Summer Assignment: AB Calculus

AB Calculus has 36 Questions

OFHS Mission Statement: Our mission is to assist students reach their full potential as proficient and organized mathematicians. We intend to do this by emphasizing the importance of perseverance, commitment, and hard work in realizing worthwhile pre-determined goals. We will emphasize our school's commitment to the Florida and College Board Standards. A central focus of our teaching will be to encourage students to take responsibility for their own learning, to assist them to problem-solve, and to help them present logical, calculated and innovative solutions to challenging problems.

Things to think of before entering AB Calculus:

- Students must be able to simplify expressions and solve equations algebraically.
- Students will be required to call upon knowledge that they have used in all past math classes
- AB Calculus students have the option to continue their Mathematics studies in BC Calculus at OFHS, or later at College. If you intend to study higher level math/engineering/ Science and Technology at College you should do BC CALCULUS. BC Calculus consolidates the work that you have done in AB Calculus and introduces several other Topics.
- Students helping other students will be encouraged, but any final work must be that of the individual student. Grades will be determined using the following ratios: Tests 85%, Homework 10%, Classwork 5%. Obviously, the latter affect the former.
- Students will be required to continue developing their critical thinking and problem-solving skills. They should make at least a good attempt at solving tough problems and not leave them blank. Go to the Internet/ Facebook/Google/Friends for help.
- Tests will be the main grade in the course. Students need to study for tests!
- Tests are cumulative and could have any problems that have been previously covered in the course. REVIEW! REVIEW! REVIEW!
- Word problems are an essential part of all of these courses.
- Students will be required to take detailed notes, produce summaries of these notes and do presentations at various times in the course. Participation is essential for learning.
- Students must have, and be able to use, a graphing calculator. Recommendation is a TI 84 Plus.

Summer Assignment

- This assignment will be due the first Friday of school when we return in August.
- All problems must be worked out step by step and an explanation must be given for each. This work must be attached to the back of the assignment and be well set out in numerical order.
- Answers are provided so you can check them. Self-Grade the assignment: The numbers in the circles, next to each question, give you the value of the question. The assignment is out of 965. Change the score to a % and clearly write the result on the front.

Try to do the assignment in the following manner.

- Do Question 1: grade it (consult the answers provided).
    - If correct and you have all the work shown, give yourself a grade out of the value of the question.

- If you got it incorrect: study the answer and see if you can work, it out. This skill will be a major help in Trig and Analytic Geometry. Write the solution out 3 times but do not change the original grade.

The assignment will be assessed based on how well you followed these steps and not on the actual score that you received when you graded it. The percentage grade is valuable for the teacher.

Your first test of the school year will include questions of the type included in the Summer Assignment. GET OFF TO A GREAT START TO THE YEAR. Problems were picked to help you get a jump start for the year.

This will count as your first major homework grade. For each day it is late: a deduction will be made to the grade.

Keep Scrolling
for questions

ARE YOU READY FOR CALCULUS?

1. Simplify: (a) $\dfrac{x^3-9x}{x^2-7x+12}$ (b) $\dfrac{x^2-2x-8}{x^3+x^2-2x}$ (c) $\dfrac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$ (d) $\dfrac{9-x^{-2}}{3+x^{-1}}$

2. Rationalize the denominator: (a) $\dfrac{2}{\sqrt{3}+\sqrt{2}}$ (b) $\dfrac{4}{1-\sqrt{5}}$ (c) $\dfrac{1}{1+\sqrt{3}-\sqrt{5}}$

3. Write each of the following expressions in the form ca^pb^q where c, p and q are numbers:
   (a) $\dfrac{(2a^2)^3}{b}$ (b) $\sqrt{9ab^3}$ (c) $\dfrac{a(2/b)}{3/a}$ (d) $\dfrac{ab-a}{b^2-b}$ (e) $\dfrac{a^{-1}}{(b^{-1})\sqrt{a}}$ (f) $\left(\dfrac{a^{2/3}}{b^{1/2}}\right)^2\left(\dfrac{b^{3/2}}{a^{1/2}}\right)$

4. Solve for x (do not use a calculator):
   (a) $5^{(x+1)}=25$ (b) $\dfrac{1}{3}=3^{2x+2}$ (c) $\log_2 x=3$ (d) $\log_3 x^2=2\log_3 4-4\log_3 5$

5. Simplify: (a) $\log_2 5+\log_2(x^2-1)-\log_2(x-1)$ (b) $2\log_4 9-\log_2 3$ (c) $3^{2\log_3 5}$

6. Simplify: (a) $\log_{10}\left(10^{1/2}\right)$ (b) $\log_{10}\left(\frac{1}{10^x}\right)$ (c) $2\log_{10}\sqrt{x}+3\log_{10}x^{1/3}$

7. Solve the following equations for the indicated variables:
   (a) $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$, for a (b) $V=2(ab+bc+ca)$, for a
   (c) $A=2\pi r^2+2\pi rh$, for positive r (d) $A=P+nrP$, for P
   (e) $2x-2yd=y+xd$, for d (f) $\dfrac{2x}{4\pi}+\dfrac{1-x}{2}=0$, for x

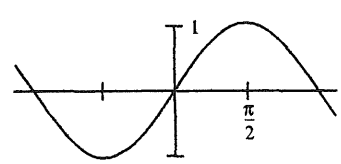
8. For the equations (a) $y=x^2+4x+3$ (b) $3x^2+3x+2y=0$ (c) $9y^2-6y-9-x=0$ complete the square and reduce to one of the standard forms $y-b=A(x-a)^2$ or $x-a=A(y-b)^2$.

9. Factor completely: (a) x^6-16x^4 (b) $4x^3-8x^2-25x+50$ (c) $8x^3+27$ (d) x^4-1

10. Find all real solutions to: (a) $x^6-16x^4=0$ (b) $4x^3-8x^2-25x+50=0$ (c) $8x^3+27=0$

11. Solve for x: (a) $3\sin^2 x=\cos^2 x$; $0\le x<2\pi$ (b) $\cos^2 x-\sin^2 x=\sin x$; $-\pi<x\le\pi$
    (c) $\tan x+\sec x=2\cos x$; $-\infty<x<\infty$

12. Without using a calculator, evaluate the following:
    (a) $\cos 210°$ (b) $\sin\dfrac{5\pi}{4}$ (c) $\tan^{-1}(-1)$ (d) $\sin^{-1}(-1)$
    (e) $\cos\dfrac{9\pi}{4}$ (f) $\sin^{-1}\dfrac{\sqrt{3}}{2}$ (g) $\tan\dfrac{7\pi}{6}$ (h) $\cos^{-1}(-1)$



13. Given the graph of $\sin x$, sketch the graphs of:
    (a) $\sin\left(x-\dfrac{\pi}{4}\right)$ (b) $\sin\left(\dfrac{x}{2}\right)$ (c) $2\sin x$ (d) $\cos x$ (e) $\dfrac{1}{\sin x}$

14. Solve the equations: (a) $4x^2+12x+3=0$ (b) $2x+1=\dfrac{5}{x+2}$ (c) $\dfrac{x+1}{x}-\dfrac{x}{x+1}=0$

15. Find the remainders on division of:
    (a) $x^5-4x^4+x^3-7x+1$ by $x+2$. (b) $x^5-x^4+x^3+2x^2-x+4$ by x^3+1.

16. (a) The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$. Find all other solutions.
(b) Solve for x, the equation $12x^3 + 8x^2 - x - 1 = 0$. (All solutions are rational and between ± 1.)

17. Solve the inequalities (a) $x^2 + 2x - 3 \leq 0$ (b) $\dfrac{2x-1}{3x-2} \leq 1$ (c) $x^2 + x + 1 > 0$

18. Solve for x: (a) $|-x+4| \leq 1$ (b) $|5x - 2| = 8$ (c) $|2x+1| = x+3$

19. Determine the equations of the following lines: (a) the line through (-1,3) and (2,-4);
(b) the line through $(-1, 2)$ and perpendicular to the line $2x - 3y + 5 = 0$;
(c) the line through $(2, 3)$ and the midpoint of the line segment from $(-1, 4)$ to $(3, 2)$.

