be combined to compare with previously established theoretical probability. Also covers 7.SP.3.6.
- **The Game of SKUNK** In this lesson, students practice decision-making skills leading to a better understanding of choice versus chance and building the foundation of mathematical probability. Also includes 7.SP.3.6.

Interactive Manipulatives/shodor.org
- **Spinner** In this activity, students adjust how many sections there are on a fair spinner then run simulated trials on that spinner as a way to develop concepts of probability.

McGraw Hill
Course 2, Chapter 9
Lesson 1
# Decoded Standard

**MAFS.7.SP.3.6**

Students collect data on chance events so that they can estimate the probability of the event. Students learn the difference between theoretical probability (probability that is calculated mathematically) and experimental probability (actual outcomes of an experiment). Seldom are the theoretical and experimental probabilities equal, although the more a simulation is repeated, the closer the theoretical and experimental probabilities become.

Relative frequency is the observed number of successful outcomes in a set number of trials. It is the observed proportion of successful events. Students learn to make predictions about the relative frequency of an event by using simulations. *(Common Core Mathematics Companion, Pg. 225)*

## Instructional Resources

### Formative Tasks

**Mathematics Formative Assessments (MFAS)**

- **Probability Cubed** Students are asked to estimate the frequency of an event given its probability and explain why an expected frequency might differ from an observed frequency.
- **Hen Eggs** Students are asked to estimate the probability of a chance event based on observed frequencies.
- **Game of Chance** students are asked to estimate the frequency of an event given its probability and explain why an expected frequency might differ from an observed frequency.

**Illustrative Mathematics Assessment Tasks**

- **Heads or Tails** This task asks students to think about how the distribution of observed outcomes from a chance experiment might differ from the theoretical distribution and to use observed data to estimate a probability.
- **Rolling Dice** Students pool the results of many repetitions of the random phenomenon (rolling dice) and compare their results to the theoretical expectation they develop by considering all possible outcomes of rolling two dice. This gives them a concrete example of what we mean by long term relative frequency.
- **Tossing Cylinders** The purpose of this task is to provide students with the opportunity to determine experimental probabilities by collecting data.

### Lesson Resources

**EngageNY**

- **Module 5, Topic A, Lesson 2** Estimate probabilities by collecting data on an outcome of a chance experiment; use given data to estimate probabilities.
- **Module 5, Topic A, Lesson 3** Determine the possible outcomes for simple chance experiments; given a description of a simple chance experiment, students determine the sample space for the experiment; given a description of a chance experiment and an event, students determine for which outcomes in the sample space the event will occur; distinguish between chance experiments with equally likely outcomes and chance experiments for which the outcomes are not equally likely.
- **Module 5, Topic B, Lesson 8** Given theoretical probabilities based on a chance experiment, students describe what they expect to see when they observe many outcomes of the experiment; students distinguish between theoretical probabilities and estimated probabilities; students understand that probabilities can be estimated based on observing outcomes of a chance experiment.

**Illuminations**

- **Adjustable Spinner** Change the number of sectors and increase or decrease their size to create any type of spinner. Then, conduct a probability experiment by spinning the spinner many times. How does the experimental probability compare with the theoretical probability?

**CPalms**

- **A Roll of the Dice** What are your chances of tossing a particular number on a number cube? Students collect data by experimenting and then converting the data in terms of probability. By the end of the lesson, students should have a basic understanding of simple events.

**MARS/Shell**

- **Evaluating Statements About Probability** This lesson unit addresses common misconceptions relating to probability of simple and compound events. The lesson will help you assess how well students understand concepts of equally likely events, randomness and sample sizes.

**McGraw Hill**

- **Course 2, Chapter 9** In Science Lab: Relative Frequency; Lesson 2 (limit content to 7.SP.3.6)
## Decoded Standard

**MAFS.7.SP.3.7**

This standard is broken into two parts (a-b). We will consider them together since they are so closely related. Overall, students develop and use probability models to find the probability of events. Uniform probability models are those where the likelihood of each outcome is equal. For example, there are 17 children in the class. What is the probability that Sam will be chosen? Using theoretical probability, students can predict frequencies of outcomes. In part b of this standard, students look at the experimental probability to develop a model. (*Common Core Mathematics Companion*, Pg. 226)

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**

- **Marble Probability** Students are asked to determine probabilities based on observed outcomes from drawing marbles from a bag and to determine if the outcomes appear to be equally likely.
- **Number Cube** Students are asked to determine probabilities based on observed outcomes from rolling a number cube and to determine if the outcomes appear to be equally likely.
- **Technical Difficulties** Students are given a scenario and asked to determine the probability of two different events.
- **Errand Runner** Students are asked to determine the probability of a chance event and explain possible causes for the difference between the probability and observed frequencies.

#### Illustrative Mathematics Assessment Tasks

- **Stay or Switch** The purpose of the task is for students to find the theoretical probability of an event by systematically recording all of the possible outcomes in the sample space and identifying those that correspond to the event.
- **How Many Buttons** This task uses student generated data to assess standard 7.SP.7. This task could also be extended to address Standard 7.SP.1 by adding a small or whole class discussion of whether the class could be considered as a representative sample of all students at your school.

## Decoded Standard

**MAFS.7.SP.3.8**

This standard is broken into three parts (a-c). We will consider them together since they are so closely related. Students move to compound events by building on their knowledge of single events. Compound events are those where two or more events are happening at once. For example, what is the probability that you forgot to study last night and there will be a surprise quiz in class today? Students select tools such as organized lists, tables, and tree diagrams to represent sample spaces for compound events. Ultimately, students design their own simulation for a compound event. (*Common Core Mathematics Companion*, Pg. 228)

*Lesson 6 on Permutations and Combinations is an unnecessary extension into the High School Standard MAFS.912.S-CP.2.9*

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**

- **Work Clothing** Students are asked to make a tree diagram to determine all possible outcomes of a compound event.

#### Engage NY

- **Module 5, Topic A, Lesson 4** Students will calculate probabilities of events for chance experiments that have equally likely outcomes.
- **Module 5, Topic A, Lesson 5** Students calculate probabilities for chance experiments that do not have equally likely outcomes.
- **Module 5, Topic B, Lesson 9** Students compare estimated probabilities to those predicted by a probability model.

#### CPalms

- **M & M Candy: I Want Green** Students compare mathematical expectations and experimental probability; then explain any difference in the two numbers. Students use colored candy pieces (such as M & M’s) for their data collection, comparisons, and explanations. “from Beacon Learning Center.
- **Likely Events: Which Bag Is It?** Students will try to make sense out of the sampled results of both physical and computer simulated experiments. They will indicate an increased confidence in their inferences as the number of trials increases.

**McGraw Hill**

Course 2, Chapter 9

Lesson 2 (limit content to 7.SP.3.7) and 3
### Accelerated Math Grade 7

#### Module 5, Topic A, Lesson 7
- Students calculate probabilities of compound events.

#### Module 5, Topic B, Lesson 10
- Students learn how to perform simulations to estimate probabilities; students use various devices to perform simulations (e.g., coin, number cube, cards).

#### Module 5, Topic B, Lesson 11
- Students design their own simulations; students learn to use two more devices in simulations: colored disks and a random number table.

#### Module 5, Topic B, Lesson 12
- Use estimated probabilities to judge whether a given probability model is plausible; students will use estimated probabilities to make informed decisions.

### Illuminations

#### Random Drawing - Sampling Distribution
This tool generates a sampling distribution by generating a large number of trials. You can choose the type of trial, how many tickets to draw, and how many times to repeat the trial. The results are shown in a histogram and table.

### CPalms

#### Pick and Roll
This lesson is designed to teach students about independent and dependent compound probability and give students opportunities to experiment with probabilities through the use of manipulatives, games, and a simulation project. Also includes MAFS.7.SP.3.8

#### Chancy Candy
In this lesson students will use candy to find the probability of independent compound events, determining the sample space from a tree diagram. They will then do an experiment to test the theoretical probability. Once the experiment is complete, the students will compare the theoretical and experimental probability.

#### How to Hit it Big in the Lottery – Probability of compound events
Students will explore a wide variety of interesting situations involving probability of compound events. Students will learn about independent and dependent events and their related probabilities.

### Virtual Manipulatives

#### Interactive Marbles
This online manipulative allows the student to simulate placing marbles into a bag and finding the probability of pulling out certain combinations of marbles. This allows exploration of probabilities of multiple events as well as probability with and without replacement.

#### Hamlet Happens
The purpose of this manipulative is to help students recognize that (1) unusual events do happen, and (2) it may take a longer time for some of them to happen. The letters are drawn at random from the beginning of Hamlet's soliloquy, “To be, or not to be.” Any word made from those letters (such as TO) can be entered in the box. When the start is pressed, letters are drawn and recorded. The process continues until the word appears.

### McGraw-Hill Textbook

**Course 2, Chapter 9**
- Inquiry Lab: Fair and Unfair Games; Inquiry Lab: Simulate Compound Events; Lessons (limit content to 7.SP.3.8) and 4
Standards/Learning Goals:

**MAFS.7.SP.1.1** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

**MAFS.7.SP.1.2** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

**MAFS.7.SP.2.3** Informally assess the degree of visual overlap of two numerical data distributions with similar variability, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

**MAFS.7.SP.2.4** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

**Open Up Resources Lessons** (be selective, you can’t use all in the time provided)
- Grade 7, Unit 8: Probability and Sampling

Decoded Standard

**MAFS.7.SP.1.1** Sampling is taught in this standard as a statistical tool used to gain information about a population without examining the entire population. Sampling is the process of taking a subset of subjects that is representative of the entire population and collecting data on that subset. The sample must have sufficient size to warrant statistical analysis. Samples need to be representative of the population in order to make valid generalizations and, therefore, should be randomly selected. A random sampling guarantees that each element of the population has an equal opportunity to be selected in the sample. An example of a random sample is taking a list of names at a school and selecting every fourth person to be in the sample to represent the population of the school. *(Common Core Mathematics Companion, Pg. 217)*

**Instructional Resources**

<table>
<thead>
<tr>
<th>Formative Tasks</th>
<th>Lesson Resources</th>
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<tbody>
<tr>
<td>Mathematics Formative Assessments (MFAS)</td>
<td>Engage NY</td>
</tr>
</tbody>
</table>
• **Ice Cream Survey** Choose a sampling method that would be most representative of a population and justify their selection.

• **Height Research** Describe a method for collecting data in order to estimate the average height of 12 year-old boys in the U.S.

• **Favorite Sport Survey** Evaluate an inference made using a biased sampling method.

**Illustrative Mathematics Assessment Tasks**

• **Mr. Briggs’ Class Likes Math** Determine whether the scenario will create a representative sample.

• **Grade 7 Module 5 Topic C Lesson 13** Students differentiate population characteristic & sample statistics.

• **Grade 7 Module 5 Topic C Lesson 14** Students understand how a sample is selected.

• **Grade 7 Module 5 Topic C Lesson 15** Students begin to develop an understanding of sampling variability.

**CPalms**

• **And the survey says...** Use data from a random sample to draw inferences about a sample population. Analyze the results of a random sample to apply generalizations to an entire population.

• **How Old are My Employees** This lesson provides activities for students to conceptually understand how to estimate an unknown characteristic of a population, the effect of sample size, the effect of multiple samples in same sizes on estimations, and the representativeness of the random sampling. The lesson consists of three tasks followed by group discussion sessions and a whole class discussion session at the end.

• **Populations and Samples** Work in pairs to gather information using a biased sample and random sample to compare data and reflect on possible misconceptions that a biased sample could produce.

**Yummy Math**

• **Peeps** Estimate the number of Peeps sold each Easter season. Consider reasonability by making guesses. Determine necessary info, problem solve and improve their original estimates. Conduct random samplings of their estimates and compare the mean of their estimates to the actual number of peeps sold each season.

**MARS/Shell**

• **Estimating Counting Trees** Solve simple problems involving ratio and direct proportion. Choose an appropriate sampling method. Collect discrete data and record them using a frequency table.

**McGraw Hill**

Course 2, Chapter 10

Lesson 1

### Decoded Standard

MAFS.7.SP.1.2

This standard connects to &.SP.1.1 by using the sample data collected to draw inferences. Generate multiple samples of the same size from a given population to examine the variation in estimates or predictions. This standard provides an introduction to variability. An example of data to collect is two random samples of 100 students about school lunch preferences.

<table>
<thead>
<tr>
<th>School Lunch Preferred</th>
<th>Sample</th>
<th>Burgers</th>
<th>Salad</th>
<th>Pizza</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>13</td>
<td>13</td>
<td></td>
<td>74</td>
<td>100</td>
</tr>
<tr>
<td>#2</td>
<td>12</td>
<td></td>
<td>11</td>
<td>77</td>
<td>100</td>
</tr>
</tbody>
</table>

*(Common Core Mathematics Companion, Pg. 218)*

### Instructional Resources

**Formative Tasks**

**Mathematics Formative Assessments (MFAS)**

• **School Days** Use data from a random sample to estimate a population parameter and explain what might be done to increase confidence in the estimate.

**Engage NY**

• **Grade 7 Module 5 Topic C Lesson 21** Random samples to draw informal references about the difference in population means.
- **Movie Genre** Use data from a random sample to draw an inference about a population.

**Illustrative Mathematics Assessment Tasks**
- **Valentine Marbles** Software was used to generate 100 random samples of size 16 from a population where the probability of obtaining a success in one draw is 33.6% (Bernoulli). Given that multiple samples of the same size have been generated, students should note that there can be quite a bit of variability among the estimates from random samples and that on average, the center of the distribution of such estimates is at the actual population value and most of the estimates themselves tend to cluster around the actual population value.

