# Grade 7 Mathematics

## 1205040

**Instructional Resource:** McGraw-Hill: *Florida Math, Course 2, ©2015*

## Course Pacing

<table>
<thead>
<tr>
<th>Unit of Instruction</th>
<th># of Days</th>
<th>Dates of Instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Review: Essential 6th Grade Standards Review</td>
<td>10</td>
<td>8.27-9.10</td>
</tr>
<tr>
<td>Unit 1: Ratios and Proportional Reasoning</td>
<td>16</td>
<td>9.11-10.2</td>
</tr>
<tr>
<td>Intervention Days</td>
<td>3</td>
<td>10.5-10.7</td>
</tr>
<tr>
<td>Unit 2: Multi-Step Percent Problems</td>
<td>9</td>
<td>10.8-10.20</td>
</tr>
<tr>
<td><strong>Cycle 1 Assessment (Units 1-2)</strong></td>
<td>1</td>
<td>10/21 (10.7 – 10.23)</td>
</tr>
<tr>
<td>Intervention Days</td>
<td>3</td>
<td>10.22-10.27</td>
</tr>
<tr>
<td>Unit 3: Rational Numbers</td>
<td>11</td>
<td>10.28-11.11</td>
</tr>
<tr>
<td>Intervention Days</td>
<td>3</td>
<td>11.12-11.16</td>
</tr>
<tr>
<td>Unit 4: Expressions</td>
<td>7</td>
<td>11.17-12.2</td>
</tr>
<tr>
<td>Thanksgiving Break 11/21 – 11/29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intervention Days</td>
<td>3</td>
<td>12.3-12.7</td>
</tr>
<tr>
<td><strong>Midterm Exam (Units 1-4)</strong></td>
<td>1</td>
<td>12/14 – 12/18</td>
</tr>
<tr>
<td>Unit 5: Multi-Step Equations and Inequalities</td>
<td>13</td>
<td>1.4-1.28</td>
</tr>
<tr>
<td>Semester 1 Ends on 1/15/21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intervention Days</td>
<td>2</td>
<td>1.29-2.1</td>
</tr>
<tr>
<td>Unit 6: Geometric Figures</td>
<td>18</td>
<td>2.2-2.26</td>
</tr>
<tr>
<td>Intervention Days</td>
<td>2</td>
<td>3.1-3.2</td>
</tr>
<tr>
<td>Unit 7: Circumference, Area, Surface Area, and Volume of</td>
<td>13</td>
<td>3.3-3.26</td>
</tr>
<tr>
<td>Compound Figures</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spring Break is 3/13-3/21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intervention Days</td>
<td>3</td>
<td>3.29-3.31</td>
</tr>
<tr>
<td>Unit 8: Probability</td>
<td>9</td>
<td>4.5-4.15</td>
</tr>
<tr>
<td>Intervention Days</td>
<td>3</td>
<td>4.16-4.19</td>
</tr>
<tr>
<td>Unit 9: Statistics</td>
<td>10</td>
<td>4.20-5.3</td>
</tr>
<tr>
<td><strong>FSA Grade 7 Math</strong></td>
<td>2</td>
<td>5/3 – 5/27</td>
</tr>
</tbody>
</table>
# Grade 7 Mathematics

## Unit 1: Ratios and Proportional Reasoning
- **MAFS.7.RP.1.1**
- **MAFS.7.RP.1.2**

## Unit 2: Multi-Step Percent Problems
- **MAFS.7.RP.1.3**

## Unit 3: Rational Numbers
- **MAFS.7.NS.1.1**
- **MAFS.7.NS.1.2**

## Unit 4: Expressions
- **MAFS.7.EE.1.1**
- **MAFS.7.EE.1.2**

## Midterm Review
- **Midterm Exam** (Units 1 - 4)
  - window: Dec. 14 - Dec. 18

## Unit 5: Multi-Step Equations & Inequalities
- Continues into Semester 2
- **MAFS.7.EE.2.3**
- **MAFS.7.EE.2.4**

## Unit 6: Geometric Figures

## Unit 7: Circumference, Area, Surface Area, and Volume of Compound Figures
- **MAFS.7.G.1.1**
- **MAFS.7.G.1.3**

## Unit 8: Probability
- **MAFS.7.SP.3.5**
- **MAFS.7.SP.3.7**

## Unit 9: Statistics
- **MAFS.7.SP.1.1**
- **MAFS.7.SP.2.3**

## Grade 7 FSA
- window: May 3 - May 28

## Binary Calendar

### August 2020
- Non-Student Day
- Non-Teacher Day
- **Building Community in the Math Classroom**
- **Essential 6th Grade Content Review**
- **MAFS.6.EE.1.3**
- **MAFS.6.EE.2.7**
- **MAFS.6.EE.1.4**
- **MAFS.6.RP.1.2**
- **MAFS.6.EE.2.6**
- **MAFS.6.RP.1.3.a,b,c**

### September 2020
- **Unit 1: Ratios and Proportional Reasoning**
- **MAFS.7.RP.1.1**
- **MAFS.7.RP.1.3**
- **MAFS.7.RP.1.2**

### October 2020
- **Cycle 1 Assessment** (Units 1 - 2)
  - window: Oct. 12 - Oct. 23

### November 2020
- **Unit 3: Rational Numbers**
- **MAFS.7.NS.1.1**
- **MAFS.7.NS.1.3**
- **MAFS.7.NS.1.2**

### December 2020
- **Unit 4: Expressions**
- **MAFS.7.EE.1.1**
- **MAFS.7.EE.1.2**

### January 2021
- **Unit 5: Multi-Step Equations & Inequalities**
  - Continues into Semester 2
- **MAFS.7.EE.2.3**
- **MAFS.7.EE.2.4**

### February 2021
- **Unit 6: Geometric Figures**
- **MAFS.7.G.1.1**
- **MAFS.7.G.1.3**
- **MAFS.7.G.1.2**
- **MAFS.7.G.2.5**

### March 2021
- **Unit 7: Circumference, Area, Surface Area, and Volume of Compound Figures**
- **MAFS.7.G.2.4**
- **MAFS.7.G.2.6**

### April 2021
- **Unit 8: Probability**
- **MAFS.7.SP.3.5**
- **MAFS.7.SP.3.7**
- **MAFS.7.SP.3.6**
- **MAFS.7.SP.3.8**

### May 2021
- **Unit 9: Statistics**
- **MAFS.7.SP.1.1**
- **MAFS.7.SP.2.3**
- **MAFS.7.SP.1.2**
- **MAFS.7.SP.2.4**

### June 2021
- **Grade 7 FSA**
  - window: May 3 - May 28
<table>
<thead>
<tr>
<th>Standards/Learning Goals:</th>
<th>Content Limits, Calculator, Assessment Types</th>
</tr>
</thead>
</table>
| **MAFS.6.EE.1.3** Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3(2+x) to produce the equivalent expression 6+3x; apply the distributive property to the expression 24x+18y to produce the equivalent expression 6(4x+3y); apply properties of operations to y+y+y to produce the equivalent expression 3y. | • Positive rational numbers, values may include exponents.  
• Variables must be included in the expression.  
• For items using distribution, coefficients may be fractions before distribution but must be integer values after simplification. Only positive rational numbers may be distributed.  
Calculator: NO  
Context: ALLOWABLE |
| **MAFS.6.EE.1.4** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y+y+y and 3y are equivalent because they name the same number regardless of which number y stands for. | • Numbers in items must be positive rational numbers.  
• Variables must be included in the expression.  
Calculator: NO  
Context: NO CONTEXT |
| **MAFS.6.EE.2.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. | • Numbers in items should not require students to perform operations with negative numbers or result in answers with negative rational numbers.  
• Expressions must contain at least one variable.  
Calculator: NO  
Context: ALLOWABLE |
| **MAFS.6.EE.2.7** Solve real-world and mathematical problems by writing and solving equations of the form x+p=q and px=q for cases in which p, q, and x are all non-negative rational numbers. | • Numbers in items should not require students to perform operations with negative numbers or result in answers with negative rational numbers.  
• Items must be one-step linear equations with one variable.  
Calculator: NO  
Context: ALLOWABLE |
| **MAFS.6.RP.1.2** Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is \( \frac{3}{4} \) cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.” | • Items using the comparison of a ratio will use whole numbers.  
• Rates can be expressed as fractions, with "/" or with words.  
• Items may involve mixed units within each system (e.g. convert hours/min to seconds).  
• Context itself does not determine the order.  
• Name the amount of either quantity in terms of the other as long as one of the values is on unit.  
Calculator: NO  
Context: REQUIRED |
| **MAFS.6.RP.1.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.  
  a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.  
  b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?  
  c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. | • Rates can be expressed as fractions, with "/" or with words.  
• Items may involve mixed units within each system (e.g. convert hours/min to seconds).  
• Percent found as a rate per 100.  
• Quadrant I only for MAFS.6.RP.1.3a  
Calculator: NO  
Context: ALLOWABLE |
Instructional Focus: Students will review solving equations and expressions with positive rational numbers. (6 days)

- **MAFS.6.EE.1**
  - 6.EE.1.3 Generate equivalent expressions using the distributive property
  - 6.EE.1.4 Identify when expressions are equivalent

- **MAFS.6.EE.2**
  - 6.EE.2.6 Use variables to represent numbers and write expressions to solve problems
  - 6.EE.2.7 Write and solve one-step equations

Instructional Focus: Students will review ratio and proportional reasoning standards prior to starting the 7th grade unit on Unit 1 Ratios and Proportional Reasoning. (4 days)

- **MAFS.6.RP.1**
  - 6.RP.1.2 Understand unit rate and use unit rate language in context
  - 6.RP.1.3(a)(b)(c) Create ratio tables and generate equivalent ratios; solve unit rate problems; and find the percent of a quantity
Semester 1  |  Unit 1: Ratios and Proportional Reasoning  |  16 days: 9/11 – 10/2

<table>
<thead>
<tr>
<th>Standards/Learning Goals:</th>
<th>Content Limits, Calculator, Assessment Types</th>
</tr>
</thead>
</table>
| **MAFS.7.RP.1.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, area and other quantities measured in like or different units. | • The item stem must include at least one fraction.  
• Ratios may be expressed as fractions, with “:” or with words.  
• Units may be the same or different across the two quantities.  
Calculator: YES  
Context: ALLOWABLE |
| **MAFS.7.RP.1.2** Recognize and represent proportional relationships between quantities. | • Ratios should be expressed as fractions, with “:” or with words.  
• Units may be the same or different across the two quantities.  
Calculator: NEUTRAL  
Context: ALLOWABLE |
| a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.  
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.  
c. Represent proportional relationships by equations. For example, if total cost $t$ is proportional to the number $n$ of items purchased at a constant price $p$, the relationship between the total cost and the number of items can be expressed as $t = pn$.  
d. Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ were $r$ is the unit rate. | |
| **MAFS.7.RP.1.3** Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. | • Units may be the same or different across the two quantities.  
Calculator: YES  
Context: ALLOWABLE |

Open Up Resources Lessons

Grade 7, Unit 2: Introducing Proportional Relationships
• Lesson 1: One of These Things is Not Like the Other  
• Lesson 2: Introducing Proportional Relationships with Tables  
• Lesson 3: More about Constant of Proportionality  
• Lesson 4: Proportional Relationships and Equations  
• Lesson 5: Two Equations for Each Relationship  
• Lesson 6: Using Equations to Solve Problems  
• Lesson 7: Comparing Relationships with Tables  
• Lesson 8: Comparing Relationships with Equations  
• Lesson 9: Solving Problems about Proportional Relationships  
• Lesson 10: Introducing Graphs of Proportional Relationships  
• Lesson 11: Interpreting Graphs of Proportional Relationships  
• Lesson 12: Using Graphs to Compare Relationships
### Decoded Standard

MAFS.7.RP.1.1

This standard focuses on computing unit rates using ratios of fractions known as complex fractions. In a complex fraction, the numerator, denominator, or both are fractions. In the standard, \( \frac{1}{2} \cdot \frac{1}{4} \) is an example of a complex fraction. Complex fractions can be interpreted as division statements. For example, \( \frac{1}{2} \cdot \frac{1}{4} \) can be thought of as \( \frac{1}{2} \div \frac{1}{4} \). Applications include situations where the quantities are measured in different units such as miles per hour, pounds per square foot, feet per second, and so on. (*Common Core Mathematics Companion*, Pg. 18)

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**

- **Unit Rate Area** Convert this ratio to a unit rate and explain what this unit rate means in the context of the problem.
- **Computing Unit Rates** Write two unit rates and explain what each unit rate means in the context of the problem.
- **Comparing Unit Rates** Express a rate as a unit rate in gallons per hour and determine which is faster.
- **Unit Rate Length** Show how you converted this ratio to a unit rate.

