CONTENT GUIDE

FOR TARGETED PRACTICE 2023-2024

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Content Guide for Targeted Practice

This guide contains additional examples of questions that are organized by question type. This content guide is organized based on the three major sections of the CLT exam (Verbal Reasoning, Grammar/Writing, and Quantitative Reasoning). As shown in the outline below, each section drills down further into domains within the section and their subdomains. Examples of each question type within a subdomain are provided within the text of this guide.

1. Section 1: Verbal Reasoning

- a. Analysis
 - i. <u>Interpretation of Evidence</u>
 - ii. <u>Textual Analysis</u>
- b. Comprehension
 - i. <u>Passage as a Whole</u>
 - ii. <u>Passage Details</u>
 - iii. Passage Relationships
- 2. Section 2: Grammar/Writing
 - a. Grammar
 - i. <u>Agreement</u>
 - ii. <u>Punctuation and Sentence Structure</u>
 - b. Writing
 - i. <u>Structure</u>
 - ii. <u>Style</u>
 - iii. <u>Word Choice</u>

3. Section 3: Quantitative Reasoning

- a. Algebra
 - i. Algebraic Expressions and Equations
 - ii. <u>Arithmetic and Operations</u>
- b. Geometry
 - i. <u>Plane Geometry</u>
 - ii. Properties of Shapes
 - iii. <u>Trigonometry</u>
- c. Mathematical Reasoning
 - i. <u>Logic</u>
 - ii. <u>Word Problems</u>

Section 1: Verbal Reasoning

Analysis: Interpretation of Evidence

Questions on interpretation of evidence test the student's ability to understand how verbal and quantitative evidence is used in a passage. A student's score in this subdomain indicates their mastery of skills including:

- Identifying evidence supporting an argument or inference from the passage
- Recognizing how different parts of a text relate to an author's intent
- Distinguishing between the purposes of different parts of a passage
- Reading and drawing inferences based on charts, graphs, tables, and other figures
- Connecting ideas in a passage to related figures

Particularly important topics in the Interpretation of Evidence subdomain are Supporting Arguments (Evidence) and Analysis of Evidence (Quantitative).

I. Supporting Arguments (Evidence) Questions. These questions test the student's ability to identify evidence from a passage for answers about an author's intent, purpose, argument, etc.

Example #1: Supporting Arguments (Evidence) Questions (The example question follows the passage. Read the passage carefully.)

Literature

This passage is adapted from Anne Brontë's Agnes Grey, first published in 1847.

[1] I spare my readers the account of my delight on coming home, my happiness while there, and my sorrow on being obliged to bid them, once more, a long adieu.

[2] I returned, however, with unabated vigor to my work—a more arduous task than anyone can imagine, who has not felt something like the misery of being charged with the care and direction of a set of mischievous, turbulent rebels, whom his utmost exertions cannot bind to their duty, while, at the same time, he is responsible for their conduct to a higher power, who exacts from him what cannot be achieved without the aid of the superior's more potent authority, which, either from indolence, or the fear of becoming unpopular with the said rebellious gang, the latter refuses to give. I can conceive few situations more harassing than that wherein, however you may long for success, however you may labor to fulfill your duty, your efforts are baffled and set at naught by those beneath you, and unjustly censured and misjudged by those above.

[3] I have not enumerated half the vexatious propensities of my pupils, or half the troubles resulting from my heavy responsibilities, for fear of trespassing too much upon the reader's patience.

[4] But I particularly remember one wild, snowy afternoon, soon after my return in January: the children had all come up from dinner, loudly declaring that they meant "to be naughty," and they had well kept their resolution, though I had talked myself hoarse, and wearied every muscle in my throat, in the vain attempt to reason them out of it. I had got Tom pinned up in a corner, whence, I told him, he should not escape till he had done his appointed task. Meantime, Fanny had possessed herself of my work-bag, and was rifling its contents—and spitting into it besides. I told her to let it alone, but to no purpose, of course.

[5] "Burn it, Fanny!" cried Tom: and this command she hastened to obey. I sprang to snatch it from the fire, and Tom darted to the door. "Mary Ann, throw her desk out of the window!" cried he: and my precious desk, containing my letters and papers, my small amount of cash, and all my valuables, was about to be precipitated from the three-storey window. I flew to rescue it. Meanwhile Tom had left the room, and was rushing down the stairs, followed by Fanny. Having secured my desk, I ran to catch them, and Mary Ann came scampering after. All three escaped me, and ran out of the house into the garden, where they plunged about in the snow, shouting and screaming in exultant glee. [6] What must I do? If I followed them, I should probably be unable to capture one, and only drive them farther away. If I did not, how was I to get them in? While I stood in this perplexity, I heard a voice behind me, in harshly piercing tones, exclaiming—

[7] "Miss Grey! Is it possible? What, in the devil's name, can you be thinking about?"

[8] "I can't get them in, sir," said I, turning round, and beholding Mr. Bloomfield, with his hair on end, and his pale blue eyes bolting from their sockets.

[9] "But I insist upon their being got in!" cried he.

[10] "Then, sir, you must call them yourself, if you please, for they won't listen to me," I replied, stepping back.

This passage has been excerpted and adapted from the original, including minor punctuation changes, spelling changes, and other modifications that have not substantially changed content or intent.