**Decoded Standard**
MAFS.7.SP.2.3
Students compare statistics on two data sets for the first time. Build on their understanding of graphs, mean, median, mean absolute deviation (MAD), and interquartile range from sixth grade. Students understand that variability is responsible for the overlap of two data sets, which can be visible when the data are presented in graphic form – two dot plots or box-and-whisker plots, for example. With two data distributions with similar variability, students will express the difference between centers (mean, median, mode) as a multiple of a measure of variability. For an example see Reproducible 4 (pg. 261).

(Common Core Mathematics Companion, Pg. 220)

**Instructional Resources**

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<tr>
<th>Formative Tasks</th>
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<tbody>
<tr>
<td><strong>Formative Assessments (MFAS)</strong></td>
<td><strong>CPalms</strong></td>
</tr>
<tr>
<td><strong>TV Ages</strong> Informally determine the degree of overlap between two box plots with the same interquartile range (IQR) by expressing the difference between their medians as a multiple of the IQR.</td>
<td><strong>Stepping Up Measures of Center</strong> Explore the use of dot plots and mean absolute deviation to compare and draw inferences from two different sets of numerical data.</td>
</tr>
<tr>
<td><strong>More TV Ages</strong> Informally determine the degree of overlap between two box plots with the same interquartile range (IQR) by expressing the difference between their medians as a multiple of the IQR.</td>
<td><strong>Who’s Taller</strong> Uses real-world data sets to guide students through representing and comparing data sets in separate dot plots. Represent and compare the data sets by using the mean and MAD (mean absolute deviation).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Illustrative Mathematics Assessment Tasks</th>
<th>Illuminations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>College Athletes</strong> Conjecture about the differences in the two groups from a strictly visual perspective and then support their comparisons with appropriate measures of center and variability.</td>
<td><strong>Mean and Median</strong> Using an interactive applet, students can compare and contrast properties of measures of central tendency, specifically the influence of changes in data values on the mean and median. As students change the data values by dragging the red points to</td>
</tr>
<tr>
<td><strong>Offensive Linemen</strong> Conjecture about the differences and similarities in the two groups from a strictly visual perspective and</td>
<td></td>
</tr>
</tbody>
</table>
then support their comparisons with appropriate measures of center and variability.

the left or right, the interactive figure dynamically adjusts the mean and median of the new data set.

- **Advanced Data Grapher** This is an online graphing utility that can be used to create box plots, bubble graphs, scatterplots, histograms, and stem-and-leaf plots.

**MARS/Shell**

- **Comparing Data** This lesson is intended to help students to make meaningful comparisons between sets of data. In particular, selecting appropriate measures of center and variability in order to summarize the important features of a set of data and using quantitative measures to justify an argument.

**McGraw Hill**

Course 2, Chapter 10
Inquiry Lab: Visual Overlap of Data Distributions; Lesson 4

<table>
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<tr>
<th>Decoded Standard</th>
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<tbody>
<tr>
<td>MAFS.7.SP.2.4</td>
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</tbody>
</table>
Draw valid comparative inferences about two populations. The inferences are drawn from using measures of center (mean, median, mode) and variability (range, mean absolute deviation, and interquartile range) from random samples. This standard differs from the previous in that students are now drawing inferences. Using the examples from the previous standard where the data were collected will unify this work. *(Common Core Mathematics Companion, Pg. 221)*

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<th>Instructional Resources</th>
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<td><strong>Formative Tasks</strong></td>
</tr>
<tr>
<td>Mathematics Formative Assessments (MFAS)</td>
</tr>
</tbody>
</table>
- **Word Lengths** Use the mean and the mean absolute deviation (MAD) to compare two distributions.
- **Overlapping Trees** Compare two distributions given side-by-side box plots.

**Illustrative Mathematics Assessment Tasks**

- **College Athletes** Conjecture about the differences in the two groups from a strictly visual perspective and then support their comparisons with appropriate measures of center and variability.
- **Offensive Linemen** Conjecture about the differences and similarities in the two groups from a strictly visual perspective and then support their comparisons with appropriate measures of center and variability.

**Illuminations**

- **Mean and Median** Using an interactive applet, students can compare and contrast properties of measures of central tendency, specifically the influence of changes in data values on the mean and median. As students change the data values by dragging the red points to the left or right, the interactive figure dynamically adjusts the mean and median of the new data set.
- **Advanced Data Grapher** This is an online graphing utility that can be used to create box plots, bubble graphs, scatterplots, histograms, and stem-and-leaf plots.

**MARS/Shell**

- **Comparing Data** This lesson is intended to help students to make meaningful comparisons between sets of data. In particular, selecting appropriate measures of center and variability in order to summarize the important features of a set of data and using quantitative measures to justify an argument.

**McGraw Hill**

Course 2, Chapter 10
Inquiry Lab: Collect Data
**Semester 1**

**Unit 6: Real Numbers**

**Thanksgiving Break 11/21 – 11/29**

<table>
<thead>
<tr>
<th>Standards/Learning Goals</th>
<th>Content Limits, Assessment Types, Calculator</th>
</tr>
</thead>
</table>
| **MAFS.8.NS.1.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. | • All irrational numbers may be used, excluding e. 
• Only rational numbers with repeating decimal expansions up to thousandths may be used. 

Calculator: NO

Context: NO CONTEXT |

| **MAFS.8.NS.1.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., \( \pi^2 \)). For example, by truncating the decimal expansion of \( \sqrt{2} \) is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. | • All irrational numbers may be used, excluding e. 
• Irrational expressions should only use one operation. 

Calculator: NO

Context: NO CONTEXT |

| **MAFS.8.EE.1.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, \( 3^2 \times 3^{-5} = 3^{-3} = 1/3^2 = 1/27 \). | • Exponents must be integers. 
• Bases must be whole numbers 
• Variables may not be used. 

Calculator: NO

Context: NO CONTEXT |

| **MAFS.8.EE.1.2** Use square root and cube root symbols to represent solutions to equations of the form \( x^2 = p \) and \( x^3 = p \), where \( p \) is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that \( \sqrt{2} \) is irrational. | • Square roots and cube roots may be used to represent solutions to equations. 
• Radicands may not include variables. 

Calculator: NEUTRAL

Context: ALLOWABLE |

| **MAFS.8.EE.1.3** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 times \( 10^8 \) and the population of the world as 7 times \( 10^9 \), and determine that the world population is more than 20 times larger. | • N/A 

Calculator: NO

Context: ALLOWABLE |

| **MAFS.8.EE.1.4** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. | • N/A 

Calculator: NO

Context: ALLOWABLE |

---

**Open Up Resources Lessons** (be selective, you can’t use all in the time provided)

- Grade 8: [Unit 7: Exponents and Scientific Notation](#)
- Grade 8, [Unit 8: Pythagorean Theorem and Irrational Numbers](#)

---

**Decoded Standard**

MAFS.8.NS.1.1

Students expand their knowledge of the Real Number System to include irrational numbers. A diagram shows the relationship of the subsets:

[see image on page 71 of the Common Core Mathematics Companion](#)
An irrational number is a decimal whose expansion does not terminate or repeat. Irrational numbers cannot be written in fraction form. Using decimal expressions, students compare rational numbers and irrational numbers to show that rational number expansion repeat and irrational numbers expansions do not. The notation “… means “continues indefinitely without repeating.” For example, 0.3 is a rational number that repeats but \( \pi = 3.1415 \ldots \) does not repeat.

To convert a decimal expansion into a fraction:

Change 0.5 to a fraction
1. Let \( x = 0.555 \ldots \)
2. Multiply both sides so that the repeating digits will be in front of the decimal. In this case, one digit repeats so both sides are multiplied by 10, giving \( 10x = 5.555 \ldots \)
3. Subtract the original equation from the new equation.
   \[
   \begin{align*}
   10x & = 5.555 \ldots \\
   -x & = 0.555 \ldots \\
   9x & = 5
   \end{align*}
   \]
4. Solve the equation by dividing both sides of the equation by 9.
5. \( x = \frac{5}{9} \)

(Common Core Mathematics Companion, Pg. 71)

### Instructional Resources

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<td><strong>Mathematics Formative Assessments (MFAS)</strong></td>
<td><strong>Engage NY</strong></td>
</tr>
<tr>
<td>- <strong>Rational Numbers</strong> Identify rational numbers from a list of real numbers.</td>
<td>- <strong>Grade 8, Module 7, Topic B, Lesson 8</strong> Decimal expansion</td>
</tr>
<tr>
<td>- <strong>Fraction to Decimal Conversion</strong> Given a fraction to convert to a decimal; determine if the decimal repeats.</td>
<td>- <strong>CPalms</strong></td>
</tr>
<tr>
<td>- <strong>Decimal to Fraction Conversion</strong> Given several terminating and repeating decimals to convert to fractions.</td>
<td>- <strong>Predicting the decimal equivalent for a fraction—terminating or repeating?</strong> Terminating and repeating decimals into fractions.</td>
</tr>
<tr>
<td><strong>Illustrative Mathematics</strong></td>
<td>- <strong>Really! I’m Rational!</strong> How repeating decimals or converted into fractions.</td>
</tr>
<tr>
<td>- <strong>Converting Decimal Representations of Rational Numbers to Fraction Representations</strong> Convert repeating decimals into fractions</td>
<td><strong>McGraw-Hill</strong></td>
</tr>
<tr>
<td>- <strong>Repeating or Terminating?</strong> Understand why terminating decimal numbers can also be written as repeating decimals where the repeating part is all 9’s.</td>
<td>Course 3, Chapter 1</td>
</tr>
<tr>
<td></td>
<td>Lesson 1</td>
</tr>
</tbody>
</table>

### Decoded Standard

MAFS.8.NS.1.2
Students compare irrational numbers and locate them on a number line by finding their rational approximations. Find rational approximations by creating lists of numbers by answering the following question: Between which two numbers will you find \( \sqrt{2} \)? Since \( 1^2 = 1 \) and \( 2^2 = 4 \), it is between 1 and 2. To be more precise, it is closer to 1 or 2? Systematically square 1.1, 1.2, 1.3, 1.4.....1.9. Between which two numbers do you find 2? Repeat the process until the degree of precision you are seeking.

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<td><strong>Engage NY</strong></td>
</tr>
<tr>
<td>- <strong>Approximating Irrational Numbers</strong> Plot the square root of eight on three number lines, scaled to progressively more precision.</td>
<td>- <strong>Grade 8, Module 7, Topic B, Lesson 11</strong> Decimal expansion of roots</td>
</tr>
<tr>
<td>- <strong>Locating Irrational Numbers</strong> Graph three different irrational numbers on number lines.</td>
<td>- <strong>Grade 8, Module 7, Topic B, Lesson 12</strong> Decimal expansions of fractions</td>
</tr>
<tr>
<td>- <strong>Comparing Irrational Numbers</strong> Estimate the value of several irrational numbers using a calculator and order them on a number line.</td>
<td>- <strong>Grade 8, Module 7, Topic B, Lesson 13</strong> Compare and order rational approximations</td>
</tr>
</tbody>
</table>

**MARS/Shell**
The Irrational Beauty of the Golden Ratio
Find and interpret lower and upper bounds of an irrational expression using a calculator.

Illustrative Mathematics
- Comparing Rational and Irrational Numbers
  Compare rational and irrational numbers without a calculator
- Irrational Numbers on the Number Line
  Label irrational numbers on a number line

Engaging Tasks
- Decimal Approximations of Roots - Open Middle
- Rational and Irrational Roots - Open Middle
- Number 18 - Which One Doesn’t Belong

Decoded Standard
MAFS.8.EE.1.1
Students learn how to compute using integer exponents building on their earlier experiences with adding and subtracting integers. For any non-zero real numbers a and b and integers n and m, the properties of integer exponents are as follows:

1. \( a^m \cdot a^n = a^{m+n} \)
2. \( (a^n)^m = a^{nm} \)
3. \( a^n \cdot b^n = (ab)^n \)
4. \( a^0 = 1 \)
5. \( a^{-n} = \frac{1}{a^n} \)
6. \( \frac{a^n}{a^m} = a^{n-m} \)

(Common Core Mathematics Companion, Pg. 118)

Engaging Tasks
- How Can We Make Stronger Passwords
  Determine how long it will take to crack your password.

Lesson Resources
Engage NY
- Grade 8, Module 1, Topic A, Lesson 1 Understanding exponential notation
- Grade 8, Module 1, Topic A, Lesson 2 Simplifying exponential expressions
- Grade 8, Module 1, Topic A, Lesson 3 Powers of powers
- Grade 8, Module 1, Topic A, Lesson 4 Base raised to the zero power
- Grade 8, Module 1, Topic A, Lesson 5 Negative exponents
- Grade 8, Module 1, Topic A, Lesson 6 Integer exponents

MARS/Shell
- Applying Properties of Exponents
  Apply the properties of exponents by a matching activity.

CPalms
- Exponential Chips
  Apply the properties of exponents to multiply and divide.
## Decoded Standard

### MAFS.8.EE.1.2
Students learn that squaring and cubing numbers are the inverse operations to finding square and cube roots. This standard works with perfect squares and perfect cubes, and students will begin to recognize those numbers. Equations should include rational numbers such as \( x^2 = \frac{1}{4} \) and \( x^3 = \frac{1}{64} \) and fractions where both the numerator and denominator are perfect squares or cubes: \( x^2 = \frac{1}{4} \)

\[
\sqrt{x^2} = \pm \frac{\sqrt{1}}{\sqrt{4}}
\]

\[
x = \pm \frac{1}{2}
\]

Square roots can be positive or negative because \( 2 \times 2 = 4 \) and \(-2 \times -2 = 4\).

*(Common Core Mathematics Companion, Pg. 119)*

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**

- **The Root of the Problem** Evaluate perfect square roots and perfect cube roots.
- **Dimension Needed** Solve problems involving square roots and cube roots.
- **Roots and Radicals** Use square root and cube root symbols to represent the real solutions of each equation. Then evaluate any square roots of perfect squares and cube roots of perfect cubes. Indicate if any of your solutions are irrational.