20. (a) Find the point of intersection of the lines: $3x - y - 7 = 0$ and $x + 5y + 3 = 0$
(b) Shade the region in the $x - y$ plane that is described by the inequalities $\begin{cases} 3x - y - 7 & < 0 \\ x + 5y + 3 & \geq 0 \end{cases}$

21. Find the equations of the following circles:
(a) the circle with centre at $(1, 2)$ that passes through the point $(-2, -1)$;
(b) the circle that passes through the origin and has intercepts equal to 1 and 2 on the $x-$ and $y-$ axes, respectively.

22. For the circle $x^2 + y^2 + 6x - 4y + 3 = 0$, find:
(a) the centre and radius; (b) the equation of the tangent at $(-2, 5)$

23. A circle is tangent to the y-axis at $y = 3$ and has one x-intercept at $x = 1$.
(a) Determine the other x-intercept. (b) Deduce the equation of the circle.

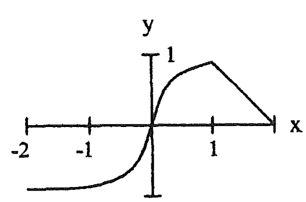
24. A curve is traced by a point $P(x, y)$ which moves such that its distance from the point $A(-1, 1)$ is three times its distance from the point $B(2, -1)$. Determine the equation of the curve.

25. (a) Find the domain of the function $f(x) = \dfrac{3x+1}{\sqrt{x^2+x-2}}$.
(b) Find the domain and range of the functions: i) $f(x) = 7$ ii) $g(x) = \dfrac{5x-3}{2x+1}$

26. Let $f(x) = \dfrac{|x|}{x}$. Show that $f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$. Find the domain and range of $f(x)$.

27. Simplify $\dfrac{f(x+h) - f(x)}{h}$, where (a) $f(x) = 2x + 3$ (b) $f(x) = \dfrac{1}{x+1}$ (c) $f(x) = x^2$.

28. The graph of the function $y = f(x)$ is given as follows:



Determine the graphs of the functions:
(a) $f(x+1)$ (b) $f(-x)$ (c) $|f(x)|$ (d) $f(|x|)$

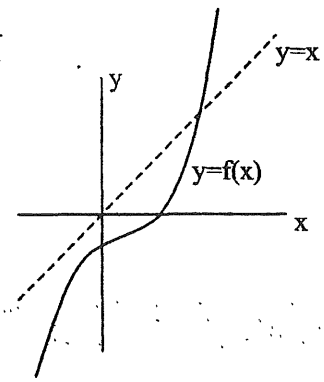
29. Sketch the graphs of the functions: (a) $g(x) = |3x + 2|$ (b) $h(x) = |x(x-1)|$

30. (a) The graph of a quadratic function (a parabola) has x-intercepts -1 and 3 and a range consisting of all numbers less than or equal to 4. Determine an expression for the function.
(b) Sketch the graph of the quadratic function $y = 2x^2 - 4x + 3$.

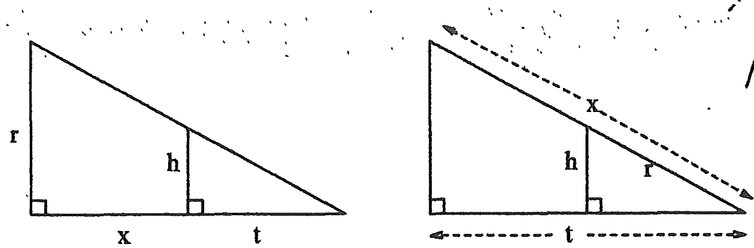
31. Write as a single equation in x and y: (a) $\begin{cases} x = t+1 \\ y = t^2 - t \end{cases}$ (b) $\begin{cases} x = \sqrt[3]{t} - 1 \\ y = t^2 - t \end{cases}$ (c) $\begin{cases} x = \sin t \\ y = \cos t \end{cases}$

32. Find the inverse of the functions: (a) $f(x) = 2x + 3$ (b) $f(x) = \dfrac{x+2}{5x-1}$
(c) $f(x) = x^2 + 2x - 1\ ,\ \ x > 0$

33. A function $f(x)$ has the graph to the right
Sketch the graph of the inverse function $f^{-1}(x)$.

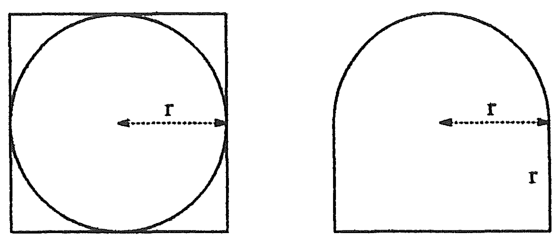


34. Express x in terms of the other variables in the picture.



(a) (b)

35. (a) Find the ratio of the area inside the square but outside the circle to the area of the square in the picture (a) below.



(a) (b)

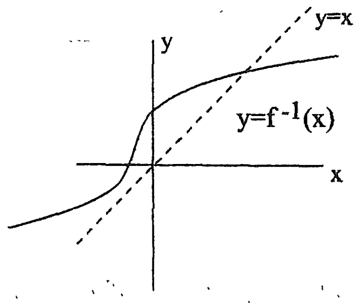
(b) Find a formula for the perimeter of a window of the shape in the picture (b) above.
(c) A water tank has the shape of a cone (like an ice cream cone without ice cream). The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep (in the middle) what is the surface area of the top of the water?
(d) Two cars start moving from the same point. One travels south at 100km/hour, the other west at 50 km/hour. How far apart are they two hours later?
(e) A kite is 100m above the ground. If there are 200m of string out, what is the angle between the string and the horizontal. (Assume that the string is perfectly straight.)

36. You should know the following trigonometric identities.
(A) $\sin(-x) = -\sin x$ (C) $\cos(x+y) = \cos x \cos y - \sin x \sin y$
(B) $\cos(-x) = \cos x$ (D) $\sin(x+y) = \sin x \cos y + \cos x \sin y$
Use these to derive the following important identities, which you should also know.
(a) $\sin^2 x + \cos^2 x \equiv 1$ (use C and $\cos 0 = 1$) (b) $\sin 2x \equiv 2 \sin x \cos x$ (c) $\cos 2x \equiv \cos^2 x - \sin^2 x$
(d) $\cos 2x \equiv 2\cos^2 x - 1$ (e) $\cos 2x \equiv 1 - 2\sin^2 x$ (f) $\left|\cos \frac{x}{2}\right| \equiv \sqrt{\frac{1+\cos x}{2}}$ (g) $\left|\sin \frac{x}{2}\right| \equiv \sqrt{\frac{1-\cos x}{2}}$

31. (a) $y = x^2 - 3x + 2$ (b) $y = x(x^2 + 3x + 3)(x+1)^3$ (c) $x^2 + y^2 = 1$

32. (a) $f^{-1}(x) = \dfrac{x-3}{2}$ (b) $f^{-1}(x) = \dfrac{x+2}{5x-1}$ (c) $-1 + \sqrt{x+2},\ x > -1$

33.



34. (a) $x = t\left(\dfrac{r-h}{h}\right)$ (b) $x = \dfrac{rt}{\sqrt{r^2 - h^2}}$

35. (a) $1 - \frac{\pi}{4}$ (b) $4r + \pi r$ (c) $\frac{9\pi}{4}$ (d) $100\sqrt{5}$ km (e) $\frac{\pi}{6}$ or $30°$

36. (a) Use B, $y = x$, A and $\cos 0 = 1$ (b) Use D. (c) Use C. (d) Use (c) then (a). (e) Use (d) then (a). (f) Replace x by $\frac{\pi}{2}$ in (d). (g) Replace x by $\frac{\pi}{2}$ in (e).