The passage makes clear that one of the main difficulties of the narrator's work is that she

- A) is consistently disrespected by the house staff.
- B) does not earn enough income to support herself.
- C) lacks support from her superior.
- D) is hated by her charges because of her looks.

(C) is the correct answer: The narrator notes in Paragraph 2, Sentence 1 that while she is responsible for the behavior of her charges to her employer, it is impossible for her to obtain their good behavior without her employer's support, which he unfortunately refuses to give her. Thus, (C) is correct. (A) cannot be correct because there is no mention of any house staff, other than the narrator herself, in the passage. There is a brief mention of the narrator's "small amount of cash" and "valuables" in Paragraph 5, Sentence 3, but it is not noted whether these amount to a sufficient income, meaning that (B) is incorrect. The behavior of the children in the fourth and fifth paragraphs does seem to indicate a lack of affection from the children towards the narrator, but as the narrator does not articulate a particular motive for their lack of affection, and there is no support in the passage for the idea that the children hate her because of her looks, (D) is incorrect.

Which lines in the passage provide the best evidence in support of the answer to the previous question?

- A) Paragraph 1, Sentence 1 ("I spare . . . adieu")
- B) Paragraph 2, Sentence 2 ("I can . . . above")
- C) Paragraph 3, Sentence 1 ("I have . . . patience")
- D) Paragraph 4, Sentence 3 ("Meantime . . . besides")

(B) is the correct answer: In Paragraph 2, Sentence 2, the narrator indicates her frustration with her employer's lack of support, which prevents her from fulfilling her "duty" despite her desire to succeed. (B) is therefore the correct answer. (A) is incorrect because it speaks of the narrator's fondness for her home. While Paragraph 3, Sentence 1 speaks to the numerous occasions where the narrator has been frustrated by her job, it does not elaborate on the source of her frustration, so (C) is incorrect. (D) is incorrect narrator's charges, and not one of the main difficulties of her job overall.

Example #2: Supporting Arguments (Evidence) Questions (The example question follows the passage. Read the passage carefully.)

Philosophy/Religion

This passage is adapted from John Paul II's On the Christian Meaning of Human Suffering, first published in 1984.

[1] Even though in its subjective dimension, suffering seems almost inexpressible and not transferable, perhaps at the same time nothing else requires as much as does suffering, in its "objective reality," to be dealt with, meditated upon, and conceived as an explicit problem. Therefore, it also requires that basic questions be asked about it and the answers sought.

[2] Medicine, as the science and also the art of healing, discovers in the vast field of human sufferings the best-known area, the one identified with greater precision and relatively more counterbalanced by the methods of "reaction" (that is, the methods of therapy). Nonetheless, this is only one area. The field of human suffering is much wider, more varied, and multi-dimensional. Man suffers in different ways, ways not always considered by medicine, not even in its most advanced specializations.

[3] Suffering is something which is still wider than sickness, more complex, and at the same time still more deeply rooted in humanity itself. A certain idea of this problem comes to us from the distinction between physical suffering and moral suffering. This distinction is based upon the double dimension of the human being and indicates the bodily and spiritual element as the immediate or direct subject of suffering. Insofar as the words "suffering" and "pain" can, up to a certain degree, be used as synonyms, physical suffering is present when "the body is hurting" in some way, whereas moral suffering is "pain of the soul." In fact, it is a question of pain of a spiritual nature, and not only of the "psychological" dimension of pain, that accompanies both moral and physical suffering. The vastness and the many forms of moral suffering are certainly no less in number than the forms of physical suffering. But at the same time, moral suffering seems as it were less identified and less reachable by therapy.

[4] It can be said that man suffers whenever he experiences any kind of evil. Suffering expresses a situation in which man experiences evil and in doing so becomes the subject of

suffering. Even when man brings suffering on himself, when he is its cause, this suffering remains something passive in its metaphysical essence.

[5] This does not, however, mean that suffering in the psychological sense is not marked by a specific "activity." This is, in fact, that multiple and subjectively differentiated "activity" of pain, sadness, disappointment, discouragement or even despair, according to the intensity of the suffering subject and his or her specific sensitivity. In the midst of what constitutes the psychological form of suffering there is always an experience of evil, which causes the individual to suffer.

[6] Thus the reality of suffering prompts the question about the essence of evil: what is evil? This question seems, in a certain sense, inseparable from the theme of suffering. Man suffers on account of evil, which is a certain lack, limitation, or distortion of good. We could say that man suffers because of a good in which he does not share, from which in a certain sense he is cut off, or of which he has deprived himself. He particularly suffers when he ought—in the normal order of things—to have a share in this good and does not have it.

This passage has been excerpted and adapted from the original, including minor punctuation changes, spelling changes, and other modifications that have not substantially changed content or intent.

The passage most strongly suggests that medicine is

- A) a science and an art that studies all forms of human suffering.
- B) a field that has no special expertise when it comes to alleviating suffering.

C) a method for curing psychological suffering, as well as some types of physical suffering.

D) a tool to combat some, but not all, forms of suffering.