#### Lesson Resources

**Engage NY**

- **Grade 8, Module 7, Topic A, Lesson 2** Square and cube roots

**Explore Learning - Gizmos**

- **Square Roots** Free 30 Day Trial Required: A resource to explore square roots using an area model.

**Learnzillion**

- **Identify perfect squares and perfect cubes by building and observing models.** Slide show and video lesson to identify perfect squares and perfect cubes.

**McGraw-Hill**

Course 3, Chapter 1

Lesson 8

## Decoded Standard

### MAFS.8.EE.1.3
This standard emphasizes scientific notation. Students write very large and very small numbers in scientific notation using positive and negative exponents. For example 123,000 written in scientific notation is \( 1.23 \times 10^5 \), and 0.008 written in scientific notation is \( 8 \times 10^{-4} \). When mastered, students use the skill to determine how many times larger (or smaller) one number written in scientific notation is than another. To compare, if the exponent increases by 1, the value increases 10 times. In the example of the U.S. and world populations, the exponent increased by 1, and the 7 is a little more than 2 times 3. So 2 x 10 makes for 20 times larger.

*(Common Core Mathematics Companion, Pg. 120)*

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**

- **Estimating Extreme Values** Estimate each value described below by writing it in the form \( a \times 10^n \) where \( a \) is a single digit number and \( n \) is an integer.
- **How Many Times** Given pairs of numbers written in exponential form to compare them multiplicatively.
- **Compare Numbers** Given pairs of numbers written in scientific notation compare them multiplicatively.

#### Lesson Resources

**Engage NY**

- **Grade 8, Module 1, Topic B, Lesson 1** Powers of 10
- **Grade 8, Module 1, Topic B, Lesson 2** Translating scientific notation and standard form
- **Grade 8, Module 1, Topic B, Lesson 13** Comparing numbers in scientific notation
- **Order Matters** Given pairs of numbers written in the form of an integer times a power of 10. Then compare the numbers in each pair using the inequality symbols.

- **Illustrated Mathematics**
  - **Ant and Elephant** Compare very small and very large quantities using metric system

- **MARS/Shell**
  - **Applying Properties of Exponents** Estimating length using scientific notation.

- **McGraw-Hill**
  - Course 3, Chapter 1
  - Lesson 6 (supplement to express how many times larger)

### Decoded Standard

**MAFS.8.EE.1.4**

This standard builds on previous standards as now students use what they know about scientific notation and properties of integer exponents to solve problems. Quantities in the problems can be expressed in scientific notation and decimal form. Students focus on the size of the measurement to determine which units are appropriate for the context such as millimeters for very small quantities. This standard also calls for students to use technology and be able to interpret the scientific notation used. The teacher needs to check the class calculators to be familiar with the notation used by those particular calculators as the notation used by calculators to express scientific notation is not standard. *(Common Core Mathematics Companion, Pg. 121)*

### Instructional Resources

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<td><strong>Mathematics Formative Assessments (MFAS)</strong></td>
<td><strong>Engage NY</strong></td>
</tr>
<tr>
<td>- <strong>Mixed Form Operations</strong> Given word problems with numbers in both standard and scientific notation to solve problems using various operations.</td>
<td>- <strong>Grade 8, Module 1, Topic B, Lesson 9</strong> Operations with numbers in scientific notation</td>
</tr>
<tr>
<td>- <strong>Sums and Differences in Scientific Notation</strong> Add and subtract numbers given in scientific notation in real-world contexts.</td>
<td>- <strong>Grade 8, Module 1, Topic B, Lesson 10</strong> Operations with numbers in scientific notation</td>
</tr>
<tr>
<td>- <strong>Scientific Multiplication and Division</strong> Multiply and divide numbers given in scientific notation in real-world contexts.</td>
<td>- <strong>Grade 8, Module 1, Topic B, Lesson 11</strong> Operations with numbers in scientific notation</td>
</tr>
<tr>
<td>- <strong>Scientific Calculator Display</strong> Given examples of calculator displays and asked to convert the notation in the display to both scientific notation and standard form.</td>
<td>- <strong>Grade 8, Module 1, Topic B, Lesson 12</strong> Measurement</td>
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</tbody>
</table>

<table>
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<th><strong>Engaging Tasks</strong></th>
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<td>Unit 7: Linear Equations in One Variable</td>
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<td>------------</td>
<td>----------------------------------------</td>
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<td><strong>Standards/Learning Goals:</strong></td>
<td><strong>Content Limits, Assessment Types, Calculator</strong></td>
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<tr>
<td>MAFS.8.EE.3.7 Solve linear equations in one variable.</td>
<td>- Numbers in items must be rational numbers.</td>
</tr>
<tr>
<td>a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form ( x = a, a = a, ) or ( a = b ) results (where ( a ) and ( b ) are different numbers).</td>
<td>Calculator: YES</td>
</tr>
<tr>
<td>b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</td>
<td>Context: ALLOWABLE</td>
</tr>
</tbody>
</table>

**Open Up Resources Lessons** (be selective, you can’t use all in the time provided)
- Grade 8: [Unit 2: Dilations, Similarity, and Introducing Slope](#)
- Grade 8, [Unit 3: Linear Relationships](#)

**Decoded Standard**
MAFS.8.EE.3.7
This standard has students solving linear equations. It is explained by 8.EE.3.7a and b. It is best to teach a and b together so that they are not considered isolated skills. These standards provide the foundation for all future work with linear equations. Students solve equations that have one, zero, or infinitely many solutions and relate those solutions to the context. If the solution is in the form \( x = a \), there is only one solution. If \( a = a \), there are infinitely many solutions. If \( a = b \) results (where \( a \) and \( b \) are different numbers), there are no solutions.

Linear equations can have fractions and decimals as coefficients and can be solved by expanding expressions with the distributive property and/or collecting like terms. (Common Core Mathematics Companion, Pg. 128)

**Instructional Resources**

**Formative Tasks**
- **Mathematics Formative Assessments (MFAS)**
  - **Counting Solutions** Worksheet includes three equations where students identify whether there is one solution, no solution, or infinitely many solutions.
  - **Equation Prototypes** Worksheet includes three questions where students are to create equations with one solution, no solution, and infinitely many solutions.
  - **Linear Equations I** Students are to solve one linear equation with only one variable that involves rational numbers (fractions). \( \frac{3}{2}x - 4 \frac{1}{2} = -8 \)
  - **Linear Equations II** Students are to solve one linear equation with only one variable that involves rational coefficients (decimals) and distributive property. \(-3.5(10x - 2) = -176.75\)
  - **Linear Equations III** Students are to solve a linear equation in one variable with rational coefficients and variables on both sides of the equation. \(-4(2x + 9) + 3x = 6 - 4(x - 3)\)

**Engaging Tasks**

**Engage NY**
- **Grade 8, Module 4, Topic A, Lesson 3** Solving equations with variables on both sides
- **Grade 8, Module 4, Topic A, Lesson 4** Solving equations with rational coefficients and variables on both sides
- **Grade 8, Module 4, Topic A, Lesson 6** Solving equations with rational coefficients, distributive property and variables on both sides
- **Grade 8, Module 4, Topic A, Lesson 7** Understanding the conditions for \( a=a, a=b \) and \( x=a \)

**MARS/Shell**
- **Solving Linear Equations in One Variable** Tasks require students to use rational coefficients, collect like terms, expand using distributive property, and categorize equations as one, none, or infinitely many solutions. Whole class instruction, small group and assessment tasks are available.
- **Classifying Solutions to Systems of Equations** Tasks require students to classify solutions that are represented graphically and use substitution to complete a table of values for linear equations.

**McGraw-Hill**
<table>
<thead>
<tr>
<th>Course 3, Chapter 2</th>
<th>Inquiry Lab: Equations with Variables on Each Side; Lesson 4 and 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>• <strong>Linear Equations with One Solutions</strong> Create a</td>
<td></td>
</tr>
<tr>
<td>linear equation with one solution — Open Middle</td>
<td></td>
</tr>
<tr>
<td>• <strong>One Solution, No Solutions, Infinite Solutions</strong></td>
<td></td>
</tr>
<tr>
<td>— Open Middle</td>
<td></td>
</tr>
<tr>
<td>• <strong>Solving Equations</strong> — Math Mistakes</td>
<td></td>
</tr>
<tr>
<td>• <strong>Solving Systems Algebraically</strong> — Math Mistakes</td>
<td></td>
</tr>
<tr>
<td>• <strong>Number 17</strong> — Which One Doesn’t Belong</td>
<td></td>
</tr>
</tbody>
</table>
Semester 2 | Unit 8: Linear Equations in Two Variables | 15 days: 1/21 – 2/10

<table>
<thead>
<tr>
<th>Standards/Learning Goals:</th>
<th>Content Limits, Assessment Types, Calculator</th>
</tr>
</thead>
</table>
| **MAFS.8.EE.2.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* | • Numbers in items must be rational numbers  
Calculator: YES  
Context: ALLOWABLE |
| **MAFS.8.EE.2.6** Use similar triangles to explain why the slope $m$ is the same distance between two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $b$. | • All triangles must be right triangles and on a coordinate grid.  
• Numbers in items must be rational numbers.  
• Functions must be linear.  
Calculator: YES  
Context: ALLOWABLE |
| **MAFS.8.EE.3.8** Analyze and solve pairs of simultaneous linear equations.  
a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.  
b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be $5$ and $6.*  
c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* | • Numbers in items must be rational numbers.  
• Coefficients of equations in standard form must be integers.  
• Items written for MAFS.8.EE.3.8a must include the graph or the equations.  
• Equations in items written for MAFS.8.EE.3.8a must be given in slope-intercept form.  
Calculator: YES  
Context: ALLOWABLE |

Open Up Resources Lessons (be selective, you can’t use all in the time provided)

- Grade 8: [Unit 3: Linear Relationships](#)
- Grade 8, [Unit 4: Linear Equations and Linear Systems](#)

Decoded Standard

**MAFS.8.EE.2.5**

Students build on their work from Grade 6 with unit rates and their work with proportional relationships in Grade 7 to compare graphs, tables, and equations of linear (proportional) relationships. Students identify the unit rate as slope in graphs, tables, and equations to compare proportional relationships presented using different representations. For example, compare the unit rate in a problem about a phone bill presented in graphic form on a Cartesian plane to a phone bill from a different company where the unit rate can be found represented in an equation or table. *(Common Core Mathematics Companion, Pg. 123)*

### Instructional Resources

<table>
<thead>
<tr>
<th>Formative Tasks</th>
<th>Lesson Resources</th>
</tr>
</thead>
</table>
| Mathematics Formative Assessments (MFAS)  
- **Interpreting Slope** Using a worksheet, graph a proportionate relationship (from a table of values), find and interpret slope.  
- **Proportional Paint** Interpret a graph through a worksheet including three questions (identify unit rate, find slope, and describe how they are related).  

Engage NY

- **Grade 8, Module 4, Topic B, Lesson 11** Constant rate problems displayed in a graph and a table

MARS/Shell
**Pinellas County Schools**  
**ACCELERATED MATH GRADE 7**  
2020-2021

- **Compare Slopes** identify, describe, and compare the slopes of two proportional relationships given the graph of one and the equation of the other.
- **Buying cars** Students will create, compare, and evaluate different representations of functions.
- **Defining Lines by Points, Slopes and Equations** Find slopes and equations using graphs and use slopes and y-intercepts to derive equations

**Illustrative Mathematics**
- **Who has the best job?** Compare the rate of change of two functions displayed as a table and an equation.
- **Peaches and Plums** Reason about the relative costs per pound of the two fruits without actually knowing what the costs are.

**McGraw-Hill**  
Course 3, Chapter 3  
Lesson 1

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### Decoded Standard

**MAFS.8.EE.2.6**
Students gain additional knowledge about slope in this standard as they use similar triangles to explain how the slope $m$ of a line is the same between any two points on a given non-vertical line. Students understand positive/negative slopes, 0 slope, and undefined slopes. Through the use of similar triangles, teachers lead students to derive the general equation $(y = mx + b)$ of a line and discover that $m$ is the slope and $b$ is the y-intercept. ([Common Core Mathematics Companion](https://example.com), Pg. 124)

The similarity portion of this standard will be addressed in Unit 6.

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### Instructional Resources

**Formative Tasks**
- **Slope Triangles** Use similar triangles to explain why the slope is the same regardless of the points used to calculate it (worksheet uses proportionality of line segments to help students visualize concept).
- **Deriving Lines II** Students are asked to derive one general equation of a line (using the slope formula) with a y-intercept of $(0,b)$.

**Illustrative Mathematics**
- **Slopes between points on a line** Help students understand why the calculated slope will be the same for any two points on a given line.

**Engage NY**
- **Grade 8, Module 4, Topic C, Lesson 15** Interpret slope as rate of change on a graph
- **Grade 8, Module 4, Topic C, Lesson 16** Use triangles to explain slope; slope formula to find slope
- **Grade 8, Module 4, Topic C, Lesson 17** Find slope of a line; Transform standard form to slope intercept form
- **Grade 8, Module 4, Topic C, Lesson 19** Proof that any point on a line is a point on the graph of the equation of that line.
- **Grade 8, Module 4, Topic C, Lesson 20** Any line is the graph of a linear equation
- **Grade 8, Module 4, Topic C, Lesson 23** Solving systems with equations in different forms

**MARS/Shell**
- **Defining Lines, by Points, Slopes, and Equations** Find slopes and equations with ordered pairs; calculate and use slope and y-intercept to derive an equation. May involve similar triangles to help define slope.

**McGraw-Hill**  
Course 3, Chapter 3  
Lesson 4

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### Decoded Standard

**MAFS.8.EE.3.8**
This standard has students solving simultaneous linear equations. It is explained by 8.EE.3.8a-c. It is best to consider $a$, $b$, and $c$ together as they are not isolated skills.