AB Calc Summer Solutions

$$\frac{\quad}{975} = \quad \%$$

1. a) $\dfrac{x^3-9x}{x^2-7x+12} = \dfrac{x(x^2-9)}{(x-4)(x-3)}$

$= \dfrac{x(x+3)\cancel{(x-3)}}{(x-4)\cancel{(x-3)}}$

$= \boxed{\dfrac{x(x+3)}{(x-4)} \quad x \neq 3, 4}$ (+5)

b) $\dfrac{x^2-2x-8}{x^3+x^2-2x} = \dfrac{(x-4)(x+2)}{x(x^2+x-2)}$

$= \dfrac{(x-4)\cancel{(x+2)}}{x\cancel{(x+2)}(x-1)}$

$= \boxed{\dfrac{x-4}{x(x-1)} \quad x \neq -2, 0, 1}$ (+5)

c) $\dfrac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}} = \dfrac{5-x}{5x} \cdot \dfrac{25x^2}{25-x^2}$

top $= \dfrac{5}{5x} - \dfrac{x}{5x}$

$= \dfrac{5-x}{5x}$

bottom $= \dfrac{25}{25x^2} - \dfrac{x^2}{25x^2}$

$= \dfrac{25-x^2}{25x^2}$

$= \dfrac{\cancel{(5-x)}\,\cancel{25}^{5}\cancel{x^2}}{\cancel{5}\cancel{x}\cancel{(5-x)}(5+x)}$

$= \boxed{\dfrac{5x}{5+x} \quad x \neq 0, \pm 5}$ (+10)

d) $\dfrac{9-x^{-2}}{3+x^{-1}} = \dfrac{9x^2-1}{x^2} \cdot \dfrac{x}{3x+1}$

top $= \dfrac{9}{1} - \dfrac{1}{x^2}$

$= \dfrac{9x^2-1}{x^2}$

bottom $= \dfrac{3}{1} + \dfrac{1}{x}$

$= \dfrac{3x+1}{x}$

$= \dfrac{(3x-1)\cancel{(3x+1)}\cancel{x}}{x^{\cancel{2}}\cancel{(3x+1)}}$

$= \boxed{\dfrac{3x-1}{x} \quad x \neq 0, -\frac{1}{3}}$ (+10)

2. a) $\frac{2}{\sqrt{3}+\sqrt{2}} \cdot \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})} = \frac{2(\sqrt{3}-\sqrt{2})}{3-2}$

$= \boxed{2(\sqrt{3}-\sqrt{2})}$ +5

b) $\frac{4}{1-\sqrt{5}} \cdot \frac{(1+\sqrt{5})}{(1+\sqrt{5})} = \frac{4(1+\sqrt{5})}{1-5}$

$= \boxed{-1-\sqrt{5}}$ +5

c) $\frac{1}{(1+\sqrt{3})-\sqrt{5}} \cdot \frac{(1+\sqrt{3})+\sqrt{5}}{(1+\sqrt{3})+\sqrt{5}} = \frac{1+\sqrt{3}+\sqrt{5}}{(1+\sqrt{3})^2-5}$

$= \frac{1+\sqrt{3}+\sqrt{5}}{1+2\sqrt{3}+3-5}$

$= \frac{1+\sqrt{3}+\sqrt{5}}{(2\sqrt{3}-1)} \cdot \frac{(2\sqrt{3}+1)}{(2\sqrt{3}+1)}$

$= \frac{(2\sqrt{3}+1)(1+\sqrt{3}+\sqrt{5})}{4 \cdot 3-1}$

$= \frac{2\sqrt{3}+2(3)+2\sqrt{15}+1+\sqrt{3}+\sqrt{5}}{11}$

$= \boxed{\frac{3\sqrt{3}+7+2\sqrt{15}+\sqrt{5}}{11}}$ +10

3. a) $\dfrac{(2a^2)^3}{b} = \boxed{8a^6b^{-1}}$ (+5)

b) $\sqrt{9ab^3} = \boxed{3a^{1/2}b^{3/2}}$ (+5)

c) $\dfrac{a\left(\frac{2}{b}\right)}{\frac{3}{a}} = \dfrac{2a}{b} \cdot \dfrac{a}{3}$

$= \boxed{\frac{2}{3}a^2b^{-1}}$ (+5)

d) $\dfrac{ab-a}{b^2-b} = \dfrac{a\cancel{(b-1)}}{b\cancel{(b-1)}}$

$= \boxed{ab^{-1}}$ (+5)

e) $\dfrac{a^{-1}}{b^{-1}\sqrt{a}} = a^{-1} \cdot a^{-1/2} b^{1}$

$= \boxed{a^{-\frac{3}{2}}b}$ (+5)

f) $\left(\dfrac{a^{2/3}}{b^{1/2}}\right)^2 \left(\dfrac{b^{3/2}}{a^{1/2}}\right) = \dfrac{a^{4/3}b^{3/2}}{a^{1/2}b}$

$= \boxed{a^{5/6}b^{1/2}}$ (+10)

4. a) $5^{x+1} = 25$

$5^{x+1} = 5^2$

$x+1 = 2$

$\boxed{x = 1}$ (+5)

b) $3^{2x+2} = 3^{-1}$

$2x+2 = -1$

$2x = -3$

$\boxed{x = -\frac{3}{2}}$ (+5)

c) $\log_2 x = 3$

$2^3 = x$

$\boxed{x = 8}$ (+5)

d) $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

$\log_3 x^2 = \log_3 \frac{4^2}{5^4}$

$x^2 = \frac{4^2}{5^4}$

$\boxed{x = \pm \frac{4}{25}}$ (+10)

5. a) $\log_2 5 + \log_2(x^2-1) - \log_2(x-1)$

$= \log_2 \frac{5(x^2-1)}{(x-1)}$

$= \log_2 \frac{5\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$

$= \boxed{\log_2 5(x+1)}$ +10

b) $2\log_4 9 - \log_2 3$

$= \log_4 9^2 - \log_2 3$

$= \frac{\log_2 9^2}{\log_2 4} - \log_2 3$

$= \frac{1}{2}\log_2 9^2 - \log_2 3$

$= \log_2 9 - \log_2 3$

$= \log_2 \frac{9}{3}$

$= \boxed{\log_2 3}$ +10

c) $3^{2\log_3 5}$

$= 3^{\log_3 5^2}$

$= 5^2$

$= \boxed{25}$

+5

6. a) $\log_{10} 10^{1/2}$

$= \frac{1}{2}\log_{10} 10$

$= \boxed{\frac{1}{2}}$

+5

b) $\log_{10}\left(\frac{1}{10^x}\right)$

$= \log_{10} 10^{-x}$

$= \boxed{-x}$

+5

c) $2\log_{10}\sqrt{x} + 3\log_{10} x^{1/3}$

$= \log_{10}(x \cdot x)$

$= \log_{10} x^2$

$= \boxed{2\log_{10} x}$

+5

7. a) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$\frac{x}{a} = \frac{1}{1} - \frac{y}{b} - \frac{z}{c}$

$\frac{x}{a} = \frac{bc - yc - zb}{bc}$

$\boxed{a = \frac{bcx}{bc - cy - bz}}$

(+10)

b) $V = 2(ab + bc + ca)$

$V - 2bc = 2ab + 2ca$

$V - 2bc = a(2b + 2c)$

$a = \frac{V - 2bc}{2b + 2c}$

$\boxed{a = \frac{V - 2bc}{2(b+c)}}$

(+10)

c) $A = 2\pi r^2 + 2\pi r h$

$0 = 2\pi r^2 + 2\pi h r - A$

$r = \frac{-2\pi h \pm \sqrt{(2\pi h)^2 - 4(2\pi)(A)}}{2(2\pi)}$

$= \frac{-2\pi h \pm \sqrt{4\pi^2 h^2 - 8\pi A}}{4\pi}$

$= \frac{-\overset{1}{\cancel{2}}\pi h \pm 2\sqrt{\pi^2 h^2 - 2\pi A}}{\underset{2}{\cancel{4}}\pi}$

$= \boxed{\frac{-\pi h \pm \sqrt{\pi^2 h^2 - 2\pi A}}{2\pi}, r > 0}$ (+10)

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = 2\pi$, $b = 2\pi h$, $c = A$