(D) is the correct answer: The author discusses the purview of medicine in the second paragraph, in which it is considered the art of healing sickness. But the author notes that beyond the scope of medicine, "[t]he field of human suffering is much wider, more varied, and multi-dimensional." Thus, (D) is correct. (A) is incorrect because while the author calls medicine "the science and also the art of healing" he states that it concerns "only one area" of suffering. (B) is incorrect because the author acknowledges medicine's "methods of therapy" for physical suffering. (C) is incorrect because the author discusses medicine primarily as a means of healing physical suffering; the passage attributes the psychological dimension of pain to both moral and physical suffering, but medicine is limited to healing physical suffering.

Which lines in the passage provide the best evidence in support of the answer to the previous question?

- A) Paragraph 2, Sentence 1 ("Medicine . . . therapy")
- B) Paragraph 2, Sentence 4 ("Man suffers . . . specializations")
- C) Paragraph 3, Sentence 2 ("A certain . . . suffering")
- D) Paragraph 3, Sentence 4 ("Insofar as . . . soul")

(B) is the correct answer: Paragraph 2, Sentence 4 provides the best support for the previous answer, as in that line the author makes explicit the argument he has been building throughout that paragraph: "Man suffers in different ways, ways not always considered by medicine." (A) is incorrect because while Paragraph 2, Sentence 1 notes that medicine reacts to suffering with "methods of therapy," it does not make clear—as Paragraph 2, Sentence 4 does—that some forms of suffering are "not always considered by medicine, not even in its most advanced specializations." (C) is incorrect because in Paragraph 3, Sentence 2 the author draws a clear distinction between physical and moral suffering but does not directly mention medicine. (D) is incorrect because Paragraph 3, Sentence 4 expands upon the author's dichotomy between physical and moral suffering without discussion of medicine.

Example #3: Supporting Arguments (Evidence) Questions (The example question follows the passage. Read the passage carefully.)

Philosophy/Religion

This passage is adapted from Jonathan Edwards' "A Farewell Sermon," given in 1750. Here, Jonathan Edwards speaks to the congregation at First Church in Northampton, MA, after he was voted out as the pastor.

[1] Ministers, and the people that have been under their care, must be parted in this world, no matter how well they have been united. If they are not separated before, they must be parted by death, and they may be separated while life is continued. We live in a world of change, where nothing is certain or stable, and where a little time, a few revolutions of the sun, brings to pass strange things and surprising alterations in particular persons, families, towns, churches, countries, and nations.

[2] It often happens that those who seem most united are, in a little time, most disunited, and at the greatest distance. Thus ministers and people, between whom there has been the greatest mutual regard and strictest union, may not only differ in their judgments and be alienated in affection, but one may rend from the other and all relation between them be dissolved. The minister may be removed to a distant place, and they may never have any more to do one with another in this world. But if it be so, there is one meeting more that they must have, and that is in the last great day of accounts.

[3] Since I have been settled in the work of the ministry in this place, I have always had a peculiar concern for the souls of the young people, and a desire that religion might flourish among them. Because I knew the special opportunity they had beyond others and that ordinarily those for whom God intended mercy were brought to fear and love him in their youth. And it has ever appeared to me a peculiarly amiable thing to see young people walking in the ways of virtue and Christian piety, having their hearts purified and sweetened with a principle of divine love. How exceedingly beautiful and conducive to the adorning and happiness of the town if the young people could be persuaded, when they

meet together, to converse as Christians and as the children of God, avoiding impurity, levity, and extravagance, keeping strictly to rules of virtue and conversing together of the things of God and Christ and heaven!

[4] This is what I have longed for, and it has been exceedingly grievous to me when I have heard of vice, vanity, and disorder among our youth. And so far as I know my own heart, it was from hence that I formerly led this church to some measures, for the suppressing of vice among our young people, which gave so great offense, and by which I became so obnoxious. I have sought the good and not the hurt of our young people. I have desired their truest honor and happiness, and not their reproach: knowing that true virtue and religion tended not only to the glory and felicity of young people in another world, but their greatest peace and prosperity and highest dignity and honor in this world, and above all things to sweeten, and render pleasant and delightful, even the days of youth.

[5] But whether I have loved you, and sought your good more or less, now committing your souls to him who once committed the pastoral care of them to me—nothing remains, but only (as I am now taking my leave of you) earnestly to beseech you, from love to yourselves, if you have none to me, not to despise and forget the warnings and counsels I have so often given you. Remember the day when you and I must meet again before the great Judge of quick and dead, when it will appear whether the things I have taught you were true, whether the counsels I have given you were good, whether I truly sought your welfare, and whether you have well improved my endeavors.

[6] I have, from time to time, earnestly warned you against some liberties commonly taken by young people in the land. And whatever some may say in justification of such liberties and customs, and may laugh at warnings against them, I now leave you my parting testimony against such things, not doubting but God will approve and confirm it in that day when we shall meet before him.

This passage has been excerpted and adapted from the original, including minor punctuation changes, spelling changes, and other modifications that have not substantially changed content or intent.

Over the course of the passage, the author expresses a particular concern for

- A) ministers who no longer have congregations.
- B) young people in the church.
- C) orphans and widows.
- D) souls that are facing judgment.

(B) is the correct answer: In the third paragraph, the author describes his particular concern for young people in the church, so (B) is correct. (A) is incorrect because the author is the one who will no longer have a congregation, but he does not express concern for himself. (C) is incorrect as the author does not mention orphans and widows. (D) is incorrect as it is too broad and is not touched on by the author specifically.

Which lines in the passage best support the answer to the previous question?