**Illustrative Mathematics Assessment Tasks**

- **Cooking with the Whole Cup** Use a recipe to find unit rates for many different pair-wise ratios.
- **Molly’s Run** Context involving constant speed provides a transition from working with ratios involving whole numbers to ratios involving fractions.
- **Molly’s Run-Assessment Variation** This task is part of three assessment tasks that address various aspects of 6.RP domain and help distinguish between 6th and 7th grade expectations.
- **Track Practice** Ask students to find the unit rates that one can compute in this context with same and different units.
- **Buying Bananas-Assessment Version** Find a unit rate for a ratio of non-whole numbers.

#### Engage New York

- **Module 1, Topic C, Lesson 11** Students use ratio tables and ratio reasoning to compute unit rates associated with ratios of fractions in the context of measured quantities such as recipes, lengths, areas, and speed.
- **Module 1, Topic C, Lesson 12** Students use ratio tables and ratio reasoning to compute unit rates associated with ratios of fractions in the context of measured quantities, e.g., recipes, lengths, areas, and speed.

#### McGraw-Hill

**Course 2, Chapter 1**

- Inquiry Lab: Unit Rates
  - Lesson 2
MAFS.7.RP.1.2
Sections a-d of this standard break down the standard to give guidance on ways to recognize and represent proportional relationships.

A. This standard emphasizes two methods for deciding whether a proportional relationship exists. One method is to use equivalent ratios in a table. If the ratios are equivalent, then you have a proportional relationship such as:

<table>
<thead>
<tr>
<th># of people in a room</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td># of hands in the room</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>?</td>
</tr>
</tbody>
</table>

The other method is to graph the relationship on a coordinate plane and observe whether the graph is a straight line that goes through the origin. Note that computation using cross-multiplication is not a part of this standard. 

(Common Core Mathematics Companion, Pg. 19)

B. This standard focuses on proportional relationships that can be represented as tables, graphs, equations, diagrams, and verbal descriptions. Students have already seen tables, graphs, and verbal descriptions. The unit rate on a graph is the point where \( x = 1 \). In an equation, it is the slope represented by the coefficient, \( m \), in the formula \( y = mx + b \). The terms unit rate, constant of proportionality, and slope are equivalent. Note that students are only required to read and interpret equations in this standard. 

(Common Core Mathematics Companion, Pg. 21)

C. In the previous standard students read equations to find the unit rates. In this standard students are given verbal descriptions of proportional relationships and are expected to create the equations in the form \( y = mx + b \). For example, in Town C if you are caught speeding, you receive a traffic ticket. The penalty is $25 for every mile over the speed limit. What is the equation if \( p \) represents the penalty and \( m \) represents the number of miles over the speed limit? The equation is \( p=25m \). 

(Common Core Mathematics Companion, Pg. 22)

D. An example of a proportional situation is: The scale on a map suggests that 1 centimeter represents an actual distance of 4 kilometers. The map distance between two towns is 8 centimeters. What is the actual distance? The graph of this relationship is represented as:

Please see image on Pg. 23 of the Common Core Mathematics Companion

Note the points (0,0) and (1,4). The point is the unit rate or slope of the line for the equation \( c \) is the number of centimeters. 

(Common Core Mathematics Companion, Pg. 23)

### Mathematics Formative Assessments (MFAS)

- **Teacher to Student Ratios** Graph four ordered pairs given in context and decide if the variables they represent are proportionally related.
- **Constant of Proportionality Trip** Identify and explain the constant of proportionality given a verbal description and a diagram representing a proportional relationship.
- **Finding Constant of Proportionality** Determine the constant of proportionality using a table and a graph and explain it within the context of the problem.
- **Deciding if Proportional** Decide if two variables are proportionally related based on data given in a table.
- **Writing an Equation** Write an equation to represent a proportional relationship depicted in a graph.
- **Identify Constant of Proportionality in Equations** Identify and explain the constant of proportionality in three different equations.
- **Graphs of Proportional Relationships** Identify the graph of a proportional relationship.
- **Babysitting Graph** Given a graph that models the hourly earnings, interpret ordered pairs in context.
- **Serving Size** Write an equation for the size of the serving and the number of calories.

### EngageNY

- **Module 1, Topic A, Lesson 1** Students compute unit rates associated with ratios of quantities measured in different units. Students use the context of the problem to recall the meaning of a ratio, equivalent ratios, rate and unit rate, relating them to the context of the experience.
- **Module 1, Topic A, Lesson 2** Students understand that two quantities are proportional to each other when there exists a constant (number) such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity.
- **Module 1, Topic A, Lesson 5** Students decide whether two quantities are proportional to each other by graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- **Module 1, Topic B, Lesson 10** Students consolidate their understanding of equations representing proportional relationships as they interpret what points on the graph of a proportional relationship mean in terms of the situation or context of the problem, including the point (0, 0).

### MARS/Shell

- **Proportion and Non-Proportion Situations** Identify when two quantities are proportional or not. Solve proportionality problems.
- **Modeling: A Race** Recognize and use proportional relationships.
<table>
<thead>
<tr>
<th>Art Class, Assessment Variation</th>
<th>Decide proportional relationship using a table, find a unit rate using non-whole numbers, and represent with an equation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buying Coffee</td>
<td>Find a unit rate in a context and to draw the graph.</td>
</tr>
<tr>
<td>Robot Races</td>
<td>Identify the points on a distance vs. time graph within context.</td>
</tr>
<tr>
<td>Robot Races, Assessment Variation</td>
<td>Explain the meaning of a point on the graph and compute and compare unit rates with fractions.</td>
</tr>
<tr>
<td>Sore Throats-Variation 1</td>
<td>Finding equivalent ratios and proportional reasoning.</td>
</tr>
<tr>
<td>Walk-a-thon 2</td>
<td>Translate information in a table (with decimals) and find unit distance and distance traveled per unit time. Translate into equations and graphs.</td>
</tr>
<tr>
<td>Cider versus Juice-Variation 1</td>
<td>Compare two rates in different units.</td>
</tr>
<tr>
<td>Proportionality</td>
<td>Make sense out of the definition of direct proportionality.</td>
</tr>
<tr>
<td>Busses Task</td>
<td>Works with a distance-time graph describing a bus journey.</td>
</tr>
<tr>
<td>Comparing Strategies for Proportion</td>
<td>This lesson unit is intended to help you assess whether students recognize relationships of direct proportion and how well they solve problems that involve proportional reasoning Problems.</td>
</tr>
</tbody>
</table>

### Decoded Standard

**MAFS.7.RP.1.3**

In this standard students solve problems involving proportional relationships. Students set up and solve proportions using cross-multiplication. For example: “Directions to make a tablecloth call for \( \frac{3}{4} \) yard of ribbon for every 2 yards of fabric. If you increase the amount of fabric used to 3 years, how much ribbon will be needed?” The proportion is \( \frac{3}{2} = \frac{x}{3} \). To cross-multiply:

\[
3 \cdot \frac{3}{4} = 2x
\]

Problems for this standard should be multi-step and include contexts with simple interest, tax, tips, commissions, percent error, percent increase/decrease, discounts, fees, markups, markdowns, discount, sales, and/or original prices. To calculate a percent increase from 2 to 10, find the difference between the two numbers, in this case, 10-2=8. Take the difference, 8, and divide by the original number: \( \frac{8}{2} = 4 \). Multiply the quotient by 100: \( 4 \times 100 = 400\% \). *(Common Core Mathematics Companion, Pg. 24)*

**Special Note:** Students will solve multistep percent problems in Unit 2.

### Formative Tasks

**Mathematics Formative Assessments (MAFS)**

- **Making Cookies** Find values given a set of rational number quantities.

**Illustrative Mathematics Assessment Tasks**

- **Friends Meeting on Bikes** Determine speed based on distance and speed approaching from opposite direction.
- **Two-School Dance** Calculate the fraction of a combined population given different ratios for two distinct populations.
- **Sale** Students need opportunities to evaluate the relative savings of advertised sales.
- **Tax and Tip** How much will the total bill be, including tax and tip?
- **Shirt Sale** A tape diagram shows the solution in a very succinct way.

### Lesson Resources

**Engage NY**

- **Module 1, Topic C, Lesson 13** Students use tables to find an equivalent ratio of two partial quantities given a part-to-part ratio and the total of those quantities, in the third column, including problems with ratios of fractions.
- **Module 1, Topic c, Lesson 15** Students use equations and graphs to represent proportional relationships arising from ratios and rates involving fractions.

**McGraw-Hill**

- **Course 2, Chapters 1** Chapter 1 Lesson 6
- **Course 2, Chapter 4** Chapter 4 Lesson 7

---

**Decoded Standard**

MAFS.7.RP.1.3

In this standard students solve problems involving proportional relationships. Students set up and solve proportions using cross-multiplication. For example: “Directions to make a tablecloth call for \( \frac{3}{4} \) yard of ribbon for every 2 yards of fabric. If you increase the amount of fabric used to 3 years, how much ribbon will be needed?” The proportion is \( \frac{3}{2} = \frac{x}{3} \). To cross-multiply:

\[
3 \cdot \frac{3}{4} = 2x
\]

Problems for this standard should be multi-step and include contexts with simple interest, tax, tips, commissions, percent error, percent increase/decrease, discounts, fees, markups, markdowns, discount, sales, and/or original prices. To calculate a percent increase from 2 to 10, find the difference between the two numbers, in this case, 10-2=8. Take the difference, 8, and divide by the original number: \( \frac{8}{2} = 4 \). Multiply the quotient by 100: \( 4 \times 100 = 400\% \). *(Common Core Mathematics Companion, Pg. 24)*

**Special Note:** Students will solve multistep percent problems in Unit 2.

### Formative Tasks

**Mathematics Formative Assessments (MAFS)**

- **Making Cookies** Find values given a set of rational number quantities.

**Illustrative Mathematics Assessment Tasks**

- **Friends Meeting on Bikes** Determine speed based on distance and speed approaching from opposite direction.
- **Two-School Dance** Calculate the fraction of a combined population given different ratios for two distinct populations.
- **Sale** Students need opportunities to evaluate the relative savings of advertised sales.
- **Tax and Tip** How much will the total bill be, including tax and tip?
- **Shirt Sale** A tape diagram shows the solution in a very succinct way.

### Lesson Resources

**Engage NY**

- **Module 1, Topic C, Lesson 13** Students use tables to find an equivalent ratio of two partial quantities given a part-to-part ratio and the total of those quantities, in the third column, including problems with ratios of fractions.
- **Module 1, Topic c, Lesson 15** Students use equations and graphs to represent proportional relationships arising from ratios and rates involving fractions.

**McGraw-Hill**

- **Course 2, Chapters 1** Chapter 1 Lesson 6
- **Course 2, Chapter 4** Chapter 4 Lesson 7
<table>
<thead>
<tr>
<th><strong>• Gotham City Taxis</strong></th>
<th>Solve a multi-step ratio problem that can be approached in many ways.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MARS/Shell</strong></td>
<td></td>
</tr>
<tr>
<td><strong>• Ice Cream Task</strong></td>
<td>Uses multi-step proportional reasoning to solve a real-world problem related to ice cream.</td>
</tr>
<tr>
<td><strong>• Short Tasks-Ratio and Proportions</strong></td>
<td>Uses several short questions from RP cluster. Most problems are multi-step.</td>
</tr>
</tbody>
</table>
## Grade 7 Math

### Unit 2: Multi-Step Percent Problems

**Semester 1**

<table>
<thead>
<tr>
<th>Standards/Learning Goals</th>
<th>Content Limits, Assessment Types, Calculator</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MAFS.7.RP.1.3</strong> Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</td>
<td>• Units may be the same or different across the two quantities.</td>
</tr>
<tr>
<td><strong>MAFS.7.EE.2.3</strong> Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: <em>If a woman making $25 an hour gets a 10% raise, she will make an additional $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.</em></td>
<td>• Items should not use variables.</td>
</tr>
</tbody>
</table>

### Open Up Resources Lessons

- **Grade 7, Unit 4: Proportional Relationships and Percentages**
  - Lesson 6: Increasing and Decreasing
  - Lesson 7: One Hundred Percent
  - Lesson 8: Percent Increase and Decrease with Equations
  - Lesson 9: More and Less than 1%
  - Lesson 10: Tax and Tip
  - Lesson 11: Percentage Contexts
  - Lesson 12: Finding the Percentage
  - Lesson 13: Measurement Error
  - Lesson 14: Percent Error
  - Lesson 15: Error Intervals

### Decoded Standard

**MAFS.7.RP.1.3**

In this standard students solve problems involving proportional relationships. Students set up and solve proportions using cross-multiplication. For example: "Directions to make a tablecloth call for $\frac{3}{4}$ yard of ribbon for every 2 yards of fabric. If you increase the amount of fabric used to 3 years, how much ribbon will be needed?" The proportion is $\frac{\frac{3}{4}}{2} = \frac{x}{3}$. To cross-multiply:

$$3 \cdot \frac{3}{4} = 2x$$

Problems for this standard should be multi-step and include contexts with simple interest, tax, tips, commissions, percent error, percent increase/decrease, discounts, fees, markups, markdowns, discount, sales, and/or original prices.