$\sqrt{4(\pi^2 h^2 - 2\pi A)}$

$2\sqrt{\pi^2 h^2 - 2\pi A}$

d) $A = P + nrP$

$A = P(1 + nr)$

$\boxed{P = \frac{A}{1 + nr}}$

(+5)

e) $2x - 2yd = y + xd$

$xd + 2yd = 2x - y$

$(x + 2y)d = 2x - y$

$\boxed{d = \frac{2x - y}{x + 2y}}$

(+5)

f) $\frac{2x}{4\pi} + \frac{1 - x}{2} = 0$

$\frac{2x + 2\pi - 2\pi x = 0}{4\pi}$

$2x + 2\pi - 2\pi x = 0$

$x(2 - 2\pi) = -2\pi$

$x = -\frac{\overset{1}{\cancel{2}}\pi}{\underset{1}{\cancel{2}} - 2\pi}$

$x = \boxed{-\frac{\pi}{\;}}$ (+10)

8. a) $y = x^2 + 4x + 3$

$y = (x^2 + 4x + \underline{4}) + 3 - \underline{4}$

$\left(\frac{4}{2}\right)^2 = 2^2 = 4$

$y = (x+2)^2 - 1$

$\boxed{y - (-1) = (x - (-2))^2}$

(+10)

b) $3x^2 + 3x + 2y = 0$

$3(x^2 + x + \underline{\tfrac{1}{4}}) - 3(\underline{\tfrac{1}{4}}) + 2y = 0$

$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$3(x + \frac{1}{2})^2 - \frac{3}{4} + 2y = 0$

$-2y + \frac{3}{4} = 3(x + \frac{1}{2})^2$

$-2(y - \frac{3}{8}) = 3(x + \frac{1}{2})^2$

$\boxed{y - \frac{3}{8} = -\frac{3}{2}\left(x - (-\tfrac{1}{2})\right)^2}$

(+10)

c) $9y^2 - 6y - 9 - x = 0$

$x = 9(y^2 - \frac{2}{3}y + \underline{\tfrac{1}{9}}) - 9 - 9(\underline{\tfrac{1}{9}})$

$\left(\frac{\frac{2}{3}}{2}\right)^2 = \frac{1}{9}$

$x = 9(y - \frac{1}{3})^2 - 10$

$\boxed{x - (-10) = 9(y - \frac{1}{3})^2}$ (+10)

9. a) $x^6 - 16x^4$

$= x^4(x^2 - 16)$

$= \boxed{x^4(x-4)(x+4)}$

(+5)

b) $(4x^3 - 8x^2)(-25x + 50)$

$4x^2(x-2) - 25(x-2)$

$(x-2)(4x^2 - 25)$

$\boxed{(x-2)(2x+5)(2x-5)}$

(+10)

c) $8x^3 + 27$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$= \boxed{(2x+3)(4x^2 - 6x + 9)}$

(+5)

d) $x^4 - 1$

$= (x^2 - 1)(x^2 + 1)$

$= \boxed{(x+1)(x-1)(x^2+1)}$

(+5)

7

0. a) $x^6 - 16x^4 = 0$

$x^4(x-4)(x+4) = 0$

$\boxed{x = 0, \pm 4}$ (+5)

b) $4x^3 - 8x^2 - 25x + 50 = 0$

$(x-2)(2x-5)(2x+5) = 0$

$\boxed{x = 2, \pm \frac{5}{2}}$ (+5)

c) $8x^3 + 27 = 0$

$(2x+3)(4x^2 - 6x + 9) = 0$

$\boxed{x = -\frac{3}{2}}$ (+5)

$4x^2 - 6x + 9 = 0$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(4)(9)}}{2(4)}$

$= \frac{6 \pm \sqrt{-108}}{8}$

not a <u>real</u> solution

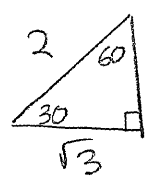
11. a) $3\sin^2 x = \cos^2 x$

$3\sin^2 x = 1 - \sin^2 x$

$4\sin^2 x = 1$

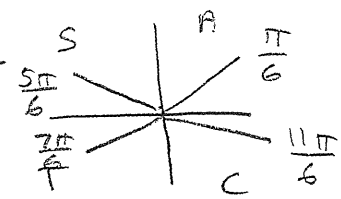
$\sin^2 x = \frac{1}{4}$

$\sin x = \pm \frac{1}{2}$



$\sin 30 = \frac{1}{2}$

$\frac{\pi}{6}$



$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$ (+10)

b) $\cos^2 x - \sin^2 x = \sin x$

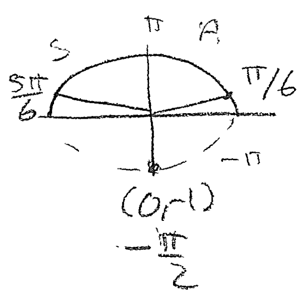
$1 - \sin^2 x - \sin^2 x = \sin x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0 \quad \sin x + 1 = 0$

$\sin x = \frac{1}{2} \quad \sin x = -1$



$-\pi < x \leq \pi$

$\boxed{x = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}}$ (+15)

11. c) $\tan x + \sec x = 2\cos x \qquad -\infty < x < \infty$

$$\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x \qquad * \cos x \neq 0$$

$$\sin x + 1 = 2\cos^2 x$$

$$\sin x + 1 = 2(1-\sin^2 x)$$

$$\sin x + 1 = 2 - 2\sin^2 x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$2\sin x - 1 = 0 \qquad \sin x + 1 = 0$

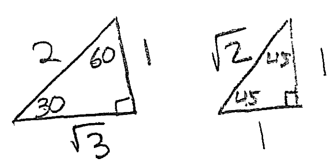
$\sin x = \frac{1}{2} \qquad \sin x = -1$

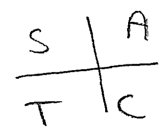
every $\frac{\pi}{6}$ & $\frac{5\pi}{6}$

$$\boxed{x = \frac{\pi}{6} \pm 2k\pi \quad x = \frac{5\pi}{6} \pm 2k\pi}$$

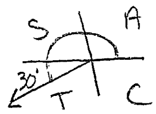
* at $\sin x = -1$ $\cos x = 0$ which isn't in the domain ∴ no solution

+20



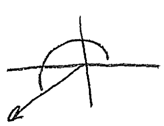


12. a) $\cos 210° = -\cos 30°$



$= \boxed{-\frac{\sqrt{3}}{2}}$ +5

b) $\sin\frac{5\pi}{4} = -\sin 45°$



$= \boxed{-\frac{\sqrt{2}}{2}}$ +5

c) $\tan^{-1}(-1) = \boxed{-\frac{\pi}{4}}$ +5

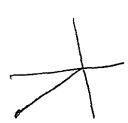
d) $\sin^{-1}(-1) = \boxed{-\frac{\pi}{2}}$ +5