- A) Paragraph 1, Sentence 1 ("Ministers . . . united")
- B) Paragraph 2, Sentence 3 ("The minister . . . world")
- C) Paragraph 3, Sentence 1 ("Since I... them")
- D) Paragraph 5, Sentence 1 ("But whether . . . you")

(C) is the correct answer: Paragraph 3, Sentence 1 in the passage states that the author has "always had a peculiar concern for the souls of the young people," providing direct support for the answer to the previous question. (A) is incorrect as Paragraph 1, Sentence 1 is a general statement on ministers and people under their care. (B) is incorrect as Paragraph 2, Sentence 3 discusses what might happen if a minister leaves his congregation, not which group the author is most concerned about. (D) is incorrect as Paragraph 5, Sentence 1 is a plea from the author to his people, not an expression of his concern about a group.

II. Analysis of Evidence (Quantitative) Questions. These questions test the student's ability to correctly read and interpret charts, graphs, tables, and other figures, and to draw conclusions from them. Examples include:

Table 1—Total Neurons in Various Species	
Species	Number of Total Neurons (in Billions)
Elephant	251
Rhesus Monkey	6
Gorilla	33
Human	86

Example #1: Analysis of Evidence (Quantitative) Questions

Table information adapted from https://mosaicscience.com/extra/does-brain-size-matter. Table and passage used under Creative Commons license. (https://creativecommons.org/licenses/by/4.0/)

Which of the following conclusions can be drawn based on the information in Table 1?

- A) The number of neurons in a species is directly proportional to its weight.
- B) Gorillas have 27 more neurons than rhesus monkeys.
- C) Elephants have almost three times as many neurons as humans.
- D) Vertebrates have, on average, more neurons than invertebrates.

(C) is the correct answer: Table 1 lists the numbers of neurons, in billions, present in four different species, including both humans and elephants. Humans possess 86 billion neurons and elephants possess 251 billion neurons. 80 billion multiplied by 3 is 240 billion. Relatively speaking, 240 billion is very close to 251 billion, and so humans possess roughly

¹/₈ the number of neurons that elephants do, making (C) the correct answer. (A) is incorrect because the table does not list the weight of the various species. While the difference between 33 and 6 is 27, (B) is incorrect because the numbers in the table are given in billions—a rhesus monkey possesses 6 billion neurons, not 6 neurons, and a gorilla possesses 33 billion neurons, not 33 neurons. (D) is incorrect because the table does not compare invertebrates to vertebrates.



Example #2: Analysis of Evidence (Quantitative) Questions

A scientist performs a new study in which she tracks the self-reported anxiety levels (from a low of 1 to a high of 10) of 50 participants while they undergo an MRI scan. She then graphs the data and comes up with a line of best fit, as shown above.

CLI 2018.

Based on the information in the graphic, it is possible to conclude that

A) participants' peak anxiety levels occurred at about 15 minutes into the MRI.

B) every participant experienced a decrease in anxiety levels during the MRI.

C) the average level of anxiety of participants was lower after 15 minutes into the MRI than at the beginning.

D) most participants were afraid of the MRI and had an unpleasant experience.

(C) is the correct answer: The graph shows that the average anxiety level at the start of the MRI was between 6 and 8, and that by 15 minutes the average anxiety level had decreased to below 4; therefore, the average anxiety level of participants was lower 15 minutes into the study than at the beginning of the study. Therefore, (C) is correct. (A) is incorrect because the peak anxiety level shown on the graph (between 6 and 8) happens at the beginning of the study. not 15 minutes into the study. (B) is incorrect because the text under the graph explains that the line is a "line of best fit," demonstrating the overall trend of the collected data, not the experience of each individual participant. Therefore, it is possible that some

participants had an experience different from that demonstrated by the line on the graph. (D) is incorrect because the graph does not measure the overall experience of the participants in the MRI, only their anxiety levels over time in the MRI.



Example #3: Analysis of Evidence (Quantitative) Questions

A group of scientific researchers compiled the data above on the size of domesticated pigs over time. At year 0, the pigs were "wild."

CLI 2022.

Based on the graph, which of the following can be concluded?

A) Pigs continuously gained weight over the course of their lives.

B) Over the course of domestication, the average weight of domesticated pigs increased.

- C) Smaller pigs were released back into the wild and not bred.
- D) Wild pigs are much larger than domesticated pigs.

(B) is the correct answer: The upward trend on the graph indicates an increase in average weight of domesticated pigs from about 110 to about 450 pounds over the period of time measured, so (B) is correct. (A) is incorrect because the graph does not show the fluctuations in weight of specific pigs over their lifespan but changes in the average weight of all domesticated pigs over several hundred years. (C) is incorrect because it suggests a factor that is outside the scope of the information conveyed in the graph. (D) is incorrect because the graph only depicts the average weight of domesticated pigs; it does not provide information to compare their size to that of wild pigs.

Analysis: Textual Analysis

Questions on textual analysis test the student's ability to make inferences from information in the passage and understand a character, narrator, or writer's point of view. A student's score in this subdomain indicates their mastery of skills including:

- Making logical inferences based on information in the text
- Understanding how different texts interact
- Comprehending point of view
- Recognizing themes or patterns in a text
- Identifying the correct viewpoint of a character, narrator, or writer based on evidence in a passage
- Using evidence in a passage to come to a logical conclusion

Particularly important topics in the Textual Analysis subdomain are Making Inferences and Themes and Point of View.