To calculate a percent increase from 2 to 10, find the difference between the two numbers, in this case, 10-2=8. Take the difference, 8, and divide by the original number: $\frac{8}{2} = 4$. Multiply the quotient by 100: $4 \times 100 = 400\%$. *(Common Core Mathematics Companion, Pg. 24)*
### Formative Tasks

#### Mathematics Formative Assessments (MFAS)
- **Finding Fees** Complete a multi-step fee percent problem.
- **Tiffany’s Tax** Calculate the amount of sales tax and total price, given prices of individual items to purchase.
- **Gasoline Prices** Calculate the percent change for gas prices.

#### Illustrative Mathematics Assessment Tasks
- **Anna in D.C.** Solve a multi-step percentage problem.
- **Lincoln’s math problem** Solve a multi-step problem involving simple interest.
- **Buying Protein Bars and Magazines** Solve a multistep problem involving sales tax.
- **Chess Club** Solve a percent increase in one part with a percent decrease in the remaining. Find the overall percent change.
- **Double Discounts** Calculate percent decreases in the context of several discounts.
- **Finding a 10% increase** Simple percent increase task.
- **Selling Computers** Calculate quantities based on percent increase.
- **Tax and Tip** Calculate the tax and tip given the subtotal.
- **Sale!** Students need opportunities to evaluate the relative savings of advertised sales.

### Lesson Resources

#### EngageNY
- **Module 1, Topic C, Lesson 14** Students will solve multi-step ratio problems including fractional markdowns, markups, commissions, fees, etc.
- **Module 4, topic B, Lesson 7** Students understand equations for markup and markdown problems and use them to solve markup and markdown problems.
- **Module 4, Topic B, Lesson 10** Students solve simple interest problems using the formula $I = Prt$.
- **Module 4, Topic B, Lesson 11** Students solve real-world percent problems involving tax, gratuities, commissions, and fees.

#### Three Act Math
- **Dueling Discounts** Which coupon should I use?

#### MARS/Shell
- **Increasing or Decreasing Quantities by Percents** Translating between percents, decimals, and fractions. Representing percent increase and decrease as multiplication. Recognizing the relationship between increases and decreases.

#### McGraw-Hill
- **Course 2, Chapter 2**
  - Inquiry Lab: Find Percents
  - Lesson 3

### Decoded Standard

MAFS.7.EE.2.3
Students solve multi-step real-world and mathematical problems. The problems should contain a combination of whole numbers, positive and negative integers, fractions, and decimals. Students will apply what they learned in previous standards about convert fractions, decimals, and percents and use properties of operations to find equivalent forms of expressions as needed. Students will be expected to check their work for reasonableness using estimation strategies, which may include but are not limited to the following:

- rounding the values in the problem up or down and then adjusting the estimate to make up for the closeness of the rounded values to the originals,
- using friendly or compatible numbers for the values in the problem that allow for common factors for multiplication or easy addition such as grouping hundreds or thousands, and
- using benchmark numbers that are easy to work with such as 2 for 1/7 to make an estimate.

*(Common Core Mathematics Companion, Pg. 108)*

**Special Note:** Students will compute with integers and positive and negative numbers in Unit 3.
### Formative Tasks

**Mathematics Formative Assessments (MFAS)**
- **Discount and Tax** Solve a multi-step problem involving percent.
- **Gas Station Equations** Solve a two-step problem involving percent.
- **Using Estimation** Assess the reasonableness of answers using estimation.

**Illustrative Mathematics Assessment Tasks**
- **Anna in D.C.** Solve a multi-step percentage problem that can be approached in many ways.
- **Discounted Books** Examine different ways of looking at percentages and turn a verbal description of several operations into mathematical symbols and identify equivalent expressions without variables.

### Lesson Resources

**EngageNY**
- **Module 3, Topic B, Lesson 7** Build an algebraic expression using the context of a word problem and use that expression to write an equation that can be used to solve the word problem.
- **Module 4, Topic D, Lesson 16** Students write and use algebraic expressions and equations to solve percent word problems related to populations of people and compilations.

**MARS/Shell**
- **Steps to Solving Equations** Form and solve linear equations involving factorizing and using the distributive law.

**McGraw-Hill**
- **Course 2, Chapter 2**
  - Inquiry Lab: Percent Diagrams
  - Lessons 1, 2, 4
  - Inquiry Lab: Percent of Change
  - Lessons 5, 6, 7
<table>
<thead>
<tr>
<th>Standards/Learning Goals:</th>
<th>Content Limits, Assessment Types, Calculator</th>
</tr>
</thead>
</table>
| **MAFS.7.NS.1.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal and vertical number line diagram.  
  a. Describe situations in which opposite quantities combine to make 0.  
  b. Understand \( p+q \) as the number located a distance \(|q|\) from \( p \), in the positive or negative direction depending on whether \( q \) is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.  
  c. Understand subtraction of rational numbers as adding the additive inverse, \( p-q=p+(-q) \). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.  
  d. Apply properties of operations as strategies to add and subtract rational numbers. |  
| **MAFS.7.NS.1.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.  
  a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as \((-1)(-1)=1\) and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.  
  b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with no-zero divisor) is a rational number. If \( p \) and \( q \) are integers, the \(-\frac{p}{q}=\frac{-p}{q}=-\frac{p}{q}\). Interpret quotients of rational numbers by describing real-world contexts.  
  c. Apply properties of operations as strategies to multiply and divide rational numbers.  
  d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. |  
| **MAFS.7.NS1.3** Solve real-world and mathematical problems involving the four operations with rational numbers.  
(Computations with rational numbers extend the rules for manipulating fractions to complex fractions.) |  
| • Complex fractions may be used, but should contain fractions with single-digit numerators and denominators. |
Open Up Resources Lessons

Grade 7 Unit 5: Rational Number Arithmetic
- Lesson 1: Interpreting Negative Numbers
- Lesson 2: Changing Temperatures
- Lesson 3: Changing Elevation
- Lesson 4: Money and Debts
- Lesson 5: Representing Subtraction
- Lesson 6: Subtracting Rational Numbers
- Lesson 7: Adding and Subtracting to Solve Problems
- Lesson 8: Position, Speed, and Direction
- Lesson 9: Multiplying Rational Numbers
- Lesson 10: Multiply!
- Lesson 11: Dividing Rational Numbers
- Lesson 12: Negative Rates

Decoded Standard

MAFS.7.NS.1.1
A. Students use real-world situation that model using opposite quantities to make zero. This prepares students for adding rational numbers with opposite signs such as $4 + (-4) = 0$. Examples can include temperature, elevation above and below sea level, owing money, and so on. (Common Core Mathematics Companion, Pg. 58)

B. This standard formalizes the concept of a positive and negative making zero from the previous standard into written equations. For example, $4 + (-4) = 0$. The 4 and (-4) are opposites because they are equidistant from 0 on the number line in opposite directions. They are also additive inverses because their sum is 0. Be sure to include examples of fractions and decimals such as $-\frac{1}{2}$ and $-4.72$ so that students are working with all types of rational numbers.

Addition of integers is modeled on a number line as in the following example: “Jose has $6 and owes Steven $5. How much money will Jose have left when he pays Steven what he owes?” see image on page 59 (Common Core Mathematics Companion, Pg. 59)

C. Subtraction of rational numbers can be thought of in terms of addition using the additive inverse (sometimes referred to as “the opposite”). For example, 6-7 can be understood as 6+(-7). The distance between two rational numbers on a number line is the same as the absolute value of the difference between the two numbers. For example, using a real-world context, if the temperature is -6 at 7a.m. and +8 at noon, how many degrees has the temperature increased between 7 a.m. and noon? The difference between $-6 - 8 = -14$. $|-14| = 14$. Shown on a number line, the distance between -6 and 8 is 14. see image on page 60 (Common Core Mathematics Companion, Pg. 60)

D. Students have previously used the commutative, associative, and additive identity properties with whole numbers. These properties apply to rational numbers. For example:

- Commutative Property of Addition: $4.5 + (-6) = (-6) + 4.5$
- Associative Property of Addition: $6.9 + (-5) + 3.1 = 6.9 + 3.1 + (-5)$
- Additive Identity Property of Addition (also called the Zero Property): $(-4.8) + 0 = (-4.8)$

See image on page 61 (Common Core Mathematics Companion, pg. 61)

Instructional Resources

Formative Assessments (MFAS)
- Exploring Additive Inverse: Describe a student-generated example of additive inverse and demonstrate on a number line.
- Adding Integers: Add integers using a vertical and horizontal number line.
- Rational Addition and Subtraction: Rewrite a subtraction problem as an equivalent addition problem and explain the equivalence using a number line.
- Finding Difference: Find the difference between two integers using a number line.

Engage NY
- Grade 7 Module 2 Topic A Lesson 1: Students explore additive inverses and quantities that combine to make 0.
- Grade 7 Module 2 Topic A Lesson 2: Students model integer addition on the number line.
- Grade 7 Module 2 topic A Lesson 3 & 4: Students understand adding integers by using arrows to show the sum of two integers.
• **Rational Water Management** Combine rational numbers, including fractions and decimals, and use the properties of operations to simplify calculations.

**Illustrative Mathematics Assessment Tasks**
- **Comparing Freezing Points** Calculate the differences of signed numbers.
- **Bookstore Account** Use algebra and the number line to understand why it makes sense that we sometimes represent debt using negative numbers.
- **Difference of Integers** Subtract integers in a real world context.
- **Differences and Distances** Connect the distance between points on a number line with the difference between numbers.
- **Distances Between Houses** Solve a problem involving distances between objects whose positions are defined relative to a specified location and to see how this kind of situation can be represented with signed numbers.
- **Rounding and Subtracting** Addresses what happens to rounding discrepancies when arithmetic is performed on rounded numbers and would be a good problem for classroom discussion.
- **Distances on a Number Line 2** Reinforce understanding of rational numbers as points on the number line and visually understand that the sum of a number and its inverse is zero.
- **Operations on the Number Line** Solidify understanding numbers as points on a number line and understand the geometric interpretation of adding and subtracting signed numbers.

**Grade 7 Module 3 Topic A Lesson 5** Students justify the rules for subtracting integers.

**Grade 7 Module 3 Topic A Lesson 8 & 9** Students use properties of operations to add and subtract rational numbers without the use of a calculator.

**MARS/Shell**
- **A Day Out Task** Analyze the results of a survey in order to plan a school trip.
- **Using Positive and Negative Numbers in Context** Use directed numbers in context. Identify and aid in ordering, comparing, adding, and subtracting positive and negative integers.

**McGraw-Hill**

**Course 2, Chapter 3 and 4**
Consider organizing the unit by operation instead of by chapter. For example, teach students how to add integers, positive and negative fractions and positive and negative mixed numbers at the same time instead of teaching all of the integer operations and then move on to the other rational numbers.

**Decoded Standard**

MAFS.7.NS.1.2
Standards 7.NS.1.2a-d break down the understandings needed to multiply and divide rational numbers.

A. **Real-world contexts help students make sense of multiplication of rational numbers.** For example, it makes sense that \( 4 \times (-6.50) = -26 \) when the context for this equations is Janene owes $6.50 to each of 4 people. How much does Janene owe altogether?

It is common to read and understand (-6) as “the opposite of six” as well as “negative six.” Use “the opposite of” wording to make sense of equations such as \( (-2) \times (-5) \) so that we read “the opposite of 2 times negative 5” or \( ((-1) \times 2) \times (-5) = -(2 \times -5) = -(-10) = 10 \).

Students should discover the rules for multiplying signed numbers, and the rules make more sense when given context. For example, the chart below shows equations with context. *see image on page 62 (Common Core Mathematics Companion, Pg. 62)*

B. **Division of rational numbers can be thought of as the inverse of multiplication relying on previous understanding of the relationship between multiplication and division.** For example, \( (-25) \div 5 = -5 \) because \( 5 \times -5 = -25 \). This preserves the relationship between multiplication and division found with whole numbers, including the fact that division by 0 is undefined. One explanation is: \( x \times 0 = 5 \), so \( 5 \div 0 = x \). There is no possible number for \( x \). The equation \( \frac{-p}{q} = \frac{-p}{-q} \) is for the teacher, no the students. Use both \( p \div (-q) \) and \( \frac{p}{-q} \) notations for division. *(Common Core Mathematics Companion, Pg. 63)*

C. **Present problems in real-world contexts that allow students to see the meaning of the properties of the operations. Properties include:**
Commutative Property of Multiplication: $3.6 \times 2 = 2 \times 3.6$
Associative Property of Multiplication: $3 \times (6 \times (-7)) \times (-2) = (3 \times 6) \times ((-7) \times (-2))$
Distributive Property: $-4(4 + (-3)) = ((-4) \times 4) + ((-4) \times (-3))$
Multiplicative Identify: $1 \times (-9) = (-9)$
Zero Property of Multiplication: $(-4.6) \times 0 = 0$

(Common Core Mathematics Companion, Pg. 64)

D. To convert rational numbers in fraction form to decimal form, use the meaning of fractions as division. For example, $\frac{4}{5} = 4 \div 5 = 0.80$. From repeated examples, students learn that the decimal form either ends in 0s (as in the example) or repeated digits/sets of digits. Students learn to use the bar above a digit/set of digits to designate digits that repeat. For example, $\frac{2}{3} = 2 ÷ 3 = 0.6\overline{6}$ and $\frac{39}{99} = 0.3\overline{9}$. This prepares students to learn about irrational numbers in Grade 8. (Common Core Mathematics Companion, Pg. 65)

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**
- **Negative Times** Given an illustration of why the product of two negatives is a positive, provide a rationale.
- **Quotients of Integers** Given an integer division problem and asked to identify fractions which are equivalent to the division problem.
- **Understanding Products** Explain why the product of a positive and a negative rational number is negative.
- **Negative Explained** Describe a real-world context for a given expression involving the product of two rational numbers.
- **Applying Rational Number Properties** Evaluate expressions involving multiplication or rational numbers and use the properties of operations to simplify calculations.
- **Integer Division** Describe a real-world context for a given expression involving the quotient of two rational integers.