e) $\cos\frac{9\pi}{4} = \cos\frac{\pi}{4}$

$= \boxed{\frac{\sqrt{2}}{2}}$ +5

f) $\sin^{-1}\frac{\sqrt{3}}{2} = \boxed{\frac{\pi}{3}}$ +5

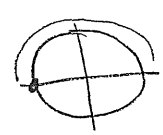
g) $\tan\frac{7\pi}{6} = \tan 30$



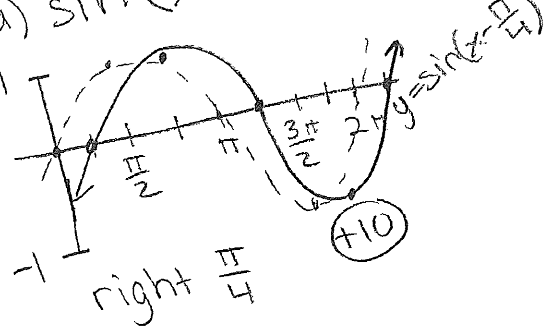
$= \frac{1}{\sqrt{3}}$

$= \boxed{\frac{\sqrt{3}}{3}}$ +5

h) $\cos^{-1}(-1) = \boxed{\pi}$ +5



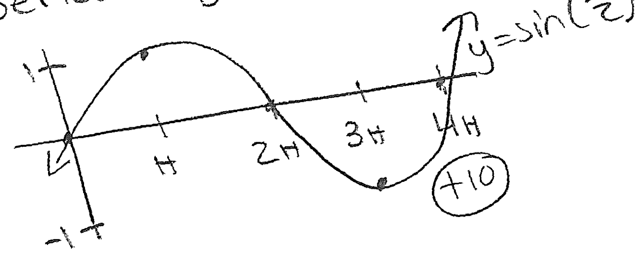
a) $\sin\left(x - \frac{\pi}{4}\right)$



right $\frac{\pi}{4}$

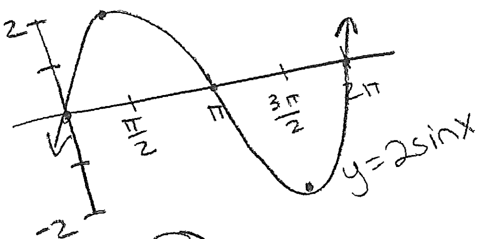
b) $\sin\left(\frac{1}{2}x\right)$

period $= \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 4\pi$



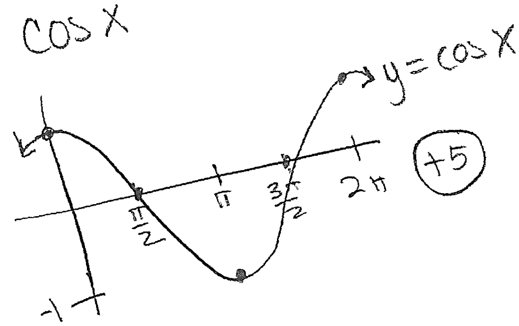
c) $y = 2\sin x$

amp $= 2$

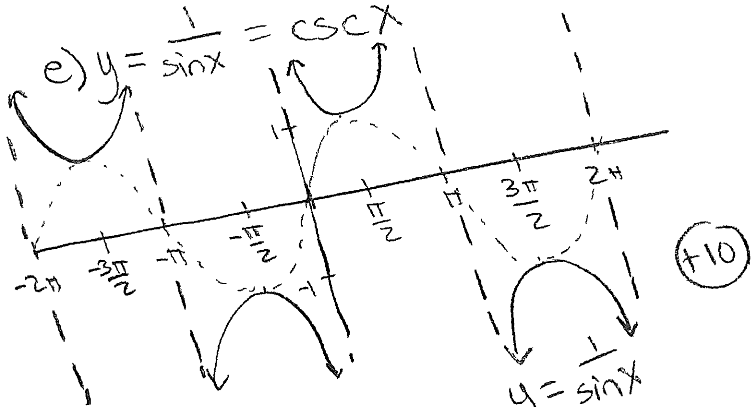


+5

d) $\cos x$



e) $y = \frac{1}{\sin x} = \csc x$



14. a) $4x^2+12x+3=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-12\pm\sqrt{12^2-4(4)(3)}}{2(4)}$$
$$=\frac{-12\pm\sqrt{96}}{8}$$
$$=\frac{-12\pm\sqrt{16\cdot 6}}{8}$$
$$=\frac{-\cancel{12}^{-3}\pm\cancel{4}^{1}\sqrt{6}}{\cancel{8}_{2}}$$
$$=\boxed{\frac{-3\pm\sqrt{6}}{2}}$$

+10

b) $2x+1=\dfrac{5}{x+2}$

$(2x+1)(x+2)=5$

$2x^2+4x+x+2-5=0$

$2x^2+5x-3=0$

$-6 \mid 5$

$-1 \mid 6$

$(2x^2-x)(+6x-3)=0$

$2x(x-1)3(2x-1)=0$

$x-1=0 \quad 2x+3=0$

$\boxed{x=1 \qquad x=-\frac{3}{2}}$

+10

c) $\left[\dfrac{x+1}{x}-\dfrac{x}{x+1}=0\right]x(x+1)$

$(x+1)^2-x^2=0$

$x^2+2x+1-x^2=0$

$2x+1=0$

$\boxed{x=-\frac{1}{2}}$ +10

15. a) $x^5-4x^4+x^3-7x+1$

Remainder Thm (s/o $x=-2$)

Rem. $=(-2)^5-4(-2)^4+(-2)^3-7(-2)+1$

$=\boxed{-89}$ +10

b)
$$\begin{array}{r|l} & x^2-x+1 \\ \hline x^3+0x^2+0x+1 & \cancel{x^5}-x^4+x^3+2x^2-x+4 \\ & -(\cancel{x^5}+0x^4+0x^3+x^2) \\ \hline & -\cancel{x^4}+x^3+x^2-x \\ & -(-\cancel{x^4}+0x^3+0x^2-x) \\ \hline & \cancel{x^3}+x^2+0x+4 \\ & -(\cancel{x^3}+0x^2+0x+1) \\ \hline & x^2\quad +3 \end{array}$$

$\boxed{\text{Remainder} = x^2+3}$

+10

16. a) $x=2$ is a solution $\therefore$ $(x-2)$ is a factor

$$\begin{array}{r|rrrr} 2 & 12 & -23 & -3 & 2 \\ & \downarrow & 24 & 2 & -2 \\ \hline & 12 & 1 & -1 & 0 \end{array}$$

$= (x-2)(12x^2+x-1)$

$= (x-2)(4x-1)(3x+1)$

$\boxed{x = \frac{1}{4}, -\frac{1}{3}}$ +15

b) $\underset{q}{12}x^3+8x^2-x-\underset{p}{1}=0$

$\frac{p}{q} = \pm\frac{1}{12}, \pm\frac{1}{2}, \pm\frac{1}{6}, \pm\frac{1}{3}, \pm\frac{1}{4}$

$x=\frac{1}{2}$ $12(\frac{1}{2})^3+8(\frac{1}{2})^2-(\frac{1}{2})-1=2$ no

$x=-\frac{1}{2}$ $12(-\frac{1}{2})^3+8(-\frac{1}{2})^2-(-\frac{1}{2})-1=0$ ✓

+10

$\therefore (x+\frac{1}{2})$ is a factor

$$\begin{array}{r|l} & 12x^2+2x-2 \\ \hline x+\frac{1}{2} & 12x^3+8x^2-x-1 \\ & -(\cancel{12x^3}+6x^2) \\ \hline & \cancel{2x^2}-x \\ & -(\cancel{2x^2}+x) \\ \hline & \cancel{-2x}-1 \\ & -(\cancel{-2x}-1) \\ \hline & 0 \end{array}$$

+10

$12x^2+2x-2$

$12x^2+6x-4x-2$

$6x(2x+1)-2(2x+1)$

$(2x+1)(6x+2)$

$x=-\frac{1}{2}$ $x=-\frac{2}{6}=-\frac{1}{3}$

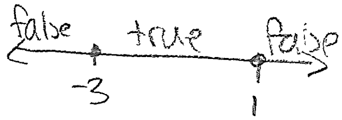
$$\begin{array}{r|l} -24 & 2 \\ \hline -4 & 6 \end{array}$$

$\boxed{\text{solutions: } x=-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{3}}$ +10

17. a) $x^2+2x-3 \le 0$

$(x+3)(x-1) \le 0$

$x=-3 \quad x=1$

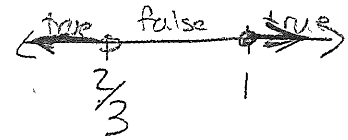


$\boxed{-3 \le x \le 1}$ (+10)

b) $\dfrac{2x-1}{3x-2} \le 1$

$2x-1 \le 3x-2$

$1 \le x \quad x \ne \frac{2}{3}$



$\boxed{x < \frac{2}{3} \cup x \ge 1}$

(+10)

c) $x^2+x+1>0$

$b^2-4ac = 1^2-4(1)(1)$

$= -3$

no real roots

above x-axis

$\boxed{\text{all reals}}$ (+5)

18. a) $|-x+4| \le 1$

$-x+4 \le 1 \qquad -x+4 \ge -1$

$-x \le -3 \qquad -x \ge -5$

$x \ge 3 \qquad x \le 5$

$\boxed{3 \le x \le 5}$ (+5)

b) $|5x-2|=8$

$5x-2=8 \qquad 5x-2=-8$

$\boxed{x=2 \qquad x=-\frac{6}{5}}$

(+5)

c) $|2x+1| = x+3$

$2x+1 = x+3 \qquad 2x+1 = -x-3$

$x=2 \qquad 3x=-4$

$x=-\frac{4}{3}$

(+5)