Students will understand that points of intersection are the solutions to pairs of simultaneous linear equations (also known as systems of linear equations). Students will solve systems graphically, algebraically, and by inspection. Examples in this standard are in real-world contexts and mathematical problems. ([Common Core Mathematics Companion](https://example.com), Pg. 129)

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### Instructional Resources

**Formative Tasks**

**Lesson Resources**
## Mathematics Formative Assessments (MFAS)

- **Identify the Solution** Two graphs are given; students are to identify the solutions of the system and justify their answer.
- **Solving Systems of Linear Equations** Three problems are given; students are asked to solve each algebraically (equations are written in both standard form and slope-intercept form).
- **Solving Systems of Linear Equations by Graphing** Solve one system of equation problem graphically (written in slope-intercept form), write the solution as an ordered pair and explain why it is the solution.  
- **How Many Solutions?** Determine the number of solutions for each of the four systems of linear equations without solving (standard form and slope-intercept form) and justify the answer.
- **Writing System Equations** Two word problems are given. Students are asked to write a system of linear equations that could be used to solve them.
- **System Solutions** One word problem (real world context) with both equations provided. Students are to solve the system of linear equations (elimination or substitution).

## Engaging Tasks

- **Candy and Chips** Solve the system to determine the cost of a new order of chips and candy.
- **Solutions of Two Linear Equations** Given a graphic, provide 4 points that represent 2 distinct lines
- **Create a System of Equations, Given 1 Equation and the Solution** Write linear equations so that the solution of the system of that line and equation is a particular point.
- **System of Equations, Special Case Infinitely Many Solutions** Fill in the boxes so that there are infinitely many solutions to the system
- **Systems of Equations, Special Case No Solution** Fill in the boxes so that there is no solution to the system
- **Solve Linear Equations with Special Cases** Complete each equation with the given number of solutions

### Engage NY

- **Grade 8, Module 4, Topic D, Lesson 24** Intro to Systems of Equations
- **Grade 8, Module 4, Topic D, Lesson 25** Solving systems by graphing
- **Grade 8, Module 4, Topic D, Lesson 26** Solving systems with parallel lines
- **Grade 8, Module 4, Topic D, Lesson 27** Solving systems with no solution
- **Grade 8, Module 4, Topic D, Lesson 28** Solving systems by elimination and substitution

### MARS/Shell

- **Solving Real Life Problems: Baseball Jerseys** Tasks require students to select appropriate mathematical methods to interpret and evaluate data generated and identify a break-even point.

### McGraw-Hill

**Course 3, Chapter 3**

**Lesson 7**
<table>
<thead>
<tr>
<th>Standards/Learning Goals:</th>
<th>Content Limits, Assessment Types, Calculator</th>
</tr>
</thead>
</table>
| **MAFS.8.F.1.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. | - Function notation may not be used.  
- Nonlinear functions may be included for identifying a function.  
Calculator: **NEUTRAL**  
Context: **ALLOWABLE** |
| **MAFS.8.F.1.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* | - Function notation may not be used.  
- Functions must be linear.  
Calculator: **YES**  
Context: **ALLOWABLE** |
| **MAFS.8.F.1.3** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points $(1, 1), (2, 4)$ and $(3, 9)$, which are not on a straight line.* | - Function notation may not be used.  
Calculator: **YES**  
Context: **ALLOWABLE** |
| **MAFS.8.F.2.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two $(x, y)$ values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. | - Function notation may not be used.  
- Functions must be linear.  
-  
Calculator: **NEUTRAL**  
Context: **ALLOWABLE** |
| **MAFS.8.F.2.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. | - Linear or nonlinear relationships may use any of the four quadrants.  
- Graph descriptions move from left to right.  
- Functional relationships must be continuous.  
Calculator: **NEUTRAL**  
Context: **ALLOWABLE** |

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**Open Up Resources Lessons**

*Grade 8, Unit 5: Functions and Volume*
- Lesson 1: Inputs and Outputs
- Lesson 2: Introduction to Functions
- Lesson 3: Equations for Functions
- Lesson 4: Tables, Equations, and Graphs of Functions
- Lesson 5: More Graphs of Functions
- Lesson 6: Even More Graphs of Functions
- Lesson 7: Connecting Representations of Functions
- Lesson 8: Linear Functions
- Lesson 9: Linear Models
- Lesson 10: Piecewise Linear Functions
MAFS.8.F.1.1
This standard is the students’ introduction to functions and involves the definition of function as a rule that assigns to each input exactly one output. Students are not required to use or recognize function notation at this grade but will be able to identify functions using tables, graphs, and equations. A relationship is not a function when there is more than one y-value associated with any x-value. Using the definition, an example of a table that does not represent a function is as follows:

See tables on page 139 of the Common Core Mathematics Companion

(Common Core Mathematics Companion, Pg. 139)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>1</td>
<td>44.5</td>
</tr>
<tr>
<td>2</td>
<td>39.00</td>
</tr>
<tr>
<td>3</td>
<td>33.50</td>
</tr>
<tr>
<td>4</td>
<td>28.00</td>
</tr>
</tbody>
</table>

Engage NY
- Grade 8, Module 5, Topic A, Lesson 2: Expressing functions by rule, and when input is used with the formula, the outcome is the output.
- Grade 8, Module 5, Topic A, Lesson 6: Determine if a function is linear and interpret the equation \(y=mx+b\) as a linear function.

CPalms
- An Introduction to Functions: How Much are Playoff Tickets?: Introduces functions with the real-world example of the cost of tickets for a playoff game.
- Desmos
- Commuting Times: This activity illustrates the relationship between a data set (which is usually not a function) and a model of the data (which—in algebra—is a function).

Better Lessons
- Function Machine: Using a real function machine to input and output values.
- Disney World Park Tickets: Use tickets to Disney World to demonstrate that multiple inputs.

McGraw-Hill
Course 3, Chapter 4
Lesson 2

MAFS.8.F.1.2
For this standard students will compare the properties of functions. One property of functions is slope. When students are given two different functions, each represented in a different form (algebraically, graphically, in a table, or by a verbal description), students should be able to determine which function has the greater slope. An example follows:

Ruth starts with a $50 gift card for Walmart. She spends $5.50 per week to buy cat food. Let \(y\) be the amount left on the card and \(x\) represent the number of weeks.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50</td>
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<td>39.00</td>
</tr>
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<td>3</td>
<td>33.50</td>
</tr>
<tr>
<td>4</td>
<td>28.00</td>
</tr>
</tbody>
</table>
Boyce rents bikes for $5 an hour. He also collects a non-refundable fee of $10.00 for a rental to cover wear and tear. Write the rule for the total cost \( (c) \) of renting a bike as a function of the number of hours \( (h) \) rented.

Solution: Ruth’s story is an example of a function with a negative slope. The amount of money left on the card decreases each week. The graph has a negative slope of -5.5, which is the amount the card balance decreases every time Ruth buys cat food.

Boyce’s bike rental is an example of a function with a positive slope. This function has a positive slope of 5, which is the amount to rent a bike for an hour. An equation for Boyce’s bikes could be \( c = 5h + 10 \).

(Common Core Mathematics Companion, Pg. 140)

<table>
<thead>
<tr>
<th>Instructional Resources</th>
<th>Formative Tasks</th>
<th>Lesson Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematics Formative Assessments (MFAS)</strong></td>
<td><strong>Engage NY</strong></td>
<td><strong>CPalms</strong></td>
</tr>
<tr>
<td>- Innovative Functions Compare the rates of change of two functions presented in different forms (an expression and a table) within a real-world context.</td>
<td>- Grade 8, Module 5, Topic A, Lesson 7 Compare 2 functions in different ways</td>
<td>- What’s My Function? Determining function rules.</td>
</tr>
<tr>
<td>- Speed Reading Compare the rates of change of two functions presented in different forms (an expression and a table) within a real-world context.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Competing Functions Recognize and compare the initial values of two functions represented in different ways.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- This House is Mine! Compare a specific value of two functions given in different forms (a graph and a verbal description) within a real-world context.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Illustrative Mathematics Assessment Tasks</th>
<th>Engaging Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Battery Charging</strong> Verbal and numerical descriptions of battery life as a function of time.</td>
<td><strong>Comparing Functions</strong> Generate five ordered pairs that represent a linear function that has a greater rate of change than the graph.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decoded Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAFS.8.F.1.3</td>
</tr>
<tr>
<td>In this standard students become familiar with the equation ( y = mx + b ) as defining a linear function that will graph as a straight line. Students distinguish between linear (functions that graph into a straight line) and nonlinear functions (functions that do not graph into a straight line such as a curve). Note that standard form and point-slope for are not studied in this grade. (Common Core Mathematics Companion, Pg. 141)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematics Formative Assessments (MFAS)</strong></td>
<td><strong>Engage NY</strong></td>
<td><strong>CPalms</strong></td>
</tr>
<tr>
<td>- What Am I? Describe a linear function, its graph, and the meaning of its parameters.</td>
<td>- Grade 8, Module 5, Topic A, Lesson 8 Determine whether an equation is linear or non-linear by examining the rate of change</td>
<td>- Beginning Linear Function Describe the concept of slope.</td>
</tr>
<tr>
<td>- Explaining Linear Functions Describe defining properties of linear functions.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Nonlinear Functions Provide an example of a nonlinear function and explain why it is nonlinear.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Linear or Nonlinear? Identify a function as either linear or nonlinear and to justify their decision.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Illustrative Mathematics Assessment Tasks</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Meal Out</strong> Use equations to solve a problem with a restaurant check.</td>
<td><strong>Linear Graphs</strong> Match equations with linear graphs.</td>
</tr>
</tbody>
</table>
### Function Rules
Connect a function described by a verbal rule with corresponding values in a table.

### Desmos
- **Card Sort: Linear or Nonlinear** Sort equations and tables of values into two categories - linear and nonlinear.

### McGraw-Hill
- **Course 3, Chapter 4** Lesson 4

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<table>
<thead>
<tr>
<th>Decoded Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAFS.8.F.2.4</td>
</tr>
<tr>
<td>Students identify the rate of change (slope) and y-intercept (initial value) from tables, graphs, equations, and verbal descriptions of linear relationships. The y-intercept is the y-value when the x-value is 0. Interpretation of slope and the initial value of the function is accomplished using real-world situations.</td>
</tr>
</tbody>
</table>

### Instructional Resources

#### Formative Tasks
- **Mathematics Formative Assessments (MFAS)**
  - **Construction Function** Construct a function to model a linear relationship between two quantities given two ordered pairs in context.
  - **Profitable Functions** Write a function to model a linear relationship given its graph.
  - **Trekking Functions** Construct a function to model a linear relationship between two quantities given a table of values.
  - **Smart TV** Determine the rate of change and initial value of a linear function given a table of values, and interpret the rate of change and initial value in terms of the situation it models.
  - **Drain the Pool** Determine the rate of change and initial value of a linear function when given a graph, and to interpret the rate of change and initial value in terms of the situation it models.

#### Engage NY
- **Grade 8, Module 6, Topic A, Lesson 1** Determine and interpret a linear function from a verbal description.
- **Grade 8, Module 6, Topic A, Lesson 2** Interpret slope and the initial value; describe the graph of the function based on its slope.
- **Grade 8, Module 6, Topic A, Lesson 3** Graph a line based on different characteristics (function, initial value, points)

#### MARS/Shell
- **Lines and Linear Functions** Interpret speed as the slope of a linear graph and translate between the equation of a line and its graphical representation.
- **Interpreting Time-Distance Graphs** Interpret distance–time graphs as if they are pictures of situations rather than abstract representations of them.

#### Desmos
- **Sugar Sugar** Use unit rates to compare the sugary-ness of five cereals.
- **The Running Game** Use proportional reasoning to predict how long it will take someone to run seven miles. Consider the meaning of several graph features in context.

#### CPalms
- **Getting Graphic with Linear Functions** Construct a linear function to model a linear relationship, determine the rates of change and initial value from a table and graph as well as interpret what the rate of change means as it relates to a situation.

### Illustrative Mathematics Assessment Tasks
- **Video Streaming** Model of a linear function.
- **High School Graduation** Estimating approximate time name called using a linear function.
- **Baseball Cards** Interpreting linear functions.