#### Illustrative Mathematics Assessment Tasks

- **Products and Quotients of Signed Rational Numbers** Provide a context for multiplying and dividing signed rational numbers, providing a means for understanding why the signs behave the way they do when taking products.
- **Why is a Negative Times a Negative Always Positive?** Understand the reason it makes sense for the product of two negative numbers to be positive.
- **Temperature Change** Provide a context for interpreting division expressions.

#### Lesson Resources

**Engage NY**
- **Grade 7 Module 2, Topic B Lesson 10** Students develop the rules for multiplying and dividing signed numbers.
- **Grade 7 Module 2 Topic B Lesson 11** Students understand the rules for multiplication of integers.
- **Grade 7 Module 2 Topic B Lesson 14** Students represent fractions as decimals (repeating and terminating decimals).
- **Grade 7 Module 2 Topic B Lesson 15** Students apply the rules for multiplying and dividing rational numbers.
- **Grade 7 Module 2 Topic B Lesson 16** Students use the properties of operations to multiply and divide rational numbers.

**MARS/Shell**
- **Increasing and Decreasing Quantities by a Percent** Interpret percent increase and decrease, and in particular, to identify and help students who have the following difficulties: Translating between percents, decimals, and fractions. Representing percent increase and decrease as multiplication. Recognizing the relationship between increases and decreases.
- **Fencing Task** Calculate the cost of building fences from fence posts and wooden panels.

**McGraw-Hill**

Course 2, Chapters 3 and 4
The concepts of multiplication and division were taught previously. Focus on the problems with signed numbers.
- Ch 3, Inquiry Lab: Multiplying Integers
- Ch 3, Lesson 4
- Ch 4, Lesson 6
- Ch 3, Inquiry Lab: Use properties to multiply
- Ch 3, Lesson 5
- Ch 4, Lesson 8
**MAFS.NS.1.3**
Extend the work with order of operations to all rational numbers. A example of a mathematical problem is \(-3 \times 2 \left(\frac{5}{6} + -\frac{1}{2}\right) = -2\). Complex fractions are fractions with a fraction in the numerator and/or a fraction in the denominator such as \(\frac{3}{4}\). Interpret the division bar to turn a complex fraction into division: \(\frac{3}{4} \div \frac{1}{2}\)

<table>
<thead>
<tr>
<th>Instructional Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Formative Tasks</strong></td>
</tr>
<tr>
<td><strong>Mathematics Formative Assessments (MFAS)</strong></td>
</tr>
<tr>
<td>• Positive and Negative Fractions  Students are asked to add, subtract, multiply, and divide positive and negative fractions.</td>
</tr>
<tr>
<td>• A Rational Number Expression Students are given a numerical expression to evaluate.</td>
</tr>
<tr>
<td>• Complex Fractions Students are asked to rewrite complex fractions as simple fractions in lowest terms.</td>
</tr>
<tr>
<td>• Monitoring Water Temperatures Students are asked to solve a word problem that involves finding the average of positive and negative decimal numbers.</td>
</tr>
<tr>
<td>• Trail Mix Munchies Students are asked to solve a word problem involving division of fractions.</td>
</tr>
</tbody>
</table>

| **Illustrative Mathematics Assessment Tasks** |
| • Comparing Freezing Points This task is appropriate for assessing student’s understanding of differences of signed numbers. |

<table>
<thead>
<tr>
<th><strong>Lesson Resources</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Engage NY</strong></td>
</tr>
<tr>
<td>• Module 2 Topic C Lesson 20 Students perform various calculations involving rational numbers to solve a problem related to the change in an investment’s balance over time.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>McGraw-Hill</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Course 2, Chapter 3 Lessons 2,3,4,5: Do Real-World Link and H.O.T. Problems from each lesson</td>
</tr>
<tr>
<td>Course 2, Chapter 4 Lessons 3,4,5,6,8: Do Real-World Link and H.O.T. Problems from each lesson</td>
</tr>
</tbody>
</table>
Standards/Learning Goals:

| MAFS.7.EE.1.1 | Apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients. |
| MAFS.7.EE.1.2 | Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, \(a + 0.05a = 1.05a\) means that “increase by 5%” is the same as “multiplying by 1.05”. |

Content Limits, Assessment Types, Calculator

- Expressions must be linear and contain a variable.
- Calculator: NEUTRAL
- Context: ALLOWABLE

Decoded Standard

MAFS.7.EE.1.1

Apply previously learned properties of operations (distributive, commutative, associative, identity, and inverse properties of addition and multiplication, as well as the zero property of multiplication) as strategies for adding, subtracting, factoring, and expanding linear expressions. Coefficients are limited to rational numbers that include integers, positive/negative fractions, and decimals. Use the properties to write equivalent expressions; for example, \(3(4a + 2) = 12a + 6\) uses the distributive property. Substituting a numerical value for the variable and then evaluating the expressions to find the same solution is a tool to determine whether two expressions are equivalent. For example, \(3(4a + 2)\) is equal to \(12a + 6\). Let \(a = 5\) and substitute 5 for \(a\) in both expressions.

\[
\begin{align*}
3(4a + 2) & = 12a + 6 \\
3(4 \cdot 5 + 2) & = (12 \cdot 5) + 6 \\
3(20 + 2) & = 60 + 6 \\
3(22) & = 66 \\
& = 66
\end{align*}
\]

(Common Core Mathematics Companion, Pg. 104)

Special Note: Be sure to supplement rational coefficient problems in this unit. The textbook does not include many problems with fractions or decimals.

Open Up Resources Lessons

Grade 7, Unit 6: Expressions, Equations, and Inequalities
- Lesson 18: Subtractions in Equivalent Expressions
- Lesson 19: Expanding and Factoring
- Lesson 20: Combining Like Terms, Part 1
- Lesson 21: Combining Like Terms, Part 2
- Lesson 22: Combining Like Terms, Part 3

Grade 7, Unit 5: Rational Number Arithmetic
- Lesson 13: Expressions with Rational Numbers

Formative Tasks

<table>
<thead>
<tr>
<th>Mathematics Formative Assessments (MFAS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalent Perimeters Students are asked to solve a geometric problem by simplifying an algebraic expression.</td>
</tr>
<tr>
<td>Equivalent Rational Expressions Students are given a polynomial with rational coefficients and asked to identify equivalent expressions from a given list.</td>
</tr>
<tr>
<td>Factored Forms Students are given two expressions and asked to rewrite each in factored form using the fewest number of terms.</td>
</tr>
</tbody>
</table>

Instructional Resources

Engage NY
- Use Properties of Operations to Generate Equivalent Expressions Students will generate equivalent expressions using the fact that addition and subtraction have the same effect. |
- Module 2, Topic C Lesson 22 Students identify and compare the sequence operations to find the solution to and algebraically solve the equation algebraically. |
- Module 2, Topic C Lesson 23
- **Identify Equivalent Multistep Expressions** Students are given an expression and are asked to identify expressions equivalent to it.

**Illustrative Mathematics Assessment Tasks**

**Writing Expressions** The instructions for two expressions sound very similar, however, the order in which the different operations are performed and the exact wording make a big difference in the final expression.

**MARS/Shell**

- **Steps to Solving Equations** Students match equations to stories and then order the steps used to solve these equations.

**McGraw-Hill**

- **Course 2, Chapter 5**
  - Lesson 1 and 2 and emphasize 6th grade content.
  - Inquiry Lab: Sequences emphasizes 6th grade content.
  - Combine Lessons 3 and 4 in preparation to teach Lesson 5.

### Decoded Standard

MAFS.7.EE.1.2

Using equivalent expressions from the previous standard, focus on how writing an equivalent statement can better show the relationship among the terms in the expressions. For example, \(6x + 15 = 3(3x + 5)\) means that three groups of \(2x + 5\) is the same as one group of \(6x\) and \(15\). *(Common Core Mathematics Companion, Pg. 105)*

### Instructional Resources

**Formative Tasks**

- **Mathematics Formative Assessments (MFAS)**
  - **Rectangular Expressions** Students are given equivalent expressions with rational coefficients and asked to explain what each expression represents within the context of the problem.
  - **Explain Equivalent Expressions** Students are given equivalent expressions with rational coefficients and asked to explain what each expression represents within the context of a problem.

**Illustrative Mathematics Assessment Tasks**

- **Ticket to Ride** The purpose of this instructional task is to illustrate how different, but equivalent, algebraic expressions can reveal different information about a situation represented by those expressions.
- **Writing Expressions** The instructions for two expressions sound very similar, however, the order in which the different operations are performed and the exact wording make a big difference in the final expression.

**Lesson Resources**

- **McGraw-Hill**
  - Lesson 5, 6, 7
  - Inquiry Lab: Factor Linear Expressions Lesson 8
  - **Lessons 6-8**, be sure to include problems with fractions and decimals from alternate resources.
## Standards/Learning Goals:

<table>
<thead>
<tr>
<th>MAFS.7.EE.2.3</th>
<th>Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAFS.7.EE.2.4</td>
<td>Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</td>
</tr>
<tr>
<td>a.</td>
<td>Solve word problems leading to equations of the form $px+q=r$ and $p(x+q)=r$, where $p, q,$ and $r$ are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is the width?</td>
</tr>
<tr>
<td>b.</td>
<td>Solve word problems leading to inequalities of the form $px+q&gt;r$ or $px+q&lt;r$, where $p, q,$ and $r$ are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.</td>
</tr>
</tbody>
</table>

## Open Up Resources Lessons

- **Lesson 1**: Relationships between Quantities
- **Lesson 2**: Reasoning about Contexts with Tape Diagrams, Part 1
- **Lesson 3**: Reasoning about Contexts with Tape Diagrams, Part 2
- **Lesson 4**: Reasoning about Equations and Tape Diagrams, Part 1
- **Lesson 5**: Reasoning about Equations and Tape Diagrams, Part 2
- **Lesson 6**: Distinguishing between Two Types of Situations
- **Lesson 7**: Reasoning about Solving Equations, Part 1
- **Lesson 8**: Reasoning about Solving Equations, Part 2
- **Lesson 9**: Dealing with Negative Numbers
- **Lesson 10**: Different Options for Solving One Equation
- **Lesson 11**: Using Equations to Solve Problems
- **Lesson 12**: Solving Problems about Percent Increase and Decrease (revisits Unit 2)
### Decoded Standard

**MAFS.7.EE.2.3**  
Students solve multi-step real-world and mathematical problems. The problems should contain a combination of whole numbers, positive and negative integers, fractions, and decimals. Students will apply what they learned in previous standards about converting fractions, decimals, and percents and use properties of operations to find equivalent forms of expressions as needed. Students will be expected to check their work for reasonableness using estimation strategies, which may include but are not limited to the following:

- Rounding the values in the problem up or down and then adjusting the estimate to make up for the closeness of the rounded values to the originals,
- Using friendly or compatible numbers for the values in the problem that allow for common factors for multiplication or easy addition such as grouping hundreds or thousands, and
- Using benchmark numbers that are easy to work with such as using $2$ for $\frac{17}{8}$ to make an estimate.

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**
- **Reeling in Expressions** Solve a multi-step problem involving rational numbers.
- **Discount and Tax** Solve a multi-step problem involving percent.

**Illustrative Mathematics Assessment Tasks**
- **Anna in D.C.** Solve a multi-step percentage problem that can be approached in many ways.
- **Discounted Books** Determine two different ways to look at percentages both as a decrease and an increase of an original amount and turn a verbal description of several operations into mathematical symbols.
- **Shrinking** Calculating and explaining percent decrease within context.
- **Who is the better batter?** Given a natural real-world context for comparing fractions, convert the fractions to decimals or describe the situation in terms of percents.
- **Gotham City Taxis** Solve a multi-step ratio problem that can be approached in many ways.

#### Lesson Resources

**EngageNY**
- **Module 3, Topic B, Lesson 8** Use properties of equality to solve word problems.

**MARS/Shell**
- **Steps to Solving Equations** Work collaboratively in pairs or threes, matching equations to stories and then ordering the steps used to solve these equations and explain their reasoning to their peers.