I. Making Inferences Questions. These questions test the student's ability to analyze information in a text and draw logical conclusions from it. Examples include:

Example #1: Making Inferences (The example question follows the passage. Read the passage carefully.)

Science

This passage is adapted from Naureen Ghani's "Inside the Mind of an Octopus," first published in 2017 on PLOS Blogs.

[1] Octopuses possess a rich behavioral repertoire and the largest nervous systems among invertebrates. Alongside squids and cuttlefish, octopuses are believed to have evolutionarily separated from humans more than 700 million years ago. Given that long divergence, we should be a very different species. And we are ... in some ways.

[2] Octopuses have striking morphological features such as camera-like eyes, an incredibly adaptive coloration system, and eight arms with more suckers than we have fingers or thumbs. And yet—they are like us. Even though octopus brains and vertebrate brains have no common anatomy, they both support similar features such as forms of short and long-term memory, versions of sleep, and the abilities to recognize individual people and explore objects through play.

[3] In an octopus, the majority of neurons are in the arms themselves—nearly twice as many as in the central brain. Each arm has about 300 suckers and each sucker contains up to 10,000 neurons. Octopus suckers attach to surfaces as a local reflex. Since chemoreceptors line the sucker rim, the octopus can taste surfaces as it moves. There are also mechanoreceptors and proprioceptors present: the former provide information on touch

and pressure, while the latter supply information about muscle activity.

[4] When contact is detected, the sucker will automatically contract and attach. The mechanoreceptors trigger the reflex to keep what is there from getting away. The taste buds on the sucker rim then more slowly provide information on whether or not the object should be repulsed or retained. In this way, octopuses can ensure that a food item like a crab cannot scuttle away.

[5] With such versatile suckers, octopuses have shown that smart can be spineless. Their arms can maneuver without input from the brain. Even an arm that is surgically removed can reach and grasp objects. How an octopus coordinates eight such arms in locomotion is still unknown.

[6] Octopuses are indeed masters of distributed control. Hyper-redundancy is what makes octopuses so intriguing to researchers. There's redundancy in the sensors of the suckers, in the information processing in the brain, and in the structure of the body. The strong yet flexible hyper-redundant arms of the octopus endow it with high maneuverability but also place a great burden on its control system. It must interface incoming sensory information with the issuing of motor commands. Since the octopus arm does not have fixed joints or fixed linkages, it has infinite degrees of freedom. In other words, each arm has virtually unlimited ways to achieve the same goal with no constraints.

[7] It is undoubtedly difficult to build a soft robot with the mechanical properties of an octopus arm. Arriving at that degree of soft manipulator form and the complexity of an octopus is dually challenging. The Grasso lab at Brooklyn College, run by psychology professor Frank Grasso, studies octopus motion instead by creating simulations in silica of what an appendage or sucker would be like. The first step is to look to biology to characterize movement. The next step is to build a model that captures what biology is doing. To do so, the Grasso lab uses neural networks inspired by octopus neuroanatomy. There are sensory inputs, motor outputs, and interneurons that model what the nervous system should do. In other words, the neural networks connect sensing in the world to outputs of the world.

[8] "We're testing how the structure of the nervous system and the information that flows through it realizes behavior input," he says.

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Table information adapted from https://mosaicscience.com/extra/does-brain size-matter. Table and passage used under Creative Commons license.

Based on the passage, which of the following may octopuses struggle to do because of their unique body design?

- A) Swim away from predators while shooting chemical defenses
- B) Consume food while digesting it in their stomach cavities
- C) Process the feel of an object while responding to that object
- D) Move their arms while swimming in the same direction

(C) is the correct answer: The passage explains in Paragraph 4, Sentence 1 that an octopus's suckers will "automatically" respond to things that they come into contact with. It is also mentioned in the fifth paragraph that each arm of an octopus can operate independently of the rest of the arm, and in the sixth paragraph that there is a "great burden on its control system." This suggests that while an octopus will be able to respond to an object it encounters, it will not as easily process the incoming sensory information, so (C) is the correct answer. (A) and (B) are incorrect because the passage does not discuss chemical defenses or the consumption of food. (D) is incorrect because while the passage indicates in the fifth paragraph that we don't understand how an octopus is able to coordinate its arms in order to move, it nowhere indicates that octopuses struggle in their actual swimming movement.

Example #2: Making Inferences (The example question follows the passage. Read the passage carefully.)

Philosophy/Religion

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[1] Even though in its subjective dimension, suffering seems almost inexpressible and not transferable, perhaps at the same time nothing else requires as much as does suffering, in its "objective reality," to be dealt with, meditated upon, and conceived as an explicit problem. Therefore, it also requires that basic questions be asked about it and the answers sought.

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- C) a method for curing psychological suffering, as well as some types of physical suffering.
- D) a tool to combat some, but not all, forms of suffering.

(D) is the correct answer: The author discusses the purview of medicine in the second paragraph, in which it is considered the art of healing sickness. But the author notes that beyond the scope of medicine, "[t]he field of human suffering is much wider, more varied, and multi-dimensional." Thus, (D) is correct. (A) is incorrect because while the author calls medicine "the science and also the art of healing," he states that it concerns "only one area" of suffering. (B) is incorrect because the author acknowledges medicine's "methods of therapy" for physical suffering. (C) is incorrect because the author discusses medicine primarily as a means of healing physical suffering; the passage attributes the psychological dimension of pain to both moral and physical suffering, but medicine is limited to healing physical suffering.