### McGraw-Hill
- **Course 3, Chapter 4** Lesson 1
Decoded Standard

MAFS.8.F.2.5
Given a graph, students will provide a verbal description of the function, including whether the graph is linear or nonlinear or where the function is increasing or decreasing. Given a function’s verbal description, students will be able to sketch the graph displaying qualitative properties of that function. The quantitative features of the graph are not displayed (specific quantities on the axes). (Common Core Mathematics Companion, Pg. 144)

Instructional Resources

<table>
<thead>
<tr>
<th>Formative Tasks</th>
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</thead>
<tbody>
<tr>
<td><strong>Mathematics Formative Assessments (MFAS)</strong></td>
<td><strong>Engage NY</strong></td>
</tr>
<tr>
<td>• Jet Fuel Describe the relationship between two linearly related quantities.</td>
<td>• Grade 8, Module 6, Topic A, Lesson 4 Describe and sketch qualitatively function relationships.</td>
</tr>
<tr>
<td>• Population Trend Describe the relationship between two quantities in a nonlinear function.</td>
<td>• Grade 8, Module 6, Topic A, Lesson 5 Qualitatively sketch and describe function relationship</td>
</tr>
<tr>
<td>• Graph the Ride Given a verbal description of the relationship between two quantities and are asked to sketch a graph to model the relationship.</td>
<td>MARS/Shell</td>
</tr>
<tr>
<td>• Bacterial Growth Graph Given a verbal description of the relationship between two quantities and are asked to sketch a graph to model the relationship.</td>
<td>• Modeling Situation with Linear Equations Explore relationships between variables in everyday situations.</td>
</tr>
<tr>
<td><strong>Illustrative Mathematics Assessment Tasks</strong></td>
<td>Desmos</td>
</tr>
<tr>
<td>• Tides Interpreting the graph of a function in terms of the relationship between quantities that it represents.</td>
<td>• Polygraph: Lines Identify important features of lines precisely describe these features</td>
</tr>
<tr>
<td>• Distance Interpret two graphs that look the same but show very different quantities.</td>
<td>CPalms</td>
</tr>
<tr>
<td>• Bike Race Interpret two distance-time graphs in terms of the context of a bicycle race.</td>
<td>• Tides Interpreting the graph of a function in terms of the relationship between quantities that it represents.</td>
</tr>
<tr>
<td><strong>Engaging Tasks</strong></td>
<td>• Are We There Yet? Write and interpret linear functions that represent real world situations, noting the importance of slope and y-intercept.</td>
</tr>
<tr>
<td>• Joules Three Acts Math - Do you think Joules will work as advertised?</td>
<td>McGraw-Hill</td>
</tr>
<tr>
<td></td>
<td>Course 3, Chapter 4</td>
</tr>
<tr>
<td></td>
<td>Lesson 9</td>
</tr>
</tbody>
</table>
Semester 2  |  Unit 10: Triangles and Pythagorean Theorem  |  10 days: 3/22 – 4/6

<table>
<thead>
<tr>
<th>Standards/Learning Goals</th>
<th>Content Limits, Assessment Types, Calculator</th>
</tr>
</thead>
</table>
| **MAFS.8.G.1.5** Use informal arguments to establish facts about the angle sum & exterior angle of triangles and about the angle created when parallel lines are cut by a transversal. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so. | • Items must not include shapes beyond triangles.  
Calculator: NEUTRAL  
Context: NO CONTEXT |
| **MAFS.8.G.2.6** Explain a proof of the Pythagorean Theorem and its converse. | • For the converse, only perfect roots should be used.  
Calculator: NEUTRAL  
Context: ALLOWABLE |
| **MAFS.8.G.2.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. | • If the triangles is part of a 3-dimensional figure, a graphic of the 3-dimensional figure must be included.  
• Points on the coordinate grid must be where grid lines intersect.  
Calculator: YES  
Context: ALLOWABLE |
| **MAFS.8.G.2.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. | • If the triangles is part of a 3-dimensional figure, a graphic of the 3-dimensional figure must be included.  
• Points on the coordinate grid must be where grid lines intersect.  
Calculator: YES  
Context: ALLOWABLE |

ASSESSED with MAFS.8.G.2.7

Open Up Resources Lessons
Grade 8, Unit 1: Rigid Transformations and Congruence  
Grade 8, Unit 8: Pythagorean Theorem and Irrational Numbers

Decoded Standard
MAFS.8.G.1.5
Students are expected to make informal arguments while exploring facts about the sum of the angles of a triangle, exterior angles of triangles, angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similar triangles. The example demonstrates how these facts are interrelated. Note that formal two-column proofs are not expected at this grade.

See image on page 184 of the Common Core Mathematics Companion.
(Common Core Mathematics Companion, Pg. 184)

Institutional Resources

<table>
<thead>
<tr>
<th>Formative Tasks</th>
<th>Lesson Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematics Formative Assessments (MFAS)</strong></td>
<td><strong>Engage NY</strong></td>
</tr>
<tr>
<td>• Same Side Interior Angles Given same side interior angles, describe relationship and provide justification when not required to find angle measurement.</td>
<td><strong>Grade 8, Module 2, Topic C, Lesson 12</strong> Understand equivalent angle relationships when lines are parallel. Understand angle relationships related to translations and rotations. Present informal arguments about angles formed from parallel lines cut by a transversal.</td>
</tr>
<tr>
<td>• Justifying Angle Relationships Describe the relationship between alternate interior angle and provide justification.</td>
<td>• <strong>Grade 8, Module 2, Topic C, Lesson 13</strong> Informal arguments about Angle Sum Theorem for triangles</td>
</tr>
<tr>
<td>• Justifying the Exterior Angle Theorem Justify when it is not required to find angle measurement.</td>
<td></td>
</tr>
</tbody>
</table>
• **What is the Triangle Relationship?** Describe the relationship between similar triangles.

• **Justifying the Triangle Sum Theorem** Provide proof using a triangle.

**Illustrative Mathematics**

• **Congruence of Alternate Interior Angles via Rotations** Experiment with rigid motions to help visualize why alternate interior angles (made by a transverse connecting two parallel lines) are congruent.

• **Find the Angle** The task is an example of a direct but non-trivial problem in which students have to reason with angles and angle measurements (and in particular, their knowledge of the sum of the angles in a triangle) to deduce information from a picture.

**Engaging Tasks**

Transversals, Tape and Stickies Place sticky notes in their assigned location based on a description

**Decoded Standard**

MAFS.8.G.2.6

There are many proofs of the Pythagorean Theorem. Students will work through one to understand the meaning of \(a^2 + b^2 = c^2\) and its converse. The converse statement is as follows: If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle. (Common Core Mathematics Companion, Pg. 186)

**Formative Tasks**

Mathematics Formative Assessments (MFAS)

• **Pythagorean Squares** Demonstrate knowledge of the square root and right triangle in the Pythagorean theorem.

• **Explaining a Proof of the Pythagorean Theorem** Proof of similar triangles.

• **Converse of the Pythagorean Theorem** Teacher scenario to prove that the teacher is correct and prove that triangles are congruent.

**Engaging Tasks**

• **How can we correct the Scarecrow?** How can we correct the Scarecrow’s statement so it is mathematically precise?

**Instructional Resources**

**Lesson Resources**

Engage NY

• **Grade 8, Module 2, Topic D, Lesson 15** Know the Pythagorean Theorem, show an informal proof of the theorem and use it to find the length of a hypotenuse.

• **Grade 8, Module 7, Topic C, Lesson 15** Explain the proof of the Pythagorean Theorem.

• **Grade 8, Module 7, Topic C, Lesson 16** Explain the proof of the converse of the Pythagorean Theorem.

Shodor

• **Squaring the Triangle** Use the applet to explore right triangles and the Pythagorean Theorem

**CPalms**

• **Keep Calm and Hypotenuse On** Provides guiding questions to guide students in finding proof (includes PowerPoint).
The Pythagorean Theorem: Square Areas

Use the area of right triangles to deduce the areas of other shapes.

McGraw-Hill
Course 3, Chapter 5
Inquiry Lab: Proofs about Pythagorean Theorem

Decoded Standards

MAFS.8.G.2.7
Students solve problems where they must apply the Pythagorean Theorem. Problems may be real-world or mathematical, and they may involve two- and three-dimensional situations. (Common Core Mathematics Companion, Pg. 187)

Formative Tasks

Mathematics Formative Assessments (MFAS)

- New Television Using the measurement given, show if the TV can fit in the space provided.
- How Far to School Use the Pythagorean theorem to find distance.
- Three Dimensional Diagonal Apply the Pythagorean theorem to a rectangular prism’s diagonal.
- Pyramid Height Find the height of a pyramid.

Illustrative Mathematics

- Running on the Football Field Reason how to use the Pythagorean Theorem to find the distance ran by Ben Watson and Champ Bailey.
- Area of a Trapezoid Decompose the given trapezoid into other polygons and use the Pythagorean Theorem to find the unknown side-lengths of a trapezoid in order to determine the area.
- Spiderbox Visualize and apply the Pythagorean Theorem to determine the length of a spider’s path around the outside of a box.

Engaging Tasks

- Viewmongous TV Is the 80” TV double, triple or quadruple the viewing area of a 55” TV?
- Pythagorean Theorem Problems Math Mistakes examines several student errors with utilizing the Pythagorean Theorem.

Lesson Resources

Engage NY

- Grade 8, Module 2, Topic D, Lesson 16 Use Pythagorean Theorem to find missing side lengths.
- Grade 8, Module 7, Topic C, Lesson 17 Use the Pythagorean Theorem to determine the distance between two points on a coordinate plane.
- Grade 8, Module 7, Topic C, Lesson 18 Apply the Pythagorean Theorem to real world and mathematical problems in two dimensions

YummyMath

- Watson Save Determine who ran the greater distance by using the Pythagorean Theorem.

CPalms

- Alas, Poor Pythagoras, I Knew You Well! Using different activities, find real life uses for the Pythagorean Theorem.

McGraw-Hill
Course 3, Chapter 5
Lesson 6

Decoded Standards

MAFS.8.G.2.8
Use the Pythagorean Theorem to find the distance between two points. Problems can best be modeled in a coordinate system. (Common Core Mathematics Companion, Pg. 188)

Formative Tasks

Mathematics Formative Assessments (MFAS)

- Distance Between Two Points Find the distance between two points on a coordinate grid.

Lesson Resources

Engage NY
• **Distance on the Coordinate Plane** Find the distance between two points on a coordinate plane.

• **Coordinate Plane Triangle** Graph the given coordinates and find the lengths of each side of the triangle.

• **Calculate Triangle Sides** Graph the given coordinates to find the lengths of each side of the triangle.

**Engaging Tasks**

• **Where’s the Nearest Toys R Us?** Determine how store locators measure distance and calculate several distances.

• **Pythagorean Theorem Problems** Math Mistakes examines several student errors with utilizing the Pythagorean Theorem

**CPalms**

• **Bike Club Trip** Plot points on coordinate grid and use the Pythagorean Theorem to find distance between points.

**McGraw-Hill**

Course 3, Chapter 5
Lesson 7
<table>
<thead>
<tr>
<th>Standards/Learning Goals:</th>
<th>Content Limits, Assessment Types, Calculator</th>
</tr>
</thead>
</table>
| **MAFS.8.G.1.1** Verify experimentally the properties of rotations, reflections, and translations: | • The coordinate plane should not be used until MAFS.8.G.1.3  
• Limit sequences to no more than two transformations.  
• A pre-image and image should not include apostrophe notation as this would give away the identification of similarity and congruence.  
• No reference to the definition of congruence or symbols relating to the definition should be used (HS Geometry) |
| a. Lines are taken to lines, and line segments to line segments of the same length. |  |
| b. Angles are taken to angles of the same measure. |  |
| c. Parallel lines are taken to parallel lines. |  |
| **MAFS.8.G.1.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. | • The coordinate plane should not be used until MAFS.8.G.1.3  
• Limit sequences to no more than two transformations.  
• A pre-image and image should not include apostrophe notation as this would give away the identification of similarity and congruence.  
• No reference to the definition of congruence or symbols relating to the definition should be used (HS Geometry) |
| **MAFS.8.G.1.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. | • Coordinate values of x and y must be integers.  
• The number of transformations should be no more than two.  
• In items that require the student to draw a transformed figure using a dilation or a rotation, the center of the transformation must be given. |
| **MAFS.8.G.1.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. | • Items should not include the coordinate plane as the coordinate plane is needed in 8.G.1.3.  
• Limit the sequence to no more than two transformations.  
• 2-dimensional figures are limited to no more than 7 sides.  
• A pre-image or image should not include apostrophe notation as this would give away the identification of similarity and congruence.  
• No reference to the definition of congruence or symbols relating to the definition should be used (HS Geometry). |
| **MAFS.8.G.1.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angle created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so. | • Items must not include shapes beyond triangles. |

**Unit 11: Transformations, Congruence and Similarity**

**8 days: 4/7 – 4/16**

**Semester 2**

**Standards/Learning Goals:**

- **MAFS.8.G.1.1** Verify experimentally the properties of rotations, reflections, and translations:
  - Lines are taken to lines, and line segments to line segments of the same length.
  - Angles are taken to angles of the same measure.
  - Parallel lines are taken to parallel lines.

- **MAFS.8.G.1.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

- **MAFS.8.G.1.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

- **MAFS.8.G.1.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

- **MAFS.8.G.1.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angle created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.
MAFS.8.EE.2.6 Use similar triangles to explain why the slope \( m \) is the same between two distinct points on a non-vertical line in the coordinate plane; derive the equation \( y = mx \) for a line through the origin and the equation \( y = mx + b \) for a line intercepting the vertical axis at \( b \).

- All triangles must be right triangles and on a coordinate grid.
- Numbers in items must be rational numbers.
- Functions must be linear.

Context: ALLOWABLE

Open Up Resources Lessons

Grade 8, Unit 1: Rigid Transformations and Congruence
Grade 8, Unit 2: Dilations, Similarity, and Introducing Slope

Decoded Standard

MAFS.8.G.1.1 – students need multiple opportunities to explore the transformation of figures

Eighth graders add rotations, reflections, and translations to their study of transformations from Grade 7 dilations. Students verify through experimentation with figures on a coordinate plane that lines are taken to lines and line segments to line segments of the same length; angles are taken to angles of the same measure, and parallel lines are taken to parallel lines. This standard is an introduction, and students should spend time exploring these transformations.

See image on page 178 of the Common Core Mathematics Companion.

(Common Core Mathematics Companion, Pg. 178)

Instructional Resources

Formative Tasks

Mathematics Formative Assessments (MFAS)
- **Segment Transformations** Translation, rotation, and reflection
- **Angle Transformations** Students will need rulers and transparent paper. Students experimentally verify the properties of angle transformations.
- **Parallel Line Transformations** Students experimentally verify properties of parallel lines transformation.

Engaging Tasks
- **How did they make Ms. Pac-Man** Describe Ms. Pac-Man’s movements with academic vocabulary
- **How do Skytypers Write Messages?** Use transformation applications to create skytyping messages and translate it into a set of coordinates.
- **Naming Coordinates, Feedback and Revision** Revise a student error involving translating a quadrilateral
- **Best Reflection** Students compare 4 images with their reflection to determine which one is the best.
- **Pool Bounce** Determine where each shot will hit using reflections.
- **Transformations – Shortest Sequence** What’s the fewest number of transformations needed to take pre-image ABCT to \( A'B'C'D' \).