19. a) $(-1,3)$ $(2,-4)$

$m = \frac{y_2-y_1}{x_2-x_1}$ $\quad y-y_1 = m(x-x_1)$

$= \frac{-4-3}{2--1}$ $\quad y-3 = -\frac{7}{3}(x+1)$

$\quad y-3 = -\frac{7}{3}x - \frac{7}{3}$

$= -\frac{7}{3}$ $\quad \boxed{y = -\frac{7}{3}x + \frac{2}{3}}$ +10

b) $(-1,2)$ $\perp$ to $2x-3y+5=0$

$m_\perp = -\frac{3}{2}$ $\quad -3y = -2x-5$

$y-2 = -\frac{3}{2}(x+1)$ $\quad y = \frac{2}{3}x + \frac{5}{3}$

$y-2 = -\frac{3}{2}x - \frac{3}{2}$

$\boxed{y = -\frac{3}{2}x + \frac{1}{2}}$ +10

c) $\underline{(2,3)}$ midpoint of $(-1,4)$ & $(3,2)$ $\quad$ Midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

vertical line through 3 $\quad = \left(\frac{-1+3}{2}, \frac{4+2}{2}\right)$

$\boxed{y=3}$ +10 $\quad = \underline{(1,3)}$

20. a) $y = 3x-7$ *substitution used but could do elimination*

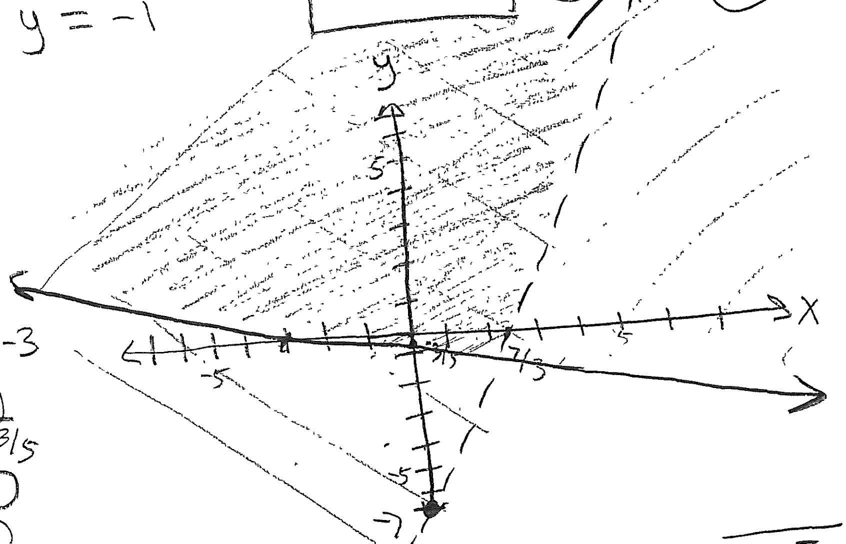
$x+5(3x-7)+3=0$ $\quad y = 3(2)-7$ $\quad \boxed{(2,-1)}$ +5

$16x-32=0$ $\quad y=-1$

$x=2$

shaded = solution +10

b) $\begin{cases} 3x-y-7<0 \\ x+5y+3 \geq 0 \end{cases}$



$3x-y<7$

x	y
0	-7
$\frac{7}{3}$	0

$x+5y \geq -3$

x	y
0	$-3/5$
-3	0

(0,0)

14

21. a) (1,2) through (-2,-1)

$(x-h)^2+(y-k)^2=r^2$

$\boxed{(x-1)^2+(y-2)^2=18}$

+10

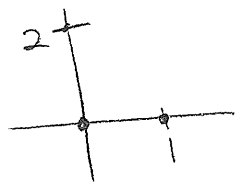
$r=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(-2-1)^2+(-1-2)^2}$

$=\sqrt{9+9}$

$=\sqrt{18}$

b)



Passes through (0,0), (0,2), (1,0)

$(x-a)^2+(y-b)^2=r^2$

substitute as (x,y)

$\boxed{(x-\frac{1}{2})^2+(y-1)=\frac{5}{4}}$

+20

set up 3 equations

(0,0): $a^2+b^2=r^2$ ①

(1,0): $(1-a)^2+b^2=r^2$ ②

(0,2): $a^2+(2-b)^2=r^2$ ③

use ① & ② to solve by substitution

$(1-a)^2+b^2=a^2+b^2$

$1-2a+\cancel{a^2}+\cancel{b^2}=\cancel{a^2}+\cancel{b^2}$

$-2a=-1$

$a=\frac{1}{2}$

use ① & ③ to solve by substitution

$a^2+(2-b)^2=a^2+b^2$

$\cancel{a^2}+4-4b+\cancel{b^2}=\cancel{a^2}+\cancel{b^2}$

$-4b=-4$

$b=1$

sub $a=\frac{1}{2}$ & $b=1$ into ① & solve for r

$r^2=\frac{1}{4}+1$

$r^2=\frac{5}{4}$

30

22. $x^2+y^2+6x-4y+3=0$

15

a) $(x^2+6x+\underline{9})+(y^2-4y+\underline{4})=-3+\underline{9}+\underline{4}$

$\left(\frac{6}{2}\right)^2=3^2=9$ $\quad\left(\frac{-4}{2}\right)^2=(-2)^2=4$

$(x+3)^2+(y-2)^2=10$

$\boxed{\begin{array}{l}\text{center } (-3,2)\\ \text{radius}=\sqrt{10}\end{array}}$ +10

b) tangent & radius are $\perp$

$(-3,2)$ & $(-2,5)$

$m=\frac{5-2}{-2--3}=\frac{-3}{1}=-3$

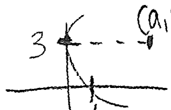
$m_\perp=-\frac{1}{3}$ & $(-2,5)$

$y-5=-\frac{1}{3}(x+2)$

$y-5=-\frac{1}{3}x-\frac{2}{3}$

$\boxed{y=-\frac{1}{3}x+\frac{13}{3}}$ +15

23.

b) 

$b=3$

$(x-a)^2+(y-b)^2=r^2$

$(x-a)^2+(y-3)^2=r^2$

$(0,3)]\quad a^2+(3-3)^2=r^2 \Rightarrow a^2=r^2 \quad \therefore r=a$

$(1,0)]\quad (1-a)^2+(0-3)^2=a^2 \qquad r=5$

$1-2a+\cancel{a^2}+9=\cancel{a^2}$

$-2a=-10$

$a=5$

$\boxed{(x-5)^2+(y-3)^2=25}$ +15

a) x-int $\Rightarrow y=0$

$(x-5)^2+9=25$

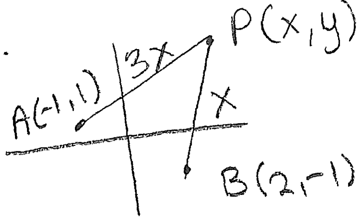
$(x-5)^2=16$

$x-5=\pm 4$

$x-5=4 \qquad x-5=-4$

$\boxed{x=9}$ +10 $\qquad x=1$

16

24. 