Example #3: Making Inferences (The example question follows the passage. Read the passage carefully.)

Historical/Founding Documents

This passage is adapted from Benjamin Franklin's "On luxury, idleness, and industry," first published in 1806.

[1] It has been computed by some political arithmetician, that if every man and woman would work for four hours each day on something useful, that labor would produce sufficient to procure all the necessaries and comforts of life, want and misery would be banished out of the world, and the rest of the twenty-four hours might be leisure and pleasure.

[2] What creates, then, so much want and misery? It is the employment of men and women in works that produce neither the necessaries nor conveniences of life, who, with those who do nothing, consume necessaries raised by the laborious.

[3] To explain this: The first elements of wealth are obtained by labor from the earth and waters. I have land, and raise corn. With this, if I feed a family that does nothing, my corn will be consumed, and at the end of the year I shall be no richer than I was at the beginning. But if, while I feed them, I employ them, some in spinning, others in making bricks for building, the value of my corn will be arrested and remain with me, and at the end of the year we may all be better clothed and better lodged.

[4] Almost all the parts of our bodies require some expense. The feet demand shoes, the legs stockings, the rest of the body clothing, and the belly a good deal of victuals. Our eyes, though exceedingly useful, ask, when reasonable, only the cheap assistance of spectacles, which could not much impair our finances. But the eyes of other people are the eyes that ruin us. If all but myself were blind, I should want neither fine clothes, fine houses, nor fine furniture.

This passage has been excerpted and adapted from the original, including minor punctuation changes, spelling changes, and other modifications that have not substantially changed content or intent.

The author of this passage would likely agree that which of the following situations would lead to the most wealth and leisure over time?

A) A farmer who tills the land while his children first go to school and then begin to market luxury goods

B) A fisherwoman who brings food home to her family, while her kids and husband work the land and produce goods to sell at market

C) A man with a well-paying job in an urban environment supporting his out-of-work brothers, sisters, and cousins

D) A woman who owns a great deal of unused but fertile lands who plans to bequeath her wealth to her daughter when she passes

(B) is the correct answer: In the third paragraph of the passage, the author explains that "the first elements of wealth are obtained by labor from the earth and waters." He then gives the following example: if, while a member of the family is raising corn, the other members are engaged in some other valuable work, they can increase their wealth at the end of the year. (B) is correct because it offers a parallel example: if a fisherwoman is engaged in labor from the earth and waters-bringing food home to her family-and her family is engaged in other productive activities, they will increase their wealth over time. (A) is incorrect because the activities of the family in this choice (going to school and marketing luxury goods) do not represent labor that will help the family increase their wealth and leisure; indeed, "marketing luxury goods" may be considered the kind of work that "produce(s) neither the necessaries nor conveniences of life" that the author references in the second paragraph. (C) is incorrect because the example of an urban worker supporting his unemployed brother and family is a parallel example of Paragraph 3, Sentence 2, where the author states: "With this, if I feed a family that does nothing, my corn will be consumed, and at the end of the year I shall be no richer than I was at the beginning." (D) is incorrect because in this example, the woman is not using the land, and thus is not producing anything valuable that will lead to further wealth and leisure.

II. Themes and Point of View Questions. These questions test the student's ability to identify themes in a passage, recognize an author's point of view, or draw conclusions about the reactions and viewpoints of various characters or people in the passage based on textual evidence. Examples include:

Example #1: Themes and Point of View (The example question follows the passage. Read the passage carefully.)

Philosophy/Religion

This passage is adapted from Jonathan Edwards's "A Farewell Sermon," given in 1750. Here, Jonathan Edwards speaks to the congregation at First Church in Northampton, MA, after he was voted out as the pastor.

[1] Ministers, and the people that have been under their care, must be parted in this world, no matter how well they have been united. If they are not separated before, they must be parted by death, and they may be separated while life is continued. We live in a world of change, where nothing is certain or stable, and where a little time, a few revolutions of the sun, brings to pass strange things and surprising alterations in particular persons, families, towns, churches, countries, and nations.

[2] It often happens that those who seem most united are, in a little time, most disunited, and at the greatest distance. Thus ministers and people, between whom there has been the greatest mutual regard and strictest union, may not only differ in their judgments and be alienated in affection, but one may rend from the other and all relation between them be dissolved. The minister may be removed to a distant place, and they may never have any more to do one with another in this world. But if it be so, there is one meeting more that they must have, and that is in the last great day of accounts.

[3] Since I have been settled in the work of the ministry in this place, I have always had a peculiar concern for the souls of the young people, and a desire that religion might flourish among them. Because I knew the special opportunity they had beyond others and that ordinarily those for whom God intended mercy were brought to fear and love him in their youth. And it has ever appeared to me a peculiarly amiable thing to see young people walking in the ways of virtue and Christian piety, having their hearts purified and sweetened with a principle of divine love. How exceedingly beautiful and conducive to the adorning and happiness of the town if the young people could be persuaded, when they meet together, to converse as Christians and as the children of God, avoiding impurity, levity, and extravagance, keeping strictly to rules of virtue and conversing together of the things of God and Christ and heaven!