Lesson Resources

Engage NY
- **Grade 8, Module 2, Topic A, Lesson 1** Rigid Motion
- **Grade 8, Module 2, Topic A, Lesson 2** Translations
- **Grade 8, Module 2, Topic A, Lesson 3** Parallel Lines
- **Grade 8, Module 2, Topic A, Lesson 4** Reflections
- **Grade 8, Module 2, Topic A, Lesson 5** Rotations

CPalms
- **A Transformation’s Adventure with Patty Paper: exploring Translations, Reflections and Rotations** Explore reflections, translations, and rotations with patty paper.

MARS/Shell
- **Representing and Combining Transformations** Combining rigid transformations

McGraw-Hill
Course 3, Chapter 6
- Inquiry Lab: Transformations
- Lesson 1, 2 and 3
Course 3, Chapter 7
- Lesson 1
### Decoded Standard

**MAFS.8.G.1.2**

Students use what they previously learned about transformations to determine congruency between figures. Congruent figures share the same size and shape. When given two congruent figures, students describe the sequence of transformations that occurred to create the congruent figure. Note that dilations cannot be used for congruent figures. *(Common Core Mathematics Companion, Pg. 180)*

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**

- **Proving Congruence** Students are asked to explain congruence in terms of rigid motions.
- **Rigid Motion 1** Students are asked to describe the motion and determine if the shapes are congruent. Translation
- **Rigid Motion II-Reflection** Describe a rigid motion to demonstrate two polygons are congruent.
- **Rigid Motion III** Describe a rigid motion to demonstrate two polygons are congruent.
- **Multistep Congruence** Describe a sequence of rigid motions to demonstrate the congruence of two polygons.

#### Illustrative Mathematics

- **Cutting a Rectangle Into Two** Shows the congruence of two triangles in a particular geometric context arising by cutting a rectangle in half along the diagonal.
- **Congruent Triangles** Develop an understanding of rigid motions in the context of demonstrating congruence and reflections refined by orientation.

#### Engaging Tasks

- **Naming Coordinates, Feedback and Revision** Revise a student error involving translating a quadrilateral
- **Best Reflection** Students compare 4 images with their reflection to determine which one is the best.
- **Transformations** - Shortest Sequence What’s the fewest number of transformations needed to take pre-image ABCT to A’B’C’D’.
- **Transformations – Three Sequences** List three sequences of transformations that take pre-image ABCT to image A’B’C’D’

### Lesson Resources

**Engage NY**

- **Grade 8, Module 2, Topic B, Lesson 10** Mapping one figure onto another
- **Grade 8, Module 2, Topic C Lesson 11** Congruence through Rigid Motion

**Desmos**

- **Polygraph: Transformations** Designed to spark vocabulary-rich conversations about transformation.
- **Polygraph: Translations** Designed to spark vocabulary-rich conversations about translations.
- **Laser Challenge** Use angles to adjust lasers and mirrors as they seek to hit all three targets in a series of challenges.

**CPalms**

- **Polygon Transformers** Introduces students to the concept that congruent polygons can be formed using a series of transformations. As a culminating activity, students will create a robot out of transformed figures.

**MARS/Shell**

- **Transforming 2D Figures** Describe in words the transformation that maps an object to a transformed image. Given a geometric figure and a rotation, reflection or translation, draw the transformed figure (or the original figure if the image is given.) Describe transformations as algebraic functions that take points in the plane as inputs and give other points as outputs.

**McGraw-Hill**

- **Course 3, Chapter 7 Lesson 2**

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### Decoded Standard

**MAFS.8.G.1.3**

Students continue looking at two-dimensional figures on the coordinate plane, concentrating on the coordinates of the resulting figure after transformations, including dilations learned in Grade 7. *(Common Core Mathematics Companion, Pg. 181)*

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**

- **Translation Coordinates** Two problems both require students to graph a two-dimensional figure’s translation and identify the new coordinates.
- **Rotation Coordinates** Two problems both require students to graph a two-dimensional figure’s rotation and identify the new coordinates.

#### Lesson Resources

**Engage NY**

- **Grade 8, Module 3, Topic A, Lesson 6** Dilations using Coordinates

**Desmos**

- **Blue Point** Observe a red point transform into a blue point by way of a mystery transformation. Write about that transformation
Pinellas County Schools  ACCELERATED MATH GRADE 7  2020-2021

- **Reflection Coordinates** Two problems both require students to graph a two-dimensional figure’s reflection and identify the new coordinates.

**Engaging Tasks**
- **How do Skytypers Write Messages?** Use transformation applications to create skytyping messages and translate it into a set of coordinates.
- **Naming Coordinates, Feedback and Revision** Revise a student error involving translating a quadrilateral.
- **Transformations** – Shortest Sequence What’s the fewest number of transformations needed to take pre-image ABCT to A’B’C’D’.

**MARS/Shell**
- **Representing and Combining Transformations** Students will recognize and visualize transformations of 2D shapes. They will translate, reflect and rotate shapes, and combine these transformations.

**McGraw-Hill**
- **Course 3, Chapter 6**
  - Lesson 1 (Translations), Lesson 2 (Reflections), Lesson 3 (Rotations) and Lesson 4 (Dilations)

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**Decoded Standard**

**MAFS.8.G.1.4**
With this standard, students move from congruence to similarity. Students develop the understanding that similar figures can be created by a series of transformations, including rotations, reflections, dilations, and translation, and can identify those transformations given an image and a pre-image.

*See image on page 182 of the Common Core Mathematics Companion.*

*(Common Core Mathematics Companion, Pg. 182)*

**Instructional Resources**

**Formative Tasks**

**Mathematics Formative Assessments (MFAS)**
- **Proving Similarity** Explain similarity in terms of transformations.
- **Similarity I** Describe a sequence of transformations to show that two polygons are similar.
- **Similarity II** Describe a sequence of transformations to show that two polygons are similar.
- **Similarity III** Describe a sequence of transformations that demonstrates two polygons are similar.

**Illustrative Mathematics**
- **Are they Similar?** Provide experience applying transformations to show that two polygons are similar.
- **Creating Similar Triangles** Provide experience applying transformations to show that two polygons are similar.

**Engaging Tasks**
- **Right Triangles – Trapezoids** What question comes to mind for the given image?

**Lesson Resources**

**Engage NY**
- **Grade 8, Module 2, Topic B, Lesson 7** Sequencing transformations that enjoy the same properties as a single translation with respect to lengths of segments and angle degrees.
- **Grade 8, Module 3, Topic B, Lesson 8** Sequence of Transformations that lead to Similarity

**CPalms**
- **Dilly Dally with Dilations** Students will understand the concept of dilation by constructing similar polygons on a coordinate grid using coordinate notation of dilation.

**McGraw-Hill**
- **Course 3, Chapter 7**
  - Lesson 4

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**Decoded Standard**

**MAFS.8.G.1.5**
Students are expected to make informal arguments while exploring facts about the sum of the angles of a triangle, exterior angles of triangles, angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similar triangles. The example demonstrates how these facts are interrelated. Note that formal two-column proofs are not expected at this grade.

*See image on page 184 of the Common Core Mathematics Companion.*

*(Common Core Mathematics Companion, Pg. 184)*

**Instructional Resources**
### Formative Tasks

#### Mathematics Formative Assessments (MFAS)
- **Same Side Interior Angles** Describe and justify the relationship between same side interior angles.
- **Justifying Angle Relationships** Describe and justify the relationship between corresponding angles and alternate interior angles.
- **Justifying the Exterior Angle Theorem** Apply the Exterior Angle of a Triangle Theorem and provide an informal justification.
- **What is the Triangle Relationship?** Describe the relationship between two triangles given that two pairs of corresponding angles are congruent, and provide an informal justification of the relationship (similarity).
- **Justifying the Triangle Sum Theorem** Provide an informal justification of the Triangle Sum Theorem.

#### Illustrative Mathematics
- **Rigid motions and congruent angles** Given parallel lines cut by a transversal, prove congruence between angle pairs.
- **A Triangle’s Interior Angles** Given parallel lines with a triangle drawn with its transversals, prove that \(a+b+c = 180\).
- **Find the Missing Angle** Find the measure of a missing angle between parallel lines.
- **Congruence of Alternate Interior Angles via Rotations** Explain why rotating a pair of parallel lines cut by a transversal demonstrates that angles are congruent.
- **Street Intersections** Apply facts about angles in order to calculate angle measures in the context of a map.

### Engage NY
- **Grade 8, Module 2, Topic C, Lesson 12** Angle Relationships of Parallel Lines
- **Grade 8, Module 2, Topic C, Lesson 13** Angle Sum Theorem Triangles
- **Grade 8 Module 2, Topic C, Lesson 14** Missing Angle Measures Triangles

### Desmos
- **Lines, Transversals, and Angles** Explore the relationship among angles formed by a transversal and a system of two lines, in particular, when the two lines are parallel vs when they are not.
- **Polygraph: Figure It Out** Introduces geometric notation and vocabulary. Uses these figures: points, lines, rays, segments, parallel, perpendicular, angles, congruence, midpoints, bisectors, betweenness, collinearity, and more.

### CPalms
- **Help me Find my Relationship!** Students will identify angles, find angle measures, and they will use the free application GeoGebra (see download link under Suggested Technology) to provide students with a visual representation of angles relationships.
- **Special Angle Pairs Discovery Activity** Students identify angle pairs and the relationship between the angles.

### Mcgraw-Hill
- **Course 3, Chapter 7, Lesson 6**

### Decoded Standard

**MAFS.8.EE.2.6**

Students gain additional knowledge about slope in this standard as they use similar triangles to explain how the slope \(m\) of a line is the same between any two points on a given non-vertical line. Students understand positive/negative slopes, 0 slope, and undefined slopes. Through the use of similar triangles, teachers lead students to derive the general equation \((y = mx + b)\) of a line and discover that \(m\) is the slope and \(b\) is the \(y\)-intercept. (Common Core Mathematics Companion, Pg. 124)

### Instructional Resources

#### Formative Tasks
- **Slope with similar Triangles** Use similar triangles to explain why the slope is the same regardless of the points used to calculate it.
- **Deriving Lines I- Using the slope formula** Derive the general equation of a line containing the origin.
- **Deriving Lines II Using the slope formula** Derive the general equation of a line with a \(y\)-intercept of \((0, b)\).

#### Illustrative Mathematics
- **Slopes between points on a line** Understand why the calculated slope will be the same for any two points on a given line.

#### Engage NY
- **Grade 8, Module 4, Topic C, Lesson 16** Use similar triangles to explain slope and calculate the slope between two distinct points on a non-vertical line.

#### CPalms
- **Designing a Skateboard Kicker Ramp** Real life application. Students design a skateboard kicker ramp.

#### Mcgraw-Hill
- **Course 3, Chapter 7, Lesson 6**
# ACCELERATED MATH GRADE 7

## Semester 2

### Unit 12: Volume

<table>
<thead>
<tr>
<th>Standards/Learning Goals:</th>
<th>Content Limits, Assessment Types, Calculator</th>
</tr>
</thead>
</table>
| **MAFS.8.G.3.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. | • Graphics of three-dimensional figures can be included.  
• Dimensions must be given as rational numbers.  
• Figures must not be composite  
Calculator: **YES**  
Context: **ALLOWABLE** |

## Open Up Resources Lessons

*Grade 8, Unit 5: Functions and Volume*

### Decoded Standard

**MAFS.8.G.3.9**  
This standard has two distinct parts. First, students learn the volume formulas for cones, cylinders, and spheres. Then they apply this knowledge to solve real-world and mathematical problems. The formulas should be taught through experiments where students figure out the formulas. (*Common Core Mathematics Companion*, Pg. 190)

### Instructional Resources

#### Formative Tasks

<table>
<thead>
<tr>
<th>Mathematics Formative Assessments (MFAS)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cone Formula</strong> Write the formula for the volume of a cone, explain what each variable represents, and label the variables on a diagram.</td>
<td></td>
</tr>
<tr>
<td><strong>Cylinder Formula</strong> Write the formula for the volume of a cylinder, explain what each variable represents, and label the variables on a diagram.</td>
<td></td>
</tr>
<tr>
<td><strong>Sphere Formula</strong> Write the formula for the volume of a sphere, explain what each variable represents, and label the variables on a diagram.</td>
<td></td>
</tr>
<tr>
<td><strong>Sugar Cone</strong> Solve a problem that requires calculating the volume of a cone.</td>
<td></td>
</tr>
<tr>
<td><strong>Platinum Cylinder</strong> Solve a problem that requires calculating the volume of a cylinder.</td>
<td></td>
</tr>
<tr>
<td><strong>Burning Sphere</strong> Solve a problem that requires calculating the volume of a sphere.</td>
<td></td>
</tr>
</tbody>
</table>

#### Illustrative Mathematics Assessment Tasks

<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Comparing Snow Cones</strong> Find the volume of a cone.</td>
</tr>
<tr>
<td><strong>Glasses</strong> Use volume formulas for cylinders, cones and spheres.</td>
</tr>
<tr>
<td><strong>Flower Vases</strong> Use volume formulas for cylinders, cones and spheres.</td>
</tr>
</tbody>
</table>

#### Engaging Tasks

<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Coca Cola Pool</strong> How many bottles of Coca Cola did they buy to fill up the pool.</td>
</tr>
<tr>
<td><strong>Guatemalan Sinkhole</strong> How much material will they need to fill the sinkhole</td>
</tr>
<tr>
<td><strong>Penny Wars</strong> Which container is worth the most money?</td>
</tr>
<tr>
<td><strong>How Many Gumballs Fit In the Gumball Machine?</strong> Calculate the volume of the sphere with and without its inner globe to determine its capacity.</td>
</tr>
</tbody>
</table>

#### Lesson Resources

<table>
<thead>
<tr>
<th>Engage NY</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Grade 8, Module 5, Topic B, Lesson 10</strong> Volume of Cylinders and Cones; Solve real-world volume problems</td>
<td></td>
</tr>
<tr>
<td><strong>Grade 8, Module 5, Topic B, Lesson 11</strong> Volume of Spheres; Solve real-world volume problems</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NCTM Illuminations</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Popcorn Anyone?</strong> Construct objects and determine the resulting volume.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>YouTube</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volume of a Cylinder</strong> Tutorial Video</td>
<td></td>
</tr>
<tr>
<td><strong>Volume of a Cone</strong> Tutorial Video</td>
<td></td>
</tr>
<tr>
<td><strong>Volume of a Sphere</strong> Tutorial Video</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>McGraw-Hill</th>
<th></th>
</tr>
</thead>
</table>
| **Course 3, Chapter 8**  
Lesson 1 (Cylinders), Lesson 2 (Cones – skip ex. 3), and Lesson 3 (Spheres – skip ex. 4) |  |
Standards/Learning Goals:

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAFS.8.SP.1.1</td>
<td>Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.</td>
</tr>
<tr>
<td>MAFS.8.SP.1.2</td>
<td>Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.</td>
</tr>
<tr>
<td>MAFS.8.SP.1.3</td>
<td>Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope as an intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.</td>
</tr>
<tr>
<td>MAFS.8.SP.1.4</td>
<td>Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?</td>
</tr>
</tbody>
</table>

Content Limits, Assessment Types, Calculator

- Numbers in items must be rational numbers.
- Trend/association is based on visual inspection.
- Line of best fit must be informally assessed.
- Trend/association must be linear.
- Calculator: NEUTRAL
- Context: ALLOWABLE

Open Up Resources Lessons

Grade 8, Unit 6: Associations in Data

Decoded Standard

MAFS.8.SP.1.1

Students study scatter plots of bivariate data by constructing and interpreting them in terms of patterns they can see. They look for the patterns of clustering, outliers, positive or negative association, and linear or nonlinear association. Examples of scatter plots below show positive and negative associations, clustering, and an outlier.