$PA = \sqrt{(x+1)^2+(y-1)^2}$ $\qquad$ $dist = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$PB = \sqrt{(x-2)^2+(y+1)^2}$

$$3(PB) = PA$$

$$\left(3\sqrt{(x-2)^2+(y+1)^2}\right)^2 = \left(\sqrt{(x+1)^2+(y-1)^2}\right)^2$$

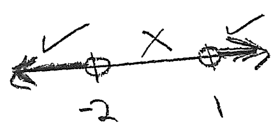
$$9\left[(x-2)^2+(y+1)^2\right] = (x+1)^2+(y-1)^2$$

$$9[x^2-4x+4+y^2+2y+1] = x^2+2x+1+y^2-2y+1$$

$$9x^2-36x+36+9y^2+18y+9 = \cancel{x^2}\,\cancel{+2x}+1+\cancel{y^2}-2y+1$$

$$\boxed{\begin{array}{l} 8x^2-38x+8y^2+20y+43=0 \\ \text{a circle} \end{array}}$$ (+20)

25. a) $f(x) = \dfrac{3x+1}{\sqrt{x^2+x-2}}$

domain: $\sqrt{x^2+x-2}$ 

$(x+2)(x-1)=0$ $\qquad$ x=0 doesn't give real root

$x=-2 \quad x=1$

$\boxed{\text{Domain}=\{x : x<-2 \cup x>1\}}$ (+10)

b) i) $f(x)=7$

$\boxed{\begin{array}{l}\text{Domain}=\{x:\text{all reals}\} \\ \text{Range}=\{y=7\}\end{array}}$

(+5)

ii) $g(x) = \dfrac{5x-3}{2x+1}$ $\qquad$ $2x+1=0$, $x=-\frac{1}{2}$ va $\qquad$ ha $\frac{5}{2}$

$\boxed{\begin{array}{l}\text{Domain}=\{x:\text{all reals}, x\neq -\frac{1}{2}\} \\ \text{Range}=\{y:\text{all reals}, y\neq \frac{5}{2}\}\end{array}}$ (+5) (+10)

$\overline{\quad 50 \quad}$

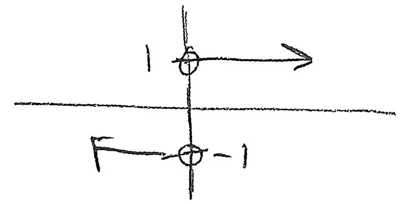
17

26. $f(x) = \frac{|x|}{x}$

$f(x) = \frac{x}{x}$ or $f(x) = \frac{-x}{x}$

$= 1$ $\quad$ $= -1$

$x \neq 0$



$$\boxed{\begin{array}{l} \text{Domain} = \{x : \text{all reals}, x \neq 0\} \\ \text{Range} = \{y : y = 1 \text{ or } y = -1\} \end{array}}$$

+10

27. $\frac{f(x+h) - f(x)}{h}$

a) $f(x) = 2x + 3$

$f(x+h) = 2(x+h) + 3$

$= 2x + 2h + 3$

$$\frac{(\cancel{2x} + 2h \cancel{+3}) - (\cancel{2x} \cancel{+3})}{h}$$

$= \frac{2h}{h}$

$= \boxed{2}$

+10

b) $f(x) = \frac{1}{x+1}$

$f(x+h) = \frac{1}{(x+h)+1}$

$= \frac{1}{x+h+1}$

$$\frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h}$$

$$= \frac{\frac{(\cancel{x}\cancel{+1}) - (\cancel{x} + h \cancel{+1})}{(x+1)(x+h+1)}}{h}$$

$$= \frac{-\cancel{h}}{(x+1)(x+h+1)} \cdot \frac{1}{\cancel{h}}$$

$$= \boxed{-\frac{1}{(x+1)(x+h+1)}}$$

+10

c) $f(x) = x^2$

$f(x+h) = (x+h)^2$

$= x^2 + 2hx + h^2$

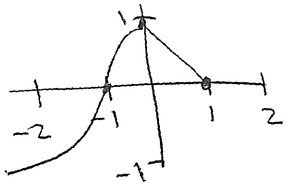
$$\frac{(\cancel{x^2} + 2\cancel{h}x + h^{\cancel{2}}) - \cancel{x^2}}{\cancel{h}}$$

$= \boxed{2x + h}$

+10

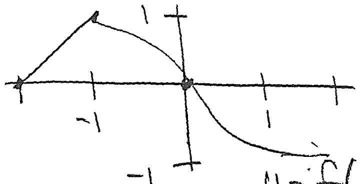
$\overline{}$
40

28. a) left 1



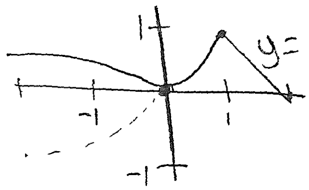
$y=f(x+1)$ (+5)

b) reflect over y-axis



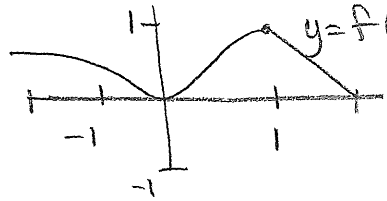
(+5)

c) absolute value of total graph



(+5)

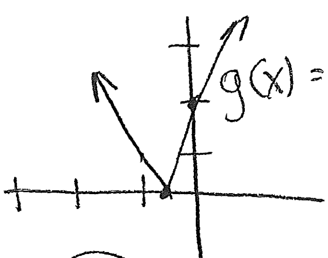
d) $f(|x|)$



(+5)

29. a) $g(x) = |3x+2|$

x	y
0	2
$-\frac{2}{3}$	0
-1	1

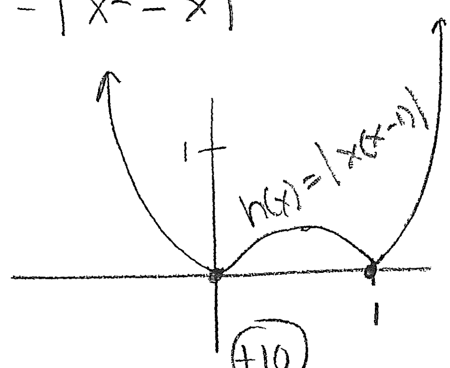


(+10)

b) $h(x) = |x(x-1)|$

$= |x^2 - x|$

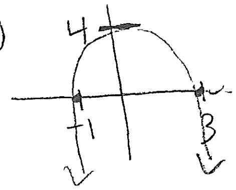
x	y
0	0
1/2	1/4
1	0



(+10)

40

30. a)



going down
so a is negative

x-intercepts ⇒ factors

$\therefore (x--1)(x-3)$

$=(x+1)(x-3)$

$= x^2-2x-3$

$y=-(x^2-2x-3)$

$y=-ax^2+2x+3$

need to find a

vertex $= \left(\frac{-1+3}{2}, 4\right)$

$= (1, 4)$
x y

$4=-a(1)^2+2(1)+3$

$-1=-a$

$a=1$

$\boxed{y=-x^2+2x+3}$

(+15)

b) $y=2x^2-4x+3$

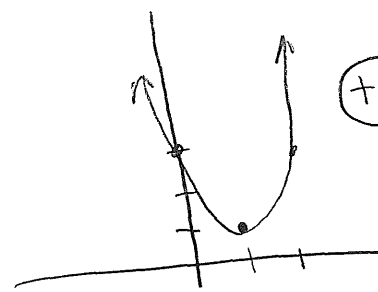
$y=2(x^2-2x+\underline{1})+3-2(\underline{1})$

$\left(\frac{-2}{2}\right)^2=1$

$y=2(x-1)^2+1$

vertex (1,1)

y-int (0,3) & reflect (2,3)



(+10)

31. a) $x=t+1 \rightarrow t=x-1$

$y=t^2-t$

$y=(x-1)^2-(x-1)$

$y=x^2-2x+1-x+1$

$\boxed{y=x^2-3x+2}$

(+5)

b) $x=\sqrt[3]{t}-1 \rightarrow (x+1)^3=t$

$y=t^2-t$

$y=(x+1)^6-(x+1)^3$

$=(x+1)^3\left[(x+1)^3-1\right]$

$=(x+1)^3\left[(x+1)(x^2+2x+1)-1\right]$

$=(x+1)^3\left[x^3+2x^2+x+x^2+2x+1-1\right]$

$=(x+1)^3(x^3+3x^2+3x)$

$\boxed{y=x(x^2+3x+3)(x+1)^3}$

(+10)

40

31. continue

c) $x = \sin t \rightarrow x^2 = \sin^2 t$
$y = \cos t \quad y^2 = \cos^2 t$

$\sin^2 t + \cos^2 t = x^2 + y^2$

$\boxed{x^2 + y^2 = 1}$ (+10)