**McGraw-Hill**
- **Course 2, Chapter 6**  
  Problem-Solving Investigation: Work Backward
MAFS.7.EE.2.4

A. Students will become fluent in solving equations. Students use the arithmetic from the problem to generalize an algebraic solution. Use word problems that lend themselves to equations in the forms of \( px + q = r \) and \( p(x + q) = r \). Two examples are as follows:

1. Three consecutive even numbers add up to 48. What is the lowest number of the three? \( x + x + 2 + x + 4 = 3x + 6 = 48 \) \( (px + q = r) \)
2. Ms. Thomas had $25 to spend on party favors. She had $10.40 left after buying 10 balloons. How much did she spend on each balloon? \( 0.1(25 - 10.40) = r \) \( (p(x + q) = r) \)

Students should develop fluency solving word problems that can be modeled by linear equations in the form \( px + q = r \). Integers, fractions, and decimals should be included as values in the word problems.

(Common Core Mathematics Companion, Pg. 109)

B. In this standard, students move from solving word problems with equations to word problems with inequalities. Inequalities follow a similar form to those of the equations, \( px + q > r \) and \( px + q < r \). Students graph the solution set of the inequality on a number line and describe what it means of the context of the word problem. Be aware that sometimes the solution set to the inequality contains values that do not make sense as solutions for the word problems. For example, in the word problem, “Donna has at most $60 to spend on a shopping spree. She wants to buy a dress for $22 dollars and spend the rest on bracelets. Each Bracelet costs $8. How many bracelets can she purchase?” we see a solution of

\[ \begin{align*}
     60 - 22 &= 38 \\
     8x &\leq 38 \\
     8x &\leq 38 \\
     8 &\leq 8 \\
     x &\leq 4.75 \\
\end{align*} \]

The number of bracelets is less than or equal to 4.75. However, Donna cannot buy .75 of a bracelet, so when we graph the inequality as below:

see image on page 110 of the Common Core Mathematics Companion

we see that the only viable solutions to the word problems are 4, 3, 2, 1, or no bracelets.

(Common Core Mathematics Companion, Pg. 110)

*Emphasis should be placed on two-step equations. Teachers will need to supplement the text to fully address the standard.

---

**Instructional Resources**

**Formative Tasks**

- **Solve Equations** Solve two multistep equations involving rational numbers.
- **Squares** Write and solve an equation of the form \( p(x + q) = r \) in the context of a problem about the perimeter of a square.
- **Write and Solve an Equation** Write and solve a two-step equation to model the relationship among variables in a given scenario.
- **Algebra or Arithmetic?** Compare an arithmetic solution to an algebraic solution of a word problem.

**Illustrative Mathematics Assessment Tasks**

- **Fishing Adventures 2** Write and solve inequalities, and represent the solutions graphically.
- **Bookstore Account** Use algebra and the number line to understand why we sometimes represent debt using negative numbers.
- **Gotham City Taxis** Solve a multi-step ratio problem that can be approached in many ways.
- **Sports Equipment Set** An instructional task with context that can naturally be represented with an inequality; explore the relationship

**EngageNY**

- **Module 2, Topic C, Lesson 17** Students use tape diagrams to solve equations of the form \( px + q = r \) and \( p(x + q) = r \), where \( p, q, \) and \( r \), are small positive integers, and identify the sequence of operations used to find the solution.
- **Module 3, Topic B, Lesson 8** and
- **Module 3, Topic B, Lesson 9** Students understand and use the addition, subtraction, multiplication, division, and substitution properties of equality to solve word problems leading to equations of the form \( px + q = r \) and \( p(x + q) = r \) where \( p, q, \) and \( r \) are specific rational numbers.
- **Module 3, Topic B, Lesson 13** Students understand that an inequality is a statement that one expression is less than (or equal to) or greater than (or equal to) another expression, such as \( 2x + 3 < 5 \) or \( 3x + 50 \geq 100 \). Students interpret a solution to an inequality as a number that makes the inequality true when substituted for the variable.
- **Module 3, Topic B, Lesson 14** Students solve word problems leading to inequalities that compare \( px + q \) and \( r \), where \( p, q, \) and \( r \) are specific rational numbers. Students interpret the solutions in the context of the problem.

**Lesson Resources**

- **Common Core Mathematics Companion**
between the context and the mathematical representation of that context.

<table>
<thead>
<tr>
<th>Pinellas County Schools</th>
<th>GRADE 7 MATH</th>
</tr>
</thead>
<tbody>
<tr>
<td>2020-2021</td>
<td></td>
</tr>
</tbody>
</table>

- **Module 3, Topic B, Lesson 15** Students graph solutions to inequalities taking care to interpret the solutions in the context of the problem.

**MARS/Shell**
- **Steps to Solving Equations** Work collaboratively in pairs or threes, matching equations to stories and then ordering the steps used to solve these equations and explain their reasoning to their peers.

**McGraw-Hill**

Course 2, Chapter 6
- Lessons 1, 2, and 3: Focus on the problems with signed rational numbers; students were taught one step equations in 6th grade. The only new content in these sections is solving equations with rational numbers (integers, signed fractions and decimals).
- Inquiry Lab: Solve Two-Step Equations
  - Lesson 4
- Inquiry Lab: More Two-Step Equations
  - Lesson 5
- Inquiry Lab: Solve Inequalities
  - Lessons 6, 7, and 8
Grade 7, Unit 6: Geometric Figures

Standards/Learning Goals:  

**MAFS.7.G.1.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.  
- Geometric figures must be two-dimensional polygons.  
- Calculator: YES  
- Context: ALLOWABLE

**MAFS.7.G.1.2** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, notice when the conditions determine a unique triangle, more than one triangle, or no triangle.  
- Given conditions should not focus on similarity or congruence or that the sum of angles in a triangle is 180 degrees.  
- Be aware of the scoring capabilities for the GRID tool when designing these items.  
- To distinguish from other grades, conditions should include factors other than parallel/perpendicular lines and angle measure, such as symmetry and side length.  
- Calculator: NEUTRAL  
- Context: ALLOWABLE

**MAFS.7.G.1.3** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.  
- Spheres, cones, and cylinders are allowed.  
- Slicing is limited to horizontal or vertical slices.  
- Bases of prisms and pyramids can be a triangle (any type); a square; a rectangle; or a regular pentagon or hexagon.  
- Items should not use composite figures.  
- Calculator: NEUTRAL  
- Context: ALLOWABLE

**MAFS.7.G.1.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.  
- Items should use angles measured in degrees only.  
- Calculator: YES  
- Context: ALLOWABLE

Open Up Resources Lessons

Grade 7, Unit 1: Scale Drawings  
- Lesson 1: [What are Scale Copies?](#)  
- Lesson 2: [Corresponding Parts and Scale Factors](#)  
- Lesson 3: [Making Scaled Copies](#)  
- Lesson 4: [Scaled Relationships](#)  
- Lesson 5: [The Size of the Scale Factor](#)  
- Lesson 6: [Scaling and Area](#)  
- Lesson 7: [Scale Drawings](#)  
- Lesson 8: [Scale Drawings and Maps](#)  
- Lesson 9: [Creating Scale Drawings](#)  
- Lesson 10: [Changing Scales in Scale Drawings](#)  
- Lesson 11: [Scales without Units](#)  
- Lesson 12: [Units in Scale Drawings](#)

Grade 7, Unit 7: Angles, Triangles, and Prisms  
- Lesson 1: [Relationships of Angles](#)  
- Lesson 2: [Adjacent Angles](#)  
- Lesson 3: [Nonadjacent Angles](#)  
- Lesson 4: [Solving for Unknown Angles](#)  
- Lesson 5: [Using Equations to Solve for Unknown Angles](#)  
- Lesson 6: [Building Polygons, Part 1](#)  
- Lesson 7: [Building Polygons, Part 2](#)  
- Lesson 8: [Triangles with 3 Common Measures](#)  
- Lesson 9: [Drawing Triangles, Part 1](#)
Decoded Standard

**MAFS.7.G.1.1**
Students work with scale drawings. They learn how to read them, calculate the scale, compute the actual lengths from the scale in the drawings, and reproduce a scale drawing using another scale. Scale drawings are proportional to one another. Problems should center on experiences in the students’ own lives. Examples include but are not limited to scale drawings of student rooms at home, the classroom, and comic book strips. The term scale factor should be used when students are asked to reproduce a scale drawing at a different scale. A scale factor is a number that multiplies some quantity. For example, doubling the length of a window that is 3 ft long corresponds to a scale factor of 2 (2 \times 3 = 6). (Common Core Mathematics Companion, Pg. 165)

* Special Note: This would be an excellent place to introduce the conversion tables on the reference sheet.

**Formative Tasks**
**Mathematics Formative Assessments (MFAS)**
- **Flying Scale** Find the length and area of an object when given a scale drawing of the object.
- **Space Station Scale** Find the ratio of the area of an object in a scale drawing to its actual area and then relate this ratio to the scale factor in the drawing.
- **Garden Design** Reproduce a scale drawing using a different scale.

**Illustrative Mathematics Assessment Tasks**
- **Floor Plan** Translate between measurements given in a scale drawing and the corresponding measurements of the object represented by the scale drawing. If used in an instructional setting, it would be good for students to have an opportunity to see other solution methods, perhaps by having students with different approaches explain their strategies to the class.
- **Map Distance** Translate between information provided on a map that is drawn to scale and the distance between two cities represented on the map.
- **Rescaling Washington Park** Think critically about the effect that changing from one scaling to another has on an image, and then to physically produce the desired image.

**Lesson Resources**
**Engage NY**
- **Grade 7 Module 1 Topic D Lesson 16** Students understand scale drawings.
- **Grade 7 Module 1 Topic D Lesson 18** Students compute the lengths of pictures using a scale drawing.
- **Grade 7 Module 1 Topic D Lesson 19** Given a scale drawing students compute the area of the actual picture.
- **Grade 7 Module 1 Topic D Lesson 20** Students create their own scale drawings of a room or building.
- **Grade 7 Module 1 Topic D Lesson 21** Students produce scale drawings at a different scale.

**MARS/Shell**
- **Drawing to Scale: Designing a Garden** Interpret and use scale drawings to plan a garden layout.

**McGraw-Hill**
**Course 2, Chapter 7**
Inquiry Lab: Scale Drawing; Lesson 4

---

Decoded Standard

**MAFS.7.G.1.2**
Students practice drawing geometric shapes using technology (computer programs both commercial and free on the Internet), rulers and protractors, and free hand. While giving practice with multiple shapes, focus on triangles and constructing them from three given angles or sides. Students should determine, by looking at the given measures, whether one, more than one, or no triangles can be created. Angles need to add up to 180° to make a triangle. The sum of two side lengths of a triangle is always greater than the third side. If this is true for all three combinations of added side lengths, then you will have a triangle. (Common Core Mathematics Companion, Pg. 166)

**Formative Tasks**
**Mathematics Formative Assessments (MFAS)**
- **Drawing Triangles AAA** Draw a triangle with given angle measures, and explain if these conditions determine a unique triangle.

**Lesson Resources**
**Engage NY**
- **Grade 7 Module 6 Topic B Lesson 6** Students use tools to draw geometric shapes based on given conditions.
- **Grade 7 Module 6 Topic B Lesson 7** Lesson 6 & 7 -
- **Drawing Triangles AAS** Draw a triangle given the measures of two angles and a non-included side and to explain if these conditions determine a unique triangle.
- **Drawing Triangles ASA** Draw a triangle given the measures of two angles and their included side and to explain if these conditions determine a unique triangle.
- **Drawing Triangles SAS** Draw a triangle given the measures of two sides and their included angle and to explain if these conditions determine a unique triangle.
- **Drawing Triangles SSA** Draw a triangle given the lengths of two of its sides and the measure of a non-included angle and to decide if these conditions determine a unique triangle.
- **Sides of Triangles** Determine if given lengths will create a triangle.

- **Grade 7 Module 6 Topic B Lesson 8** Students draw triangles under different conditions to explore if it forms many, few or one triangle
- **Grade 7 Module 6 Topic B Lesson 9**
- **Grade 7 Module 6 Topic B Lesson 10** Lesson 9 & 10: Students explore conditions of triangles.
- **Grade 7 Module 6 Topic B Lesson 11** Students understand that three given lengths determine a triangle, provided the largest length is less than the sum of the other two lengths; otherwise, no triangle can be formed
- **Grade 7 Module 6 Topic B Lesson 12** Students explore unique triangles
- **Grade 7 Module 6 Topic B Lesson 13** Students use conditions to determine a unique triangle to determine when two triangles are identical.

- **MARS/Shell**
  - **Possible Triangle Constructions** Recall, sketch, construct and apply triangle properties and to determine whether given conditions describe a unique triangle, more than one possible triangle or does not describe a possible triangle.