See the image on page 238 of the Common Core Mathematics Companion (Common Core Mathematics Companion, Pg. 238)

Instructional Resources

Formative Tasks

- **Sleepy Statistics** Describe the association between scores on the Epworth Sleepiness Scale and scores on the math test.
- **Population Density** Describe the relationship between population and land area.
- **Infectious Statistics** Describe the association between the passage of time and the number of bacteria.

Lesson Resources

- **Engage NY**
  - Grade 8, Module 6, Topic B, Lesson 6
  - Constructing Scatter Plots
  - Grade 8, Module 6, Topic B, Lesson 7
  - Patterns in Scatter Plots
  - Grade 8, Module 6, Topic C, Lesson 11
  - Scatter plots; Fit line to data; Interpret slope
**Cheesy Statistics** Describe the association between time spent watching advertisements and the percent of each group willing to buy the company’s cheese crackers.

**Bungee Cord Data** Construct a scatterplot corresponding to a given set of data.

**Illustrative Mathematics Assessment Tasks**
- **Birds’ Eggs** Identify a correlation and use it to make interpolative predictions.
- **Texting and Grades I** Describe the relationship between number of text messages sent and GPA.

**Engaging Tasks**
- **Positive Correlation** Create a set of points that have specific characteristics
- **Interpreting Graphs Where Up Isn’t Good** Interpret data on a scatterplot that appears unconventional.

### Decoded Standard

**MAFS.8.SP.1.2**

Students focus on linear patterns of association in scatter plots and understand that linear models (straight lines) are commonly used to model linear relationships. Then they begin to informally fit a straight line to the data and learn to assess its fit by judging the closeness of the line to the data points. The most appropriate line is the one that comes closest to most data points. The use of linear regression is not expected at this grade. *(Common Core Mathematics Companion, Pg. 239)*

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**
- **Two Scatterplots** Compare how well each line fits its set of data. Explain your reasoning.
- **Three Scatterplots** (Informally assess three lines fitted to data to determine which fit is the best.)
- **Line of Good Fit I** Fit a line to model the relationship between two quantitative variables and to assess how well that line fits the data.
- **Line of Good Fit II** See description above.

**Illustrative Mathematics Assessment Tasks**
- **Hand Span and Height** Construct and Interpret Scatter plots by generating and recording data.
- **Animal Brains** Create scatterplots, and think critically about associations and outliers in data as well as informally fit a trend line to data.
- **Laptop Battery Charge** Find and use a linear model answer this question.

#### Engaging Tasks
- **Line of Best Fit** Create 4 points that could generate a line of best fit with the equation y=-x+8.

**Desmos**
- **Polygraphs: Scatterplots** This Custom Polygraph is designed to spark vocabulary-rich conversations about scatter plots.

**McGraw-Hill**
**Course 3, Chapter 9**
Lesson 1

**Engage NY**
- **Grade 8, Module 6, Topic B, Lesson 8** Informally fit a line to data in scatter plot
- **Grade 8, Module 6, Topic C, Lesson 9** Informally fit a line to data in scatter plot
- **Grade 8, Module 6, Topic C, Lesson 11** Scatter plots; Fit line to data; Interpret slope

**Illuminations**
- **Barbie Bungee** Model a bungee jump using a Barbie® doll and rubber bands.
- **Bouncing Tennis Ball** Collecting and recording data using the real-world situation of a bouncing tennis ball.

**Desmos**
- **Line of best fit** Visualize a line to fit a data set, then graph that line with sliders, and use it to make a prediction.

**CPalms**
- **Scattered Data** Construct and Interpret Scatter plots by generating and recording data.
- **Scatterplot Virtual Manipulative** Use manipulatives to help understand scatter plots.

**McGraw-Hill**
**Course 3, Chapter 9**
Lesson 2
**Decoded Standard**

### MAFS.8.SP.1.3

Students practice solving contextual linear problems. The problems involve situations using bivariate measurement data such as those collected in a biology experiment. This standard connects with what students have learned about models of linear equations, slope, and intercept. *(Common Core Mathematics Companion, Pg. 240)*

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**

- **Tuition** Use this equation to predict the average tuition cost at a public university.
- **Stretching Statistics** Explain the significance of a point and the y-intercept.
- **Foot Length** Interpret the slope and intercept of a linear function that models the relationship between foot length and height.
- **Developmental Data** Explain the significance of the slope of the equation in terms of a problem’s context.

#### Illustrative Mathematics Assessment Tasks

- **US Airports, Assessment Variation** Use a linear function to model a relationship between two quantities.
- **Chicken and Steak, Variation 1** Presents a real world situation that can be modeled with a linear function best suited for an instructional context.

#### Lesson Resources

**Engage NY**

- **Grade 8, Module 6, Topic C, Lesson 10** Interpret slope and initial value
- **Grade 8, Module 6, Topic C, Lesson 11** Scatter plots; Fit line to data; Interpret slope

**Illuminations**

- **Line of Best Fit Virtual Manipulative** (User enters a set of data, plot the data on a coordinate grid, and determine the equation for a line of best fit.)

**McGraw-Hill**

- **Course 3, Chapter 9 Lesson 2**

---

**Decoded Standard**

### MAFS.8.SP.1.4

This standard asks students to switch from using numerical data to categorical data and use frequencies to answer questions about possible associations (linear/nonlinear, positive/negative/no association). Students construct and interpret tables that display categorical data on two different variables from the same subjects. A two-way table is a table that shows categorical data classified in two different ways. An example of a two-way table that records possible data from the example in the standard about chores and curfews may be the following:

<table>
<thead>
<tr>
<th>CHORES</th>
<th>CURFEW</th>
</tr>
</thead>
<tbody>
<tr>
<td>YES</td>
<td>44</td>
</tr>
<tr>
<td>NO</td>
<td>20</td>
</tr>
</tbody>
</table>

One interpretation of the chart is that of the students who answered yes, they had a curfew, 44 had chores and 20 did not. Of the students who answered no, they did not have a curfew, 20 had chores and 44 did not. From this sample, there appears to be a positive correlation between having a curfew and having chores. *(Common Core Mathematics Companion, Pg. 214)*

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**

- **Two-Way Relative Frequency Table** Convert raw data to relative frequencies by both rows and columns given a two-way frequency table.
- **School Start Time** Interpret data given in a two-way table.
- **Music and Sports** Construct a two-way frequency table given a set of raw data.

**MARS/Shell**

#### Lesson Resources

**Engage NY**

- **Grade 8, Module 6, Topic D, Lesson 13** Two-way Tables; Row and Column Relative Frequencies
- **Grade 8, Module 6, Topic D, Lesson 14** Association between Two Categorical Values
**Illustrative Mathematics Assessment Tasks**

- **Sibling and Pets** Interpret data given in a two-way table.
- **What’s Your Favorite Subject?** Calculate appropriate relative frequencies using the given data.
- **Music and Sports** Investigate the association between whether a student plays a sport and whether he or she plays a musical instrument.
- **Testing a New Product** Assess how well students are able to organize, represent and analyze bivariate categorical data in an appropriate way.

**McGraw-Hill**
Course 3, Chapter 9
Lesson 3
### Number and Quantity: Reason, describe, and analyze quantitatively, using units and number systems to solve problems.

<table>
<thead>
<tr>
<th>Performance Indicators</th>
<th>Emerging</th>
<th>Progressing</th>
<th>Meets</th>
<th>Exceeds</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>G. Students will know that there are numbers that are not rational, and approximate them by rational numbers</strong> [8.NS.1.1, 8.NS.1.2]</td>
<td>i. Students can find the square roots of small perfect squares.</td>
<td>i. Students can identify square roots of non-square numbers and pi as irrational numbers; identify rational or irrational numbers and convert familiar rational numbers with one repeating digit to fraction form.</td>
<td>i. Students can place irrational numbers on a number line; identify irrational decimal expansions as approximations; identify rational and irrational numbers and convert less familiar rational numbers to fraction form.</td>
<td>i. Students can use approximations of irrational numbers to estimate the value of an expression; compare and order rational and irrational numbers without a number line.</td>
</tr>
</tbody>
</table>

### Algebra: Create, interpret, use, and analyze expressions, equations and inequalities.

<table>
<thead>
<tr>
<th>Performance Indicators</th>
<th>Emerging</th>
<th>Progressing</th>
<th>Meets</th>
<th>Exceeds</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>E. Students will solve real-life and mathematics problems using numerical and algebraic expressions and equations.</strong> [7.EE.2.3, 7.EE.2.4]</td>
<td>i. Students can solve mathematical problems posed with whole numbers.</td>
<td>i. Students can solve mathematical problems posed with positive rational numbers.</td>
<td>i. Students can solve multistep and real-world problems posed with rational numbers, using tools strategically; apply properties of operations, conversions between forms and assesses the reasonableness of answers.</td>
<td>i. Students can create a model using rational numbers using tools strategically and can justify a solution and/or analyze errors in a real-world problem.</td>
</tr>
</tbody>
</table>

**F. Students will work with radicals and integer exponents.** [8.EE.1.1, 8.EE.1.2, 8.EE.1.3, 8.EE.1.4]  
- i. Students can identify numbers in scientific notation as being a very large number (positive exponent) or a very small number (negative exponent).  
- ii. Students can evaluate square roots and solve mathematical equations in the form \( x^2 = p \), where \( p \) is a positive  
- i. Students can apply the properties of natural number exponents to generate equivalent numerical expressions.  
- ii. Students can use square root and cube root symbols to represent solutions to mathematical equations in the  
- i. Students can use multiple properties of integer exponents within an expression; analyze the reasonableness of the result of using the properties of integer exponents.  
- ii. Students can write and solve equations representing real-
### MS Math Scoring Criteria (Grade 7 Math Advanced)

#### i. Students can graph a proportional relationship given a table.

#### ii. Students can identify the slope of a line when given an equation in slope-intercept form.

#### iii. Students can determine the slope of a line given a graph.

#### iv. Students can explain, using similar triangles, why the slope is the same between any two distinct points on a nonvertical line in the coordinate plane; derive the equation $y = mx + b$ for a line intercepting the vertical axis at $b$; compare and contrast situations in which similar triangles would or would not yield the same slope between two distinct points on a nonvertical line in world situations using square root and cube root symbols; justify how square roots and cube roots relate to each other and to their radicands.

#### iii. Students can use numbers expressed in the form of a single digit times an integer power of 10 to express very large numbers.

#### iv. Students can represent very large and very small quantities in scientific notation and use units of appropriate size for measurements of very large or very small quantities.

#### Students can use numbers expressed in the form of a single digit times an integer power of 10 to express very small numbers.

#### iv. Students can perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used; interpret scientific notation generated by technology.

#### Students can perform operations and interpret values written in scientific notation within a real-world context; analyze the process and solution to given problems using scientific notation.

#### Students can graph proportional relationships, interpreting the unit rate as the slope.

#### ii. Students can determine the slope of a line given a graph.

#### i. Students can identify the unit rate as the slope; compare two different proportional relationships represented in different ways.

#### ii. Students can explain, using similar triangles, why the slope is the same between any two distinct points on a nonvertical line in the coordinate plane; derive the equation $y = mx + b$ for a line intercepting the vertical axis at $b$; compare and contrast situations in which similar triangles would or would not yield the same slope between two distinct points on a nonvertical line in world situations using square root and cube root symbols; justify how square roots and cube roots relate to each other and to their radicands.

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#### ii. Students can determine the slope of a line given a graph.

#### i. Students can identify the unit rate as the slope; compare two different proportional relationships represented in different ways.

#### ii. Students can explain, using similar triangles, why the slope is the same between any two distinct points on a nonvertical line in the coordinate plane; derive the equation $y = mx + b$ for a line intercepting the vertical axis at $b$; compare and contrast situations in which similar triangles would or would not yield the same slope between two distinct points on a nonvertical line in world situations using square root and cube root symbols; justify how square roots and cube roots relate to each other and to their radicands.

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#### iv. Students can represent very large and very small quantities in scientific notation and use units of appropriate size for measurements of very large or very small quantities.

#### Students can use numbers expressed in the form of a single digit times an integer power of 10 to express very small numbers.

#### iv. Students can perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used; interpret scientific notation generated by technology.