32. a) $f(x) = 2x+3$

$x = 2y+3$

$2y = x-3$

$y = \frac{x-3}{2}$

$\boxed{f^{-1}(x) = \frac{x-3}{2}}$

(+10)

b) $f(x) = \frac{x+2}{5x-1}$

$x = \frac{y+2}{5y-1}$

$5xy - x = y+2$

$5xy - y = x+2$

$y(5x-1) = x+2$

$y = \frac{x+2}{5x-1}$

$\boxed{f^{-1}(x) = \frac{x+2}{5x-1}}$ ✓

(+10)

c) $f(x) = x^2+2x-1 \quad x>0$

$x = y^2+2y-1$

$x = (y^2+2y+\underline{1}) - 1 - \underline{1}$

$\left(\frac{2}{2}\right)^2 = 1$

$x = (y+1)^2 - 2$

$(y+1)^2 = x+2$

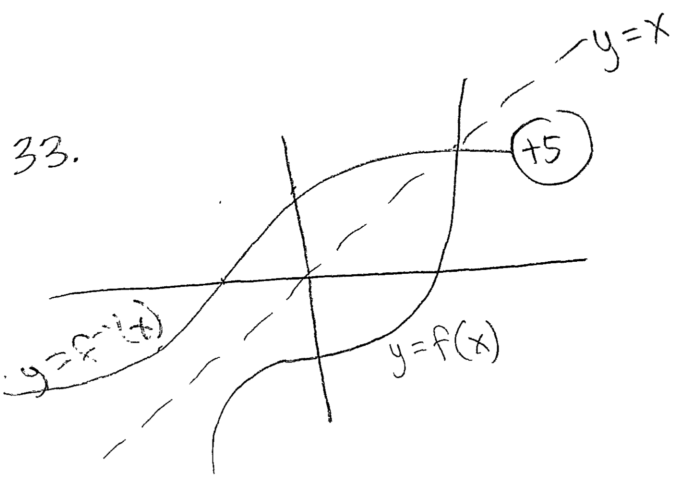
$y+1 = \pm\sqrt{x+2}$

$y = -1 \pm \sqrt{x+2}$

$\boxed{f^{-1}(x) = -1+\sqrt{x+2} \quad x>-1}$

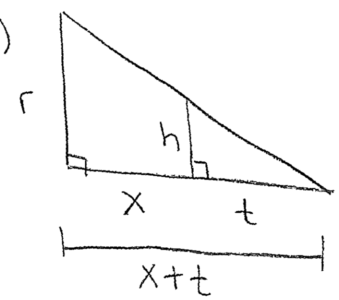
(+15)

33.



$\overline{50}$

34. a)



$$\frac{h}{r} = \frac{t}{x+t}$$

$$rt = hx + ht$$

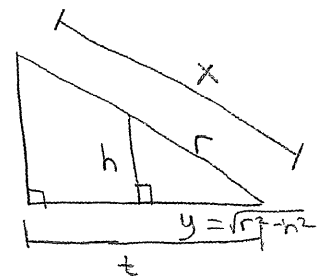
$$hx = ht - rt$$

$$hx = t(h-r)$$

$$\boxed{x = \frac{t(h-r)}{h}}$$

+10

b)



$$h^2 + y^2 = r^2$$

$$y^2 = r^2 - h^2$$

$$y = \sqrt{r^2 - h^2}$$

$$\frac{x}{r} = \frac{t}{\sqrt{r^2-h^2}}$$

$$x\sqrt{r^2-h^2} = rt$$

$$\boxed{x = \frac{rt}{\sqrt{r^2-h^2}}}$$

+10

35. a) $A_{\square} = (2r)^2$
$= 4r^2$

$$A_{\square - \circ} = 4r^2 - \pi r^2$$

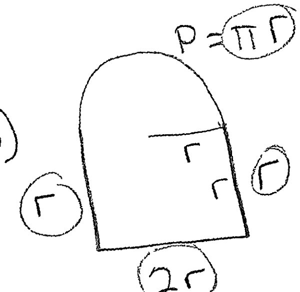
$$\text{ratio} = \frac{4\cancel{r^2} - \pi \cancel{r^2}}{4\cancel{r^2}}$$

$$= \frac{4 - \pi \div 4}{4 \div 4}$$

$$\Rightarrow \boxed{1 - \frac{\pi}{4} : 1}$$

+10

b)

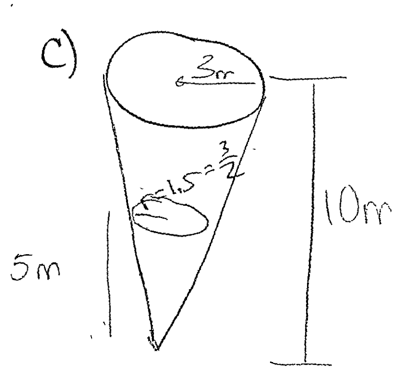


$$P = \pi r + r + 2r + r$$

$$\boxed{P = \pi r + 4r}$$

+5

c)



$$SA_{\circ} = \pi r^2$$

$$= \pi \left(\frac{3}{2}\right)^2$$

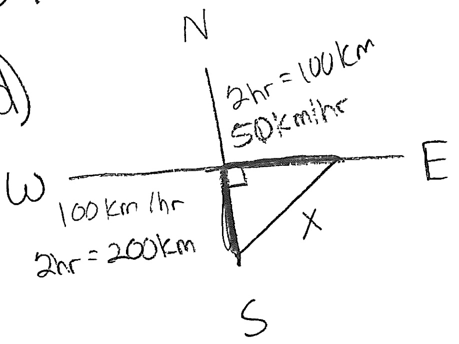
$$\boxed{A = \frac{9}{4}\pi\ m^2}$$

+10

$$\frac{3}{} = \frac{r}{}$$

31. continue

d)



$x^2 = 100^2 + 200^2$

$x^2 = 50{,}000$

$x = \sqrt{50000}$

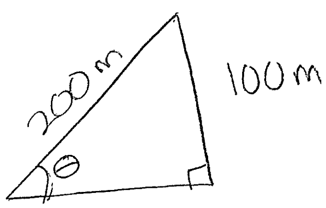
$= \sqrt{10000 \times 5}$

$x = 100\sqrt{5}$

$\boxed{\text{The cars will be } 100\sqrt{5} \text{ km apart after 2 hrs.}}$

+10

e)



$\sin\theta = \frac{o}{h}$

$\sin\theta = \frac{100}{200}$

$\sin\theta = \frac{1}{2}$

$\boxed{\theta = 30° \text{ or } \frac{\pi}{6}}$ +5

36. a) $\sin^2 x + \cos^2 x = 1$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos y = -x$

$\cos(x-x) = \cos x \cos(-x) - \sin x \sin(-x)$

$\cos 0 = \cos x \cos x - (-\sin x \sin x)$

$1 = \cos^2 x + \sin^2 x$ +10

b) $\sin 2x = 2\sin x \cos x$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

let $y = x$

$\sin(x+x) = \sin x \cos x + \cos x \sin x$

$\sin 2x = 2\sin x \cos x$ +5

30

6. continue

23

c) $\cos 2x = \cos^2 x - \sin^2 x$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

let $y = x$

$\cos(x+x) = \cos x \cos x - \sin x \sin x$

$\cos 2x = \cos^2 x - \sin^2 x$ +5

d) $\cos 2x = 2\cos^2 x - 1$

$\cos 2x = \cos^2 x - (1 - \cos^2 x)$

$\cos 2x = 2\cos^2 x - 1$

+5

e) $\cos 2x = 1 - 2\sin^2 x$

$\cos 2x = (1 - \sin^2 x) - \sin^2 x$

$\cos 2x = 1 - 2\sin^2 x$

+5

f) $\left|\cos \frac{x}{2}\right| = \sqrt{\frac{1+\cos x}{2}}$

from C let $x \to \frac{x}{2}$

$2\cos^2 x - 1 = \cos 2x$

$2\cos^2(\frac{x}{2}) - 1 = \cos x$

$\cos^2(\frac{x}{2}) = \frac{\cos x + 1}{2}$

$\left|\cos \frac{x}{2}\right| = \sqrt{\frac{\cos x + 1}{2}}$

+5

g) $\left|\sin \frac{x}{2}\right| = \sqrt{\frac{1-\cos x}{2}}$

let $x = \frac{x}{2}$

$\cos 2x = 1 - 2\sin^2 x$

$\cos x = 1 - 2\sin^2(\frac{x}{2})$

$2\sin^2(\frac{x}{2}) = 1 - \cos x$

$\sin^2(\frac{x}{2}) = \frac{1-\cos x}{2}$

$\left|\sin^2 \frac{x}{2}\right| = \sqrt{\frac{1-\cos x}{2}}$

+5