[4] This is what I have longed for, and it has been exceedingly grievous to me when I have heard of vice, vanity, and disorder among our youth.

[5] And so far as I know my own heart, it was from hence that I formerly led this church to some measures, for the suppressing of vice among our young people, which gave so great

offense, and by which I became so obnoxious. I have sought the good and not the hurt of our young people. I have desired their truest honor and happiness, and not their reproach: knowing that true virtue and religion tended not only to the glory and felicity of young people in another world, but their greatest peace and prosperity and highest dignity and honor in this world, and above all things to sweeten, and render pleasant and delightful, even the days of youth.

[6] But whether I have loved you, and sought your good more or less, now committing your souls to him who once committed the pastoral care of them to me—nothing remains, but only (as I am now taking my leave of you) earnestly to beseech you, from love to yourselves, if you have none to me, not to despise and forget the warnings and counsels I have so often given you. Remember the day when you and I must meet again before the great Judge of quick and dead, when it will appear whether the things I have taught you were true, whether the counsels I have given you were good, whether I truly sought your welfare, and whether you have well improved my endeavors.

[7] I have, from time to time, earnestly warned you against some liberties commonly taken by young people in the land. And whatever some may say in justification of such liberties and customs, and may laugh at warnings against them, I now leave you my parting testimony against such things, not doubting but God will approve and confirm it in that day when we shall meet before him.

This passage has been excerpted and adapted from the original, including minor punctuation changes, spelling changes, and other modifications that have not substantially changed content or intent.

Based on the passage, how does the author likely feel about leaving his post?

- A) Melancholic and reflective
- B) Angry and resentful
- C) Relieved and content
- D) Gleeful and giddy

(A) is the correct answer: The author of the passage is thoughtful as he discusses the end of his time as a minister for the people he addresses. He reflects on some of the more unpopular actions, such as measures for "suppressing the vices of young people." He also takes care to ask the people to not forget the advice he has given them, and on the whole seems to lament the end of a position that he treasured. (A) is therefore the best answer. (B) is incorrect as the author is not angry at or resentful of the people, but instead talks about his earnest desire for their good and his joy in serving them. (C) is incorrect because the author does not express relief at the end of the post and seems sorrowful rather than content. Lastly, (D) is too positive, as the author is thoughtful and sad rather than happy and excited.

Example #2: Themes and Point of View (The example question follows the passages. Read the passages carefully.)

Historical/Founding Documents

Passage 1 is adapted from Helen Zimmern's translation of Beyond Good and Evil by Friedrich Nietzsche, first published in 1886.

Passage 2 is adapted from John F. Kennedy's commencement address at Yale University, given on June 11, 1962.

Passage 1

[1] The Will to Truth, which is to tempt us to many a hazardous enterprise, what questions has this Will to Truth not laid before us! What strange, perplexing, questionable questions! Is it any wonder if we at last grow distrustful, lose patience, and turn impatiently away? That this Sphinx teaches us at last to ask questions ourselves? Who is it really that puts questions to us here? What really is this "Will to Truth" in us? Granted that we want the truth: Why not rather untruth? And uncertainty? Even ignorance?

[2] Having kept a sharp eye on philosophers and having read between their lines long enough, I now say to myself that the greater part of conscious thinking must be counted among the instinctive functions, and it is so even in the case of philosophical thinking; one has here to learn anew, as one learned anew about heredity and "innateness." The greater part of the conscious thinking of a philosopher is secretly influenced by his instincts, and forced into definite channels. And behind all logic and its seeming sovereignty of movement, there are valuations, or to speak more plainly, physiological demands, for the maintenance of a definite mode of life.

[3] The falseness of an opinion is not for us any objection to it: it is here, perhaps, that our new language sounds most strangely. The question is, how far an opinion is life-furthering, life preserving, species-preserving, perhaps species-rearing, and we are fundamentally inclined to maintain that the falsest opinions (to which the synthetic judgments a priori belong), are the most indispensable to us. Without a recognition of logical fictions, without a comparison of reality with the purely imagined world of the absolute and immutable, without a constant counterfeiting of the world by means of numbers, man could not live—the renunciation of false opinions would be a renunciation of life, a negation of life. To recognize untruth as a condition of life; that is certainly to impugn the traditional ideas of value in a dangerous manner, and a philosophy which ventures to do so, has thereby alone placed itself beyond good and evil.

Passage 2

[1] As every past generation has had to disenthrall itself from an inheritance of truisms and stereotypes, so in our time we must move on from the reassuring repetition of stale phrases to a new, difficult, but essential confrontation with reality. For the great enemy of truth is

very often not the lie—deliberate, contrived and dishonest—but the myth: persistent, persuasive, and unrealistic. Too often we hold fast to the clichés of our forebears. We subject all facts to a prefabricated set of interpretations. We enjoy the comfort of opinion without the discomfort of thought.

[2] Mythology distracts us everywhere, in government as in business, in politics as in economics, in foreign affairs as in domestic affairs. But today I want to particularly consider the myth and reality in our national economy. In recent months many have come to feel, as I do, that the dialogue between the parties— between business and government, between the government and the public—is clogged by illusion and platitude and fails to reflect the true realities of contemporary American society.