#### Students can perform operations and interpret values written in scientific notation within a real-world context; analyze the process and solution to given problems using scientific notation.

#### Students will understand the connections between proportional relationships, lines, and linear equations. [8.EE.2.5, 8.EE.2.6]

#### i. Students can graph a proportional relationship given a table.

#### ii. Students can identify the slope of a line when given an equation in slope-intercept form.

#### i. Students can graph proportional relationships, interpreting the unit rate as the slope.

#### ii. Students can determine the slope of a line given a graph.

#### i. Students can identify the unit rate as the slope; compare two different proportional relationships represented in different ways.

#### ii. Students can explain, using similar triangles, why the slope is the same between any two distinct points on a nonvertical line in the coordinate plane; derive the equation $y = mx + b$ for a line intercepting the vertical axis at $b$; compare and contrast situations in which similar triangles would or would not yield the same slope between two distinct points on a nonvertical line in world situations using square root and cube root symbols; justify how square roots and cube roots relate to each other and to their radicands.

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| H. Students will analyze and solve linear equations and pairs of simultaneous linear equations. [8.EE.3.7, 8.EE.3.8] | i. Students can use substitution with an equation or pair of equations and a data set to determine if any number(s) from the data set makes the equation true. | i. Students can solve linear equations with integer coefficients and variables on one side. ii. Students can interpret mathematical or real-world problems given the graph, of a system of two linear equations in two variables. | i. Students can solve multistep linear equations in one variable with rational coefficients using the distributive property or collecting like terms on a given side; identify linear equations as having solution of one, infinitely many, or none by transforming the given equation into simpler forms by inspection. ii. Students can solve mathematical and real-world systems of two linear equations in two variables with integer coefficients by inspection, algebraically by multiplying only one of the equations by an integer. | i. Students can justify why an equation has one solution, infinitely many solutions, or no solution; create examples of equations that have one solution, infinitely many solutions, or no solution. ii. Students can solve and analyze a system of equations in two variables with integer and benchmark fraction coefficients; solve and analyze problems involving two linear equations in two variables with rational coefficients or constants. |

Functions: Use various forms of functions to interpret and analyze a variety of contexts.

<table>
<thead>
<tr>
<th>Scoring Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance Indicators</td>
</tr>
<tr>
<td>A. Students will define, evaluate, and compare functions. [8.F.1.1, 8.F.1.2, 8.F.1.3]</td>
</tr>
</tbody>
</table>
## MS Math Scoring Criteria (Grade 7 Math Advanced)

### B. Students will use functions to model relationships between quantities. [8.F.2.4, 8.F.2.5]

<p>| | | |</p>
<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>i. Students can determine the rate of change given points on a coordinate plane.</td>
<td>ii. Students can describe qualitatively the functional relationship between two quantities by analyzing some features of a graph to be linear and nonlinear.</td>
<td>i. Students can interpret the rate of change and initial value of a linear function in terms of a verbal description of the linear function; analyze a set of values in either a table or graph to determine changes to be made to make the relationship linear.</td>
</tr>
<tr>
<td>i. Students can determine the rate of change from two ((x, y)) values or from a graph.</td>
<td>ii. Students can interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values; construct a function to model a linear relationship between two quantities.</td>
<td>i. Students can interpret the rate of change and initial value of a linear function in terms of a verbal description of the linear function; analyze a set of values in either a table or graph to determine changes to be made to make the relationship linear.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>i. Students can interpret the rate of change and initial value of a linear function in terms of a verbal description of the linear function; analyze a set of values in either a table or graph to determine changes to be made to make the relationship linear.</td>
</tr>
</tbody>
</table>
Geometry: Understand geometric concepts and constructions, prove theorems, and apply appropriate results to solve problems.

### Scoring Criteria

<table>
<thead>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>B. Students will solve real-life and mathematics problems using numerical and algebraic expressions and equations.</strong> [7.G.1.1, 7.G.1.2, 7.G.1.3]</td>
<td>i. Students can compute scale factor from given lengths of 2 related geometric figures. ii. Students can identify and draw 2-dimensional figures.</td>
<td>i. Students can compute actual lengths given a geometric figure and a scale factor and find actual lengths given two geometric figures with some unknown side measure. ii. Students can draw polygons with given conditions. iii. Students can identify the 2-dimensional figure that results from a vertical or horizontal cut of a right rectangular prism or a right rectangular pyramid.</td>
<td>i. Students can compute actual lengths and areas from a scale drawing and reproduces a scale drawing using a different scale. ii. Students can construct geometric shapes given a combination of angle and side conditions; notices when conditions determine a unique triangle, more than one triangle, or no triangle. iii. Students can identify 2-dimensional figures that result from a vertical or horizontal cut of a 3-dimensional figure.</td>
<td>i. Students can solve problems involving scaled drawing of 2-dimensional geometric figures by creating a drawing and finding the appropriate scale. ii. Students can explain or analyze and justify the conditions of a unique triangle, more than one triangle, or no triangle. iii. Students can describe and/or draw the 2-dimensional figure that results from a vertical or horizontal slice of a 3-dimensional figure.</td>
</tr>
<tr>
<td><strong>C. Students will solve real-life and mathematics problems using numerical and algebraic expressions and equations.</strong> [7.G.2.4, 7.G.2.5, 7.G.2.6]</td>
<td>i. Students can determine the radius and/or diameter of a circle. ii. Students can identify supplementary, complementary, vertical, and adjacent angles. iii. Students can find the area of right triangles,</td>
<td>i. Students can identify the formula for the area and/or circumference of a circle. ii. Students can use facts about relationships (supplementary, complementary, vertical, and adjacent) to find the unknown angle.</td>
<td>i. Students can use the formulas and solve problems for the area and circumference of a circle given radius or diameter, or vice versa, given a graphic representation in a real-world context. ii. Students can use facts about angle</td>
<td>i. Students can use the relationship between circumference and area of a circle; use formulas and solve real-world problems without requiring graphic representations. ii. Students can find the measures of the</td>
</tr>
<tr>
<td>D. Students will understand congruence and similarity using physical models, transparencies, or geometry software. [8.G.1.1, 8.G.1.2, 8.G.1.3, 8.G.1.4, 8.G.1.5]</td>
<td>i. Students can define/explain the terms translation, rotation, reflection, and dilation.</td>
<td>i. Students can describe a rigid transformation between two congruent figures that exhibit the congruence between them.</td>
<td>i. Students can describe a sequence of up to two rigid transformations between two congruent figures.</td>
<td>i. Students can use properties of rigid and non-rigid transformations to understand the relationship between transformations and congruence.</td>
</tr>
<tr>
<td>squares, and rectangles.</td>
<td>iii. Students can find the area of triangles, quadrilaterals, and regular polygons; find the volume of cubes and right prisms.</td>
<td>iii. Students can write and solve multistep equations for an unknown angle in a figure.</td>
<td>iii. Students can solve real-world problems involving area of 2-dimensional figures composed of triangles, quadrilaterals, and polygons; solve real-world volume and surface area problems for cubes and right prisms.</td>
<td>iii. Students can describe the effect of up to two rigid transformations on two-dimensional figures using coordinates; describe the effect of two transformations, including at least one dilation, on two-dimensional figures using coordinates and coordinate notation.</td>
</tr>
<tr>
<td>measure in a figure.</td>
<td>relationships to write and solve multistep equations for an unknown angle in a figure.</td>
<td>students can solve real-world problems involving surface area and volume of composite figures; use relationships between volume and surface area of 3-dimensional shapes to solve real-world problems.</td>
<td>unknown angles in a figure.</td>
<td>iii. Students can identify a sequence of transformations and a dilation that results in similarity.</td>
</tr>
</tbody>
</table>
### MS Math Scoring Criteria (Grade 7 Math Advanced)

**E. Students will understand and apply the Pythagorean Theorem.**  
[8.G.2.6, 8.G.2.7, 8.G.2.8]

<table>
<thead>
<tr>
<th>Performance Indicators</th>
<th>Emerging</th>
<th>Progressing</th>
<th>Meets</th>
<th>Exceeds</th>
</tr>
</thead>
</table>
| i. Students can recall the equation for the Pythagorean Theorem. | i. Students can use the Pythagorean Theorem and apply to right triangles.  
ii. Students can calculate hypotenuse length using the Pythagorean Theorem, given a picture of a right triangle or the lengths of the two legs. | i. Students can model and explain the proof of the Pythagorean Theorem and its converse using a pictorial representation.  
ii. Students can calculate unknown side lengths using the Pythagorean Theorem; apply the Pythagorean Theorem to find the distance between two points in a coordinate system.  
ii. Students can find multiple leg lengths given a hypotenuse of an isosceles triangle or find multiple leg lengths when two triangles with the same hypotenuse are given; apply the Pythagorean Theorem in multistep problems; find the coordinates of a point which is a given distance (nonvertical and nonhorizontal) from another point. | i. Students can apply the Pythagorean Theorem to a real-world situation in two and three dimensions to determine unknown side lengths or the distance between two points in a coordinate system. | results in similarity. |

**F. Students will solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**  
[8.G.3.9]

<table>
<thead>
<tr>
<th>Performance Indicators</th>
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<th>Progressing</th>
<th>Meets</th>
<th>Exceeds</th>
</tr>
</thead>
<tbody>
<tr>
<td>i. Students can identify three dimensional figures as cones, cylinders or spheres.</td>
<td>i. Students can recall the formulas for finding the volume of cones, cylinders and spheres.</td>
<td>i. Students can use the formulas for the volume of cones, cylinders, and spheres to solve real-world and mathematical problems.</td>
<td>i. Students can explain and justify the relationship between formulas for the volume of cones, cylinders, or spheres; explain the derivation of the formulas for cones, cylinders, and spheres.</td>
<td>results in similarity.</td>
</tr>
</tbody>
</table>

### Statistics and Probability: Interpret and apply statistics and probability to analyze data, reach and justify conclusions, and make inferences.

<table>
<thead>
<tr>
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<th>Meets</th>
<th>Exceeds</th>
</tr>
</thead>
<tbody>
<tr>
<td>C. Students will use random sampling to draw inferences</td>
<td>i. Students can define random sample.</td>
<td>i. Students can identify that a random sample produces the most valid</td>
<td>i. Students can use statistical data to draw inferences about a population based</td>
<td>i. Students can generate and/or use multiple samples to gauge variations</td>
</tr>
<tr>
<td>D. Students will apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. [7.SP.2.3, 7.SP.2.4]</td>
<td>i. Students can find the measures of central tendency.</td>
<td>i. Students can use basic measures of central tendency to compare two different populations.</td>
<td>i. Students can use measures of central tendency and/or variability to draw comparisons about two different populations.</td>
<td>i. Students can use measures of variability for numerical data from random samples to draw comparative inferences about two populations in any context.</td>
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</tr>
<tr>
<td>E. Students will draw informal comparative inferences about two populations. [7.SP.3.5, 7.SP.3.6]</td>
<td>i. Students can define probability.</td>
<td>i. Students can identify that the probability of a chance event is a number between 0 and 1. ii. Students can make approximations of probability for a chance event.</td>
<td>i. Students can identify the probability of a chance event as equally likely or unlikely (0.5); represent the probability as a fraction, decimal, or percent. ii. Students can use the results of an experiment to make approximations of probability for an event; predict the approximate relative frequency given the probability.</td>
<td>i. Students can compare the probabilities of two or more events and justify the likelihood of each event. ii. Students can compare and connect the relative frequency of an event to the theoretical probability of the event; justify why the experimental probability approaches the theoretical probability as the relative frequency of an event increases.</td>
</tr>
<tr>
<td>F. Students will investigate chance processes to develop, use, and evaluate probability models.</td>
<td>i. Students can explain the difference between experimental and theoretical probability.</td>
<td>i. Students can determine and develop a theoretical probability model of a simple event; determine the sample space for compound events.</td>
<td>i. Students can design a simulation to generate frequencies for compound events; use observed frequencies to</td>
<td>i. Students can use observed frequencies to create a probability model for the data from a chance process</td>
</tr>
</tbody>
</table>
## MS Math Scoring Criteria (Grade 7 Math Advanced)

<table>
<thead>
<tr>
<th>[7.SP.3.7, 7.SP.3.8]</th>
<th>create a uniform probability model to determine theoretical probabilities of events.</th>
<th>where outcomes may not be uniform; compare probabilities from a model to observed frequencies; explain possible sources of any discrepancy.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ii. Students can compare and justify the experimental and theoretical probability in a given situation; compare different simulations of compound events to see which best predicts the probability.</td>
<td></td>
</tr>
</tbody>
</table>

### G. Students will understand congruence and similarity using physical models, transparencies, or geometry software. [8.G.1.1, 8.G.1.2, 8.G.1.3, 8.G.1.4, 8.G.1.5]

<table>
<thead>
<tr>
<th>Students can define/explain the terms translation, rotation, reflection, and dilation.</th>
<th>Students can describe a rigid transformation between two congruent figures that exhibit the congruence between them.</th>
</tr>
</thead>
<tbody>
<tr>
<td>i. Students can describe the effect of a reflection or translation on two-dimensional figures using coordinates.</td>
<td></td>
</tr>
<tr>
<td>ii. Students can describe a sequence of up to two rigid transformations between two congruent figures.</td>
<td></td>
</tr>
<tr>
<td>iii. Students can describe the effect of a dilation, translation, rotation, or reflection on two-dimensional figures using coordinates and coordinate notation.</td>
<td></td>
</tr>
<tr>
<td>iv. Students can describe the effect of two transformations, including at least one dilation, on two-dimensional figures using coordinates and...</td>
<td></td>
</tr>
<tr>
<td>v. Students can describe the effect of two transformations, including at least one dilation, on two-dimensional figures using coordinates and...</td>
<td></td>
</tr>
</tbody>
</table>
coordinate notation.
i. Students can describe a sequence of transformations and a dilation that results in similarity.