- **McGraw-Hill**
  - Course 2, Chapter Inquiry Lab: Investigate Online Maps and Scale Drawings; Inquiry Lab: Create Triangles; Lesson 3

### Decoded Standard

MAFS.7.G.1.3

Students relate the two-dimensional shape that results from slicing a three-dimensional figure. Three-dimensional shapes will include right rectangular prisms and right rectangular pyramids. *(Common Core Mathematics Companion, Pg. 167)*

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**

- **Square Pyramid Slices** Sketch and describe the two-dimensional figures that result from slicing a square pyramid.
- **Rectangular Prism Slices** Sketch and describe two-dimensional figures that result from slicing a rectangular prism.
- **Cylinder Slices** Sketch and describe the two-dimensional figures that result from slicing a cylinder.
- **Cone Slices** Sketch and describe the two-dimensional figures that result from slicing a cone.

#### Illustrative Mathematics Assessment Tasks

- **Cube Ninjas!** Explore various cross sections of a cube and use precise language to describe the shape of the resulting faces.

#### Engage NY

- **Grade 7 Module 6 Topic C Lesson 16** Students describe rectangular regions that result from slicing a right rectangular prism by a plane perpendicular to one of the faces.
- **Grade 7 Module 6 Topic C Lesson 17** Students describe polygonal regions that result from slicing a right rectangular pyramid by a plane perpendicular to the base and by another plane parallel to the base.
- **Grade 7 Module 6 Topic C Lesson 18** Students describe polygonal regions that result from slicing a right rectangular prism or pyramid by a plane.
- **Grade 7 Module 6 Topic C Lesson 19** Students describe three-dimensional figures built from cubes by looking at horizontal slicing planes

#### Virtual Manipulative

- **Shodor: Cross Section Flyer** Explore cross sections of various cones, cylinders, prisms, and pyramids.

- **McGraw-Hill**
  - Course 2, Chapter 7
  
  Lesson 6 (limit content to 7.G.1.3)
MAFS.7.G.2.5
Explore supplementary, complementary, vertical, and adjacent angles and their relationships to one another. These facts are used in multi-step problems.

*see images on page 170 of the Common Core Mathematics Companion

(Common Core Mathematics Companion, Pg. 170)

### Instructional Resources

<table>
<thead>
<tr>
<th>Formative Tasks</th>
<th>Lesson Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematics Formative Assessments (MFAS)</strong></td>
<td><strong>Engage NY</strong></td>
</tr>
<tr>
<td>- <strong>Solve for the Angle</strong> Write and solve equations to determine unknown angle measures in supplementary and complementary angle pairs.</td>
<td>- <strong>Grade 7 Module 6 Topic A Lesson 1</strong> Students solve for unknown angles in word problems and in diagrams involving complementary and supplementary angles.</td>
</tr>
<tr>
<td>- <strong>Find the Angle Measure</strong> Use knowledge of angle relationships to write and solve equations to determine unknown angle measures.</td>
<td>- <strong>Grade 7 Module 6 Topic A Lesson 2</strong> Students solve for unknown angles in word problems and in diagrams involving complementary, supplementary, vertical, and adjacent angles.</td>
</tr>
<tr>
<td>- <strong>Straight Angles</strong> Write and solve equations to determine unknown angle measures in supplementary angle relationships.</td>
<td>- <strong>Grade 7 Module 6 Topic A Lesson 3</strong> Students solve for unknown angles in word problems and in diagrams involving all learned angle facts.</td>
</tr>
<tr>
<td>- <strong>What Is Your Angle?</strong> Use knowledge of angle relationships to write and solve equations to determine unknown angle measures.</td>
<td>- <strong>Grade 7 Module 6 Topic A Lesson 4</strong> Students solve for unknown angles in word problems and in diagrams involving all learned angle facts.</td>
</tr>
</tbody>
</table>

**MARS/Shell**
- **Applying Angle Theorems** Use geometric properties to solve problems using the measures of the interior and exterior angles of polygons.

**McGraw-Hill**
Course 2, Chapter 7

Lessons 1 & 2 with an emphasis on supplementary, complementary, vertical, and adjacent.
## GRADE 7 MATH

### Quarter 2

**Unit 7: Circumference, Area, Surface Area, and Volume of Compound Figures**

13 days: 3/3 – 3/26

**Standards/Learning Goals:**

<table>
<thead>
<tr>
<th>Standards/Learning Goals</th>
<th>Content Limits, Assessment Types, Calculator</th>
</tr>
</thead>
</table>
| **MAFS.7.G.2.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. | **•** Circles are limited to whole circles and semicircles.  
 **Calculator:** YES  
 **Context:** ALLOWABLE |
| **MAFS.7.G.2.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. | **•** Three-dimensional shapes may include right prisms and right pyramids.  
 **•** When the base of a figure has more than four sides, the area of the base must be given.  
 **Calculator:** YES  
 **Context:** ALLOWABLE |

### Open Up Resources Lessons

**Grade 7, Unit 3: Measuring Circles**
- Lesson 1: How Well Can You Measure?
- Lesson 2: Exploring Circles
- Lesson 3: Exploring Circumference
- Lesson 4: Applying Circumference
- Lesson 5: Circumference and Wheels
- Lesson 6: Estimating Areas
- Lesson 7: Exploring the Area of a Circle
- Lesson 8: Relating Area to Circumference
- Lesson 9: Applying Area of Circles

**Grade 7, Unit 7: Angles, Triangles, and Prisms**
- Lesson 11: Slicing Solids
- Lesson 12: Volume of Right Prisms
- Lesson 13: Decomposing Bases for Area
- Lesson 14: Surface Area of Right Prisms
- Lesson 15: Distinguishing Volume and Surface Area
- Lesson 16: Applying Volume and Surface Area

### Decoded Standard

**MAFS.7.G.2.4**

Students learn formulas for area \((A = \pi r^2)\) and circumference \((C = 2\pi r)\) of circles and then solve problems (mathematical and real-world) using these formulas. Students participate in discovering the relationship between the two formulas. (Common Core Mathematics Companion, Pg. 169)

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**
- **Circumference Formula** Students are asked to write the formula for the circumference of a circle, explain what each symbol represents, and label the variables on a diagram.
- **Circle Area Formula** Students are asked to write the formula for the area of a circle, explain what each symbol represents, and label the radius on a diagram.
- **Eye on Circumference** Students are asked to write the formula for the area of a circle, explain what each symbol represents, and label the radius on a diagram.

#### Lesson Resources

**Engage NY**
- **Module 3, Topic C, Lesson 16** Students know the formula for circumference \(C\) of a circle of diameter \(d\) and radius \(r\). Students discover that the ratio of the circumference to the diameter of a circle is called \(\pi\), written \(\pi\).
- **Module 3, Topic C, Lesson 17** Students know the formula for the area of a circle and use it to solve problems.
- **Center Circle Area** Students are asked to solve a problem involving the area of a circle.
- **Broken Circles** Students are asked to complete and explain an informal derivation of the relationship between the circumference and area of a circle.

**Illustrative Mathematics Assessment Tasks**

- **The Circumference of a Circle and the Area of the Region it Encloses** The purpose of this task is to help students differentiate between a circle and the region inside of the circle so that they understand what is being measured when the circumference and area are being found. This task is best used as a lead-in to the formulas for circumference and area of a circle.
- **Approximating the area of a circle** Use formulas for the area of squares and triangles to estimate.
- **Circumference of a Circle** The goal of this task is to study the circumferences of different sized circles, both using manipulatives and from the point of view of scaling.
- **Eight Circles** The purpose of this task is to strengthen students' understanding of area.
- **Measuring the area of a circle** This goal of this task is to give students familiarity using the formula for the area of a circle while also addressing measurement error while looking at the cross-section of a pipe.

**Decoded Standard**

MAFS.7.G.2.6

This standard pulls together much of what the students know and can do in geometry through problems solving of both mathematical and real-world problems. Students will work with two- and three-dimensional objects and apply what they know about area, volume and surface area. *(Common Core Mathematics Companion, Pg. 171)*

**Special Note:** Questions should not include cylinders, spheres or cones for this standard.

**Instructional Resources**

<table>
<thead>
<tr>
<th>Mathematics Formative Assessments (MFAS)</th>
<th>Engage NY</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Composite Polygon Area</strong> Students are asked to find the area of a composite figure.</td>
<td><strong>Module 3, Topic C, Lesson 19</strong> Students find the areas of triangles and simple polygonal regions in the coordinate plane with vertices at grid points by composing into rectangles and decomposing into triangles and quadrilaterals.</td>
</tr>
<tr>
<td><strong>Octagon Area</strong> Students are asked to find the area of a composite figure.</td>
<td><strong>Module 3, Topic C, Lesson 20</strong> Students find the area of regions in the plane with polygonal boundaries by decomposing the plane into triangles and quadrilaterals, including regions with polygonal holes.</td>
</tr>
<tr>
<td><strong>Cube Volume and Surface Area</strong> Students are asked to calculate the volume and surface area of a cube.</td>
<td><strong>Module 3, Topic C, Lesson 21</strong> Students find the surface area of three-dimensional objects whose surface area is composed of triangles and quadrilaterals. They use polyhedron nets to understand that surface area is simply the sum of the area of the lateral faces and the area of the base(s).</td>
</tr>
<tr>
<td><strong>Chilling Volumes</strong> Students are asked to solve a problem involving the volume of a composite figure.</td>
<td><strong>Module 3, Topic C, Lesson 22</strong> Students find the surface area of three-dimensional objects whose surface area is composed of triangles and quadrilaterals, specifically focusing on pyramids.</td>
</tr>
<tr>
<td><strong>Composite Surface Area</strong> Students are asked to find the surface area of a composite figure.</td>
<td><strong>Module 3, Topic C, Lesson 23</strong> Students use the known formula for the volume of a right rectangular prism (length x width x height).</td>
</tr>
<tr>
<td><strong>Prismatic Surface Area</strong> Students are asked to determine the surface area of a right triangular prism and explain the procedure.</td>
<td><strong>Module 3, Topic C, Lesson 24</strong> Students use the formula for the volume of a right rectangular prism to answer questions about the capacity of tanks. Students compute volumes of right prisms involving fractional values for length.</td>
</tr>
<tr>
<td>Pinellas County Schools</td>
<td></td>
</tr>
<tr>
<td>------------------------</td>
<td>--</td>
</tr>
<tr>
<td><strong>GRADE 7 MATH</strong></td>
<td><strong>2020-2021</strong></td>
</tr>
</tbody>
</table>

- **Stained Glass** The purpose of this task is for students to find the area and perimeter of geometric figures whose boundaries are segments and fractions of circles and to combine that information to calculate the cost of a project.

- **Module 3, Topic C, Lesson 25** Students solve real-world and mathematical problems involving volume and surface areas of three-dimensional objects composed of cubes and right prisms.

**MARS/Shell**
- **Maximizing Area: Gold Rush** Students will Explore the effects on a rectangle’s area of systematically varying the dimensions whilst keeping the perimeter constant. Interpret and evaluate the data generated, identifying the optimum case.

**McGraw-Hill Textbook**
Course 2, Chapter 8
<table>
<thead>
<tr>
<th>Standards/Learning Goals</th>
<th>Content Limits, Assessment Types, Calculator</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MAFS.7.SP.3.5</strong> Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ( \frac{1}{2} ) indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.</td>
<td>N/A</td>
</tr>
<tr>
<td><strong>MAFS.7.SP.3.6</strong> Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. <em>For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but the probably not exactly 200 times.</em></td>
<td>Long-run frequency should be greater than or equal to 300.</td>
</tr>
<tr>
<td><strong>MAFS.7.SP.3.7</strong> Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.</td>
<td>N/A</td>
</tr>
<tr>
<td>a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. <em>For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.</em></td>
<td></td>
</tr>
<tr>
<td>b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. <em>For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?</em></td>
<td></td>
</tr>
<tr>
<td><strong>MAFS.7.SP.3.8</strong> Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.</td>
<td>Numbers in items must be rational numbers.</td>
</tr>
<tr>
<td>a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.</td>
<td></td>
</tr>
<tr>
<td>b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.</td>
<td></td>
</tr>
<tr>
<td>c. Design and use a simulation to generate frequencies for compound events. <em>For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?</em></td>
<td></td>
</tr>
</tbody>
</table>

**Semester 2**

**Unit 8: Probability**

| 9 days: 4/5 – 4/15 | 2020-2021 |

**Context:** REQUIRED

**Calculator:** NEUTRAL

**ASSESSED with MAFS.7.SP.3.7**
Open Up Resources Lessons

Grade 7, Unit 8: Probability and Sampling

- Lesson 1: Mystery Bags
- Lesson 2: Chance Experiments
- Lesson 3: What Are Probabilities
- Lesson 4: Estimating Probabilities Through Repeated Experiments
- Lesson 5: More Estimating Probabilities
- Lesson 6: Estimating Probabilities Using Simulation
- Lesson 7: Simulating Multi-Step Experiments
- Lesson 8: Keeping Track of All Possible Outcomes
- Lesson 9: Multi-Step Experiments
- Lesson 10: Designing Simulations

Decoded Standard

MAFS.7.SP.3.5
This standard introduces students to the concept of chance with events that are likely, unlikely, or neither likely nor unlikely. Students learn to use a scale from 0-1 representing probabilities that range from impossible to certain as in the scale from 0-1 below:

See image on page 224 of the Common Core Mathematics Companion

Numerical probabilities are numbers from 0-1, and the larger the number (the closer to 1), the more likely the event is to occur. A number near 0 (i.e., \( \frac{1}{50} \)) indicates an unlikely event and a number in the middle (\( \approx 0.5 \)) is neither likely nor unlikely. A 0 probability is an impossible event, and a 1 is a certainty. Probabilities are expressed as ratios of the number of times an event occurs to the total number of trials performed. Probabilities can be represented as fractions, decimals, and percents. (Common Core Mathematics Companion, Pg. 224)

Instructional Resources

**Formative Tasks**

Mathematics Formative Assessments (MFAS)

- **Probability or Not?** Students are asked to determine whether or not a given number could represent the probability of an event.
- **Likely or Unlikely?** Students are asked to determine the likelihood of an event given a probability.
- **Likelihood of an Event** Students are asked to determine the likelihood of an event given a probability.

**Lesson Resources**

Engage NY

- **Module 5, Topic A, Lesson 1** A probability is a number between 0 and 1 that represents the likelihood that an event will occur; interpret a probability as the proportion of the time that an event occurs when a chance experiment is repeated many times.

MARS/Shell

- **Probability Games** In this lesson students confront and overcome common probability misconceptions. The will count equally likely outcomes using diagrams, discuss relationships between theoretical probabilities, observe outcomes and sample sizes and calculate probabilities of independent events.

Interactive Manipulatives/shodar.org

- **Spinner** In this activity, students adjust how many sections there are on a fair spinner then run simulated trials on that spinner as a way to develop concepts of probability.

McGraw Hill

Course 2, Chapter 9

Lesson 1
Decoded Standard

MAFS.7.SP.3.6

Students collect data on chance events so that they can estimate the probability of the event. Students learn the difference between theoretical probability (probability that is calculated mathematically) and experimental probability (actual outcomes of an experiment). Seldom are the theoretical and experimental probabilities equal, although the more a simulation is repeated, the closer the theoretical and experimental probabilities become. Relative frequency is the observed number of successful outcomes in a set number of trials. It is the observed proportion of successful events. Students learn to make predictions about the relative frequency of an event by using simulations. *(Common Core Mathematics Companion, Pg. 225)*

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**
- **Probability Cubed**: Students are asked to estimate the frequency of an event given its probability and explain why an expected frequency might differ from an observed frequency.
- **Hen Eggs**: Students are asked to estimate the probability of a chance event based on observed frequencies.
- **Game of Chance**: Students are asked to estimate the frequency of an event given its probability and explain why an expected frequency might differ from an observed frequency.

**Illustrative Mathematics Assessment Tasks**
- **Heads or Tails**: This task asks students to think about how the distribution of observed outcomes from a chance experiment might differ from the theoretical distribution and to use observed data to estimate a probability.
- **Rolling Dice**: Students pool the results of many repetitions of the random phenomenon (rolling dice) and compare their results to the theoretical expectation they develop by considering all possible outcomes of rolling two dice. This gives them a concrete example of what we mean by long term relative frequency.
- **Tossing Cylinders**: The purpose of this task is to provide students with the opportunity to determine experimental probabilities by collecting data.

#### Lesson Resources

**EngageNY**
- **Module 5, Topic A, Lesson 2**: Estimate probabilities by collecting data on an outcome of a chance experiment; use given data to estimate probabilities.
- **Module 5, Topic A, Lesson 3**: Determine the possible outcomes for simple chance experiments; given a description of a simple chance experiment, students determine the sample space for the experiment; given a description of a chance experiment and an event, students determine for which outcomes in the sample space the event will occur; distinguish between chance experiments with equally likely outcomes and chance experiments for which the outcomes are not equally likely.
- **Module 5, Topic B, Lesson 8**: Given theoretical probabilities based on a chance experiment, students describe what they expect to see when they observe many outcomes of the experiment; students distinguish between theoretical probabilities and estimated probabilities; students understand that probabilities can be estimated based on observing outcomes of a chance experiment.

**MARS/Shell**
- **Evaluating Statements About Probability**: This lesson unit addresses common misconceptions relating to probability of simple and compound events. The lesson will help you assess how well students understand concepts of equally likely events, randomness and sample sizes.

**McGraw Hill**
- **Course 2, Chapter 9**: Inquiry Lab: Relative Frequency; Lesson 2 (limit content to 7.SP.3.6)

---

Decoded Standard

MAFS.7.SP.3.7

This standard is broken into two parts (a-b). We will consider them together since they are so closely related. Overall, students develop and use probability models to find the probability of events. Uniform probability models are those where the likelihood of each outcome is equal. For example, there are 17 children in the class. What is the probability that Sam will be chosen? Using theoretical probability, students can predict frequencies of outcomes. In part b of this standard, students look at the experimental probability to develop a model. *(Common Core Mathematics Companion, Pg. 226)*

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**
- **Marble Probability**: Students are asked to determine probabilities based on observed outcomes from drawing marbles.

**Engage NY**
- **Module 5, Topic A, Lesson 4**: Students will calculate probabilities of events for chance experiments that have equally likely outcomes.
from a bag and to determine if the outcomes appear to be equally likely.

- **Number Cube** Students are asked to determine probabilities based on observed outcomes from rolling a number cube and to determine if the outcomes appear to be equally likely.
- **Technical Difficulties** Students are given a scenario and asked to determine the probability of two different events.
- **Errand Runner** Students are asked to determine the probability of a chance event and explain possible causes for the difference between the probability and observed frequencies.

### Illustrative Mathematics Assessment Tasks

- **Stay or Switch** The purpose of the task is for students to find the theoretical probability of an event by systematically recording all of the possible outcomes in the sample space and identifying those that correspond to the event.
- **How Many Buttons** This task uses student generated data to assess standard 7.SP.7. This task could also be extended to address Standard 7.SP.1 by adding a small or whole class discussion of whether the class could be considered as a representative sample of all students at your school.

### Decoded Standard

MAFS.7.SP.3.8

This standard is broken into three parts (a-c). We will consider them together since they are so closely related. Students move to compound events by building on their knowledge of single events. Compound events are those where two or more events are happening at once. For example, what is the probability that you forgot to study last night and there will be a surprise quiz in class today? Students select tools such as organized lists, tables, and tree diagrams to represent sample spaces for compound events. Ultimately, students design their own simulation for a compound event. *(Common Core Mathematics Companion, Pg. 228)*

*Lesson 6 on Permutations and Combinations is an unnecessary extension into the High School Standard MAFS.912.S-CP.2.9*

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**

- **Work Clothing** Students are asked to make a tree diagram to determine all possible outcomes of a compound event.
- **Number List** Students are asked to make an organized list that displays all possible outcomes of a compound event.
- **Coat Count** Students are asked to design a simulation to generate frequencies for complex events.
- **Automotive Probabilities** Students are asked to find the probability of a compound event using a tree diagram and explain how the tree diagram was used to find the probability.

**Illustrative Mathematics Assessment Tasks**

- **Red Green or Blue** The purpose of this task is for students to find the probability of compound events using organized lists, tables, or tree diagrams.
- **Sitting Across From Each Other** The purpose of this task is for students to compute the theoretical probability of a seating configuration. There are 24 possible configurations of the four friends at the table in this problem. Students could draw all 24 configurations to solve the problem but this is time consuming and so they should be encouraged to look for a more systematic method.

#### Engage NY

- **Module 5, Topic A, Lesson 6** Use tree diagrams to represent outcomes in the sample space; students calculate probabilities of compound events.
- **Module 5, Topic A, Lesson 7** Students calculate probabilities of compound events.
- **Module 5, Topic B, Lesson 10** Students learn how to perform simulations to estimate probabilities; students use various devices to perform simulations (e.g., coin, number cube, cards).
- **Module 5, Topic B, Lesson 11** Students design their own simulations; students learn to use two more devices in simulations: colored disks and a random number table.
- **Module 5, Topic B, Lesson 12** Use estimated probabilities to judge whether a given probability model is plausible; students will use estimated probabilities to make informed decisions.

#### Virtual Manipulatives

- **Interactive Marbles** This online manipulative allows the student to simulate placing marbles into a bag and finding the probability of pulling out certain combinations of marbles. This allows exploration of probabilities of multiple events as well as probability with and without replacement.
- **Waiting Times** As the standards in statistics and probability unfold, students will not yet know the rules of probability for compound events. Thus, simulation is used to find an approximate answer to these questions. In fact, part b would be a challenge to students who do know the rules of probability, further illustrating the power of simulation to provide relatively easy approximate answers to wide-ranging problems.

- **Rolling Twice** A fair six-sided die is rolled twice. What is the theoretical probability that the first number that comes up is greater than or equal to the second number?

- **Tetrahedral Dice** The purpose of this task is to have students develop an organized list, table, etc. to determine all possible outcomes of a chance experiment and then to use this information to calculate various probabilities.

- **Hamlet Happens** The purpose of this manipulative is to help students recognize that (1) unusual events do happen, and (2) it may take a longer time for some of them to happen. The letters are drawn at random from the beginning of Hamlet's soliloquy, "To be, or not to be." Any word made from those letters (such as TO) can be entered in the box. When the start is pressed, letters are drawn and recorded. The process continues until the word appears.

McGraw-Hill Textbook

Course 2, Chapter 9

- Inquiry Lab: Fair and Unfair Games; Inquiry Lab: Simulate Compound Events; Lessons (limit content to 7.SP.3.8) and 4
<table>
<thead>
<tr>
<th>Standards/Learning Goals:</th>
<th>Content Limits, Assessment Types, Calculator</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAFS.7.SP.1.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.</td>
<td>• Context must be grade appropriate.</td>
</tr>
<tr>
<td>Calculator: NEUTRAL</td>
<td>Context: REQUIRED</td>
</tr>
<tr>
<td>ASSESSED with MAFS.7.SP.1.2</td>
<td></td>
</tr>
<tr>
<td>MAFS.7.SP.1.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.</td>
<td>• Context must be grade appropriate.</td>
</tr>
<tr>
<td>Calculator: NEUTRAL</td>
<td>Context: REQUIRED</td>
</tr>
<tr>
<td>MAFS.7.SP.2.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variability, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.</td>
<td>• N/A</td>
</tr>
<tr>
<td>Calculator: NEUTRAL</td>
<td>Context: REQUIRED</td>
</tr>
<tr>
<td>ASSESSED with MAFS.7.SP.2.4</td>
<td></td>
</tr>
<tr>
<td>MAFS.7.SP.2.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.</td>
<td>• N/A</td>
</tr>
<tr>
<td>Calculator: NEUTRAL</td>
<td>Context: REQUIRED</td>
</tr>
</tbody>
</table>

Open Up Resources Lessons

Grade 7, Unit 8: Probability and Sampling

- Lesson 11: Comparing Groups
- Lesson 12: Larger Populations
- Lesson 13: What Makes a Good Sample?
- Lesson 14: Sampling in a Fair Way
- Lesson 15: Estimating Population Measure of Center
- Lesson 16: Estimating Population Proportions
- Lesson 17: More about Sampling Variability
- Lesson 18: Comparing Populations Using Samples
- Lesson 19: Comparing Populations With Friends
### Decoded Standard

**MAFS.7.SP.1.1**
Sampling is taught in this standard as a statistical tool used to gain information about a population without examining the entire population. Sampling is the process of taking a subset of subjects that is representative of the entire population and collecting data on that subset. The sample must have sufficient size to warrant statistical analysis. Samples need to be representative of the population in order to make valid generalizations and, therefore, should be randomly selected. A random sampling guarantees that each element of the population has an equal opportunity to be selected in the sample. An example of a random sample is taking a list of names at a school and selecting every fourth person to be in the sample to represent the population of the school. *Common Core Mathematics Companion, Pg. 217*

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**
- **Ice Cream Survey** Choose a sampling method that would be most representative of a population and justify their selection.
- **Height Research** Describe a method for collecting data in order to estimate the average height of 12 year-old boys in the U.S.
- **Favorite Sport Survey** Evaluate an inference made using a biased sampling method.

#### Illustrative Mathematics Assessment Tasks
- **Mr. Briggs’ Class Likes Math** Determine whether the scenario will create a representative sample.

**Engage NY**
- **Grade 7 Module 5 Topic C Lesson 13** Students differentiate population characteristic & sample statistics.
- **Grade 7 Module 5 Topic C Lesson 14** Students understand how a sample is selected.
- **Grade 7 Module 5 Topic C Lesson 15** Students begin to develop an understanding of sampling variability.

**MARS/Shell**
- **Estimating Counting Trees** Solve simple problems involving ratio and direct proportion. Choose an appropriate sampling method. Collect discrete data and record them using a frequency table.

**McGraw Hill**
- **Course 2, Chapter 10**
  - **Lesson 1**

### Decoded Standard

**MAFS.7.SP.1.2**
This standard connects to &.SP.1.1 by using the sample data collected to draw inferences. Generate multiple samples of the same size from a given population to examine the variation in estimates or predictions. This standard provides an introduction to variability. An example of data to collect is two random samples of 100 students about school lunch preferences.

<table>
<thead>
<tr>
<th>School Lunch Preferred</th>
<th>Sample 1</th>
<th>Sample 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burgers</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>Salad</td>
<td>13</td>
<td>11</td>
</tr>
<tr>
<td>Pizza</td>
<td>74</td>
<td>77</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

*(Common Core Mathematics Companion, Pg. 218)*

#### Instructional Resources

**Formative Tasks**

**Mathematics Formative Assessments (MFAS)**
- **School Days** Use data from a random sample to estimate a population parameter and explain what might be done to increase confidence in the estimate.
- **Movie Genre** Use data from a random sample to draw an inference about a population.

**Illustrative Mathematics Assessment Tasks**
- **Valentine Marbles** Software was used to generate 100 random samples of size 16 from a population where the probability of obtaining a success in one draw is 33.6% (Bernoulli). Given that multiple samples of the same size have been generated, students should note that there can be quite a bit of variability among the estimates from random samples and that on average, the center of the

**Engage NY**
- **Grade 7 Module 5 Topic C Lesson 21** Random samples to draw informal references about the difference in population means.
- **Grade 7 Module 5 Topic C Lesson 22** The difference in sample means as a multiple of a measure of variability.
- **Grade 7 Module 5 Topic D Lesson 23** Students understand the meaningful difference of two sample means due to sample variability.

**MARS/Shell**
distribution of such estimates is at the actual population value and most of the estimates themselves tend to cluster around the actual population value.

- **Estimating Counting Trees** Solve simple problems involving ratio and direct proportion. Choose an appropriate sampling method. Collect discrete data and record them using a frequency table.

**McGraw Hill**  
Course 2, Chapter 10  
Inquiry Lab: Multiple Samples of Data; Lesson 2

### Decoded Standard

**MAFS.7.SP.2.3**

Students compare statistics on two data sets for the first time. Build on their understanding of graphs, mean, median, mean absolute deviation (MAD), and interquartile range from sixth grade. Students understand that variability is responsible for the overlap of two data sets, which can be visible when the data are presented in graphic form—two dot plots or box-and-whisker plots, for example. With two data distributions with similar variability, students will express the difference between centers (mean, median, mode) as a multiple of a measure of variability. For an example see Reproducible 4 (pg. 261).  
*Common Core Mathematics Companion, Pg. 220*

### Instructional Resources

#### Formative Tasks

**Mathematics Formative Assessments (MFAS)**

- **TV Ages** Informally determine the degree of overlap between two box plots with the same interquartile range (IQR) by expressing the difference between their medians as a multiple of the IQR.

- **More TV Ages** Informally determine the degree of overlap between two box plots with the same interquartile range (IQR) by expressing the difference between their medians as a multiple of the IQR.

#### Illustrative Mathematics Assessment Tasks

- **College Athletes** Conjecture about the differences in the two groups from a strictly visual perspective and then support their comparisons with appropriate measures of center and variability.

- **Offensive Linemen** Conjecture about the differences and similarities in the two groups from a strictly visual perspective and then support their comparisons with appropriate measures of center and variability.

#### Lesson Resources

**CPalms**

- **Stepping Up Measures of Center** Explore the use of dot plots and mean absolute deviation to compare and draw inferences from two different sets of numerical data.

- **Who’s Taller** Uses real-world data sets to guide students through representing and comparing data sets in separate dot plots. Represent and compare the data sets by using the mean and MAD (mean absolute deviation).

**MARS/Shell**

- **Comparing Data** This lesson is intended to help students to make meaningful comparisons between sets of data. In particular, selecting appropriate measures of center and variability in order to summarize the important features of a set of data and using quantitative measures to justify an argument.

**McGraw Hill**  
Course 2, Chapter 10  
Inquiry Lab: Visual Overlap of Data Distributions; Lesson 4

Continues on next page
## Decoded Standard

**MAFS.7.SP.2.4**

Draw valid comparative inferences about two populations. The inferences are drawn from using measures of center (mean, median, mode) and variability (range, mean absolute deviation, and interquartile range) from random samples. This standard differs from the previous in that students are now drawing inferences. Using the examples from the previous standard where the data were collected will unify this work. *(Common Core Mathematics Companion, Pg. 221)*

## Instructional Resources

### Mathematics Formative Assessments (MFAS)

- **Word Lengths** Use the mean and the mean absolute deviation (MAD) to compare two distributions.
- **Overlapping Trees** Compare two distributions given side-by-side box plots.

### Illustrative Mathematics Assessment Tasks

- **College Athletes** Conjecture about the differences in the two groups from a strictly visual perspective and then support their comparisons with appropriate measures of center and variability.
- **Offensive Linemen** Conjecture about the differences and similarities in the two groups from a strictly visual perspective and then support their comparisons with appropriate measures of center and variability.

### Lesson Resources

**MARS/Shell**

- **Comparing Data** This lesson is intended to help students to make meaningful comparisons between sets of data. In particular, selecting appropriate measures of center and variability in order to summarize the important features of a set of data and using quantitative measures to justify an argument.

**McGraw Hill**

Course 2, Chapter 10

Inquiry Lab: Collect Data
Number and Quantity: Reason, describe, and analyze quantitatively, using units and number systems to solve problems.

<table>
<thead>
<tr>
<th>Scoring Criteria</th>
<th>Performance Indicators</th>
<th>Emerging</th>
<th>Progressing</th>
<th>Meets</th>
<th>Exceeds</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>E. Students will analyze proportional relationships and use them to solve real-world and mathematical problems.</strong> [7.RP.1.1, 7.RP.1.2, 7.RP.1.3]</td>
<td>i. Students can find a unit rate.</td>
<td>i. Students can solve a unit rate problem.</td>
<td>i. Students can compute unit rate of two fractions and use unit rate to solve multistep ratio and percent problems in context.</td>
<td>i. Students can compute unit rates with mixed numbers and solve complex, multistep ratio and percent problems in context.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ii. Students can describe changes or identify characteristics occurring in a model or representation.</td>
<td>ii. Students can decide whether two quantities are proportional and identify the constant of proportionality that models a given representation or situation.</td>
<td>ii. Students can model and explain a proportional relationship and/or the constant of proportionality using graphs, diagrams, and tables.</td>
<td>ii. Students can model a representation with a context that would represent a given proportional equation.</td>
<td></td>
</tr>
</tbody>
</table>

| **F. Students will apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** [7.NS1.1, 7.NS.1.2, 7.NS.1.3] | i. Students can identify the properties of operations. | i. Students can represent the 4 mathematical foundational operations with rational numbers using number lines and other manipulatives. | i. Students can apply properties of operations with rational numbers to solve real-world problems. | i. Students can create a real-world situation to model a given algebraic equation. |
|                                                       | i. Students can apply properties of operations as strategies to add and subtract rational coefficients; factors and expand linear expressions with integer | |

Algebra: Create, interpret, use, and analyze expressions, equations and inequalities.

<table>
<thead>
<tr>
<th>Scoring Criteria</th>
<th>Performance Indicators</th>
<th>Emerging</th>
<th>Progressing</th>
<th>Meets</th>
<th>Exceeds</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>D. Students will use properties of operations to generate equivalent expressions.</strong> [7.EE.1.1, 7.EE.1.2]</td>
<td>i. Students can identify the properties of operations.</td>
<td>i. Students can apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients.</td>
<td>i. Students can apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients.</td>
<td>i. Students can apply/justify and/or analyze errors in the use of properties of operations as strategies to add, subtract, factor and expand linear expressions.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ii. Students can identify the elements of an expression.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### MS Math Scoring Criteria (Grade 7 Math)

#### Performance Indicators

<table>
<thead>
<tr>
<th>E. Students will solve real-life and mathematics problems using numerical and algebraic expressions and equations. [7.EE.2.3, 7.EE.2.4]</th>
<th>i. Students can rewrite an expression in a different form.</th>
<th>ii. Students can show that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.</th>
<th>ii. Students can explain the key terms and factors for each expression in a given problem context and/or create equivalent expressions given in the problem context.</th>
<th>i. Students can create a model using rational numbers using tools strategically and can justify a solution and/or analyze errors in a real-world problem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>i. Students can solve mathematical problems with rational numbers.</td>
<td>i. Students can solve mathematical problems posed with positive rational numbers.</td>
<td>i. Students can solve multistep and real-world problems posed with rational numbers, using tools strategically; apply properties of operations, conversions between forms and assesses the reasonableness of answers.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Geometry: Understand geometric concepts and constructions, prove theorems, and apply appropriate results to solve problems.

<table>
<thead>
<tr>
<th>Scoring Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Performance Indicators</strong></td>
</tr>
<tr>
<td>B. Students will solve real-life and mathematics problems using numerical and algebraic expressions and equations. [7.G.1.1, 7.G.1.2, 7.G.1.3]</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
### MS Math Scoring Criteria (Grade 7 Math)

| C. Students will solve real-life and mathematics problems using numerical and algebraic expressions and equations. [7.G.2.4, 7.G.2.5, 7.G.2.6] | i. Students can determine the radius and/or diameter of a circle.  
ii. Students can identify supplementary, complementary, vertical, and adjacent angles.  
iii. Students can find the area of right triangles, squares, and rectangles. | i. Students can identify the formula for the area and/or circumference of a circle.  
ii. Students can use facts about relationships (supplementary, complementary, vertical, and adjacent) to find the unknown angle measure in a figure.  
iii. Students can find the area of triangles, quadrilaterals, and regular polygons; find the volume of cubes and right prisms. | i. Students can use the formulas and solve problems for the area and circumference of a circle given radius or diameter, or vice versa, given a graphic representation in a real-world context.  
ii. Students can use facts about angle relationships to write and solve multistep equations for an unknown angle in a figure.  
iii. Students can solve real-world problems involving area of 2-dimensional figures composed of triangles, quadrilaterals, and polygons; solve real-world volume and surface area problems for cubes and right prisms. | i. Students can use the relationship between circumference and area of a circle; use formulas and solve real-world problems without requiring graphic representations.  
ii. Students can find the measures of the unknown angles in a figure.  
iii. Students solve real-world problems involving surface area and volume of composite figures; use relationships between volume and surface area of 3-dimensional shapes to solve real-world problems. |
### Scoring Criteria

**Statistics and Probability:** Interpret and apply statistics and probability to analyze data, reach and justify conclusions, and make inferences.

<table>
<thead>
<tr>
<th>Performance Indicators</th>
<th>Emerging</th>
<th>Progressing</th>
<th>Meets</th>
<th>Exceeds</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>C. Students will use random sampling to draw inferences about a population.</strong> [7.SP.1.1, 7.SP.1.2]</td>
<td>i. Students can define random sample.</td>
<td>i. Students can identify that a random sample produces the most valid representation of the entire population.</td>
<td>i. Students can use statistical data to draw inferences about a population based on representative samples.</td>
<td>i. Students can generate and/or use multiple samples to gauge variations in estimates or predictions; justify the most representative sampling method for a situation.</td>
</tr>
<tr>
<td><strong>D. Students will apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.</strong> [7.SP.2.3. 7.SP.2.4]</td>
<td>i. Students can find the measures of central tendency.</td>
<td>i. Students can use basic measures of central tendency to compare two different populations.</td>
<td>i. Students can use measures of central tendency and/or variability to draw comparisons about two different populations.</td>
<td>i. Students can use measures of variability for numerical data from random samples to draw comparative inferences about two populations in any context.</td>
</tr>
<tr>
<td><strong>E. Students will draw informal comparative inferences about two populations.</strong> [7.SP.3.5. 7.SP.3.6]</td>
<td>i. Students can define probability.</td>
<td>i. Students can identify that the probability of a chance event is a number between 0 and 1. Students can make approximations of probability for a chance event.</td>
<td>i. Students can identify the probability of a chance event as equally likely or unlikely (0.5); represent the probability as a fraction, decimal, or percent. Students can use the results of an experiment to make approximations of probability for an event; predict the approximate relative frequency given the probability.</td>
<td>i. Students can compare the probabilities of two or more events and justify the likelihood of each event. Students can compare and connect the relative frequency of an event to the theoretical probability of the event; justify why the experimental probability approaches the theoretical</td>
</tr>
</tbody>
</table>
### MS Math Scoring Criteria (Grade 7 Math)

<table>
<thead>
<tr>
<th>F. Students will investigate chance processes to develop, use, and evaluate probability models. [7.SP.3.7, 7.SP.3.8]</th>
<th>i. Students can explain the difference between experimental and theoretical probability.</th>
<th>i. Students can determine and develop a theoretical probability model of a simple event; determine the sample space for compound events.</th>
<th>i. Students can design a simulation to generate frequencies for compound events; use observed frequencies to create a uniform probability model to determine theoretical probabilities of events.</th>
<th>i. Students can use observed frequencies to create a probability model for the data from a chance process where outcomes may not be uniform; compare probabilities from a model to observed frequencies; explain possible sources of any discrepancy. Students can compare and justify the experimental and theoretical probability in a given situation; compare different simulations of compound events to see which best predicts the probability.</th>
</tr>
</thead>
</table>