[3] I speak of these matters here because of the self-evident truth that a great university is always enlisted against the spread of illusion and on the side of reality. No one has said it more clearly than your President Griswold: "Liberal learning is both a safeguard against false ideas of freedom and a source of true ones." Your role as university men, whatever your calling, will be to increase each new generation's grasp of its duties.

[4] There are three great areas of our domestic affairs in which, today, there is a danger that illusion may prevent effective action. They are, first, the question of the size and the shape of government's responsibilities; second, the question of public fiscal policy; and third, the matter of confidence, business confidence, or public confidence, or simply confidence in America.

These passages have been excerpted and adapted from their originals, including minor punctuation changes, spelling changes, and other modifications that have not substantially changed content or intent.

Which of the following best describes the different ways that the authors of Passage 1 and 2 view truth?

A) The author of Passage 1 sees truth as a worthy, immortal pursuit, while the author of Passage 2 views truth as only one part of the larger process of myth-making.

B) The author of Passage 1 sees truth as ultimately less important than the function of opinions, while the author of Passage 2 views truth as something that must be defended against encroachments of myth-making.

C) The author of Passage 1 sees truth as something that can be distorted in pursuit of a particular purpose, while the author of Passage 2 views truth as something that informs purpose.

D) The author of Passage 1 sees truth as a safeguard against the spread of illusion, while the author of Passage 2 sees truth as part of the instinctive functions within men and women.

(B) is the correct answer: The author of Passage 1 argues that the distinction between truths and untruths is unimportant, while the author of Passage 2 argues that truth must be preserved against myth. (B) is therefore the best answer. (A) and (D) can be eliminated, since the author of

Passage 1 does not hold truth in this high esteem. (C) is incorrect as, while a case could be made for the view of the author of Passage 1, the view taken by the author of Passage 2 does not clearly express what the author's ideas about truth are; indeed, the author of Passage 2 does not directly link truth and purpose.

Example #3: Themes and Point of View (The example question follows the passage. Read the passage carefully.)

Science

This passage is adapted from Vitali Sintchenko's "Salmonella Kentucky: stopping the spread of a new superbug," first published in 2011 in The Conversation and licensed under CC-BY ND.

[1] Recent outbreaks of enterohaemorrhagic *Escherichia coli* (E. *coli*) in Germany and reports about the emergence of "new superbug" *Salmonella* Kentucky have re-focused public attention on foodborne diseases. However, both pathogens aren't new and have been recognized as bio-threats for decades. *Salmonella*, for instance, is a well-known bacterial cause of foodborne disease with a significant impact on the population and health-care systems. But improvements in sanitation have drastically reduced the frequency of people in the industrialized world getting ill from bacteria. It has essentially been reduced to a condition travelers contract in countries where it occurs naturally.

[2] In the last century, microbiologists have discovered hundreds of other types of *Salmonella* that can infect humans and some warm-blooded animals. They are especially widespread in areas with sub-standard farming practices and supplies of drinking water. These types of Salmonella have been responsible for acute gastroenteritis following the ingestion of contaminated water and undercooked meat or other animal products.

[3] Salmonella enterica serotype Kentucky represents one of the non-typhoidal types of Salmonella that microbiologists and public health professionals encounter from time to time. In Australia, more than 12,000 cases of Salmonella infection were reported in 2010, but only five of them were related to Salmonella Kentucky. What makes Salmonella Kentucky stand out and explains its rather hyperbolic label of "new superbug" is that this pathogen has managed to develop resistance to some antibiotics, making it more difficult to treat. Of particular concern is the fact that many strains of this serotype circulating in countries in the Middle East and Africa display high-level resistance to ciprofloxacin, which is one of the drugs used for treatment of Salmonella disease.

[4] The development and dissemination of its antimicrobial resistance is thought to be caused by the antibiotics used in animals farmed for food. Multinational surveillance has noted a recent increase in the frequency of the detection of drug-resistant *Salmonella enterica* serotype Kentucky and its potential for global spread. A recently published report suggested this drug-resistant type spread from Africa and the Middle East to Western Europe and Asia.

[5] The global human health impact of non typhoidal Salmonella infection could be as high

as one hundred million illnesses and 155,000 deaths each year. Many cases of salmonellosis would be prevented if common outbreak sources could be rapidly identified to allow for earlier public health interventions. But changes in foodborne disease epidemiology have complicated recognition and investigation of outbreaks.

[6] The most important contributors to the evolution of disease epidemiology are many. Socioeconomic changes because of booming Southeast Asian economies have affected food production, food supply, and food consumption habits, resulting in an increase in the number of foodborne diseases like Salmonella. Increased mobility of people and complex food production, processing, and distribution systems, as well as the plethora of retail fast-food outlets, mean we need more efficient and effective ways to identify sources of foodborne outbreaks.

[7] *Salmonella*-related outbreaks are increasingly linked to a diverse range of food sources, but the mechanisms of contamination often remain poorly understood. Health, food, and agriculture authorities need to coordinate their efforts to monitor and limit the spread of the drug-resistant strains like *Salmonella* Kentucky. Increasing the timeliness of case follow-up and linking laboratory results to public health actions are critical for reducing delays in the investigation of outbreaks.

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Based on the final paragraph, the author likely views current efforts to cope with *Salmonella* outbreaks as

- A) insufficient.
- B) misguided.
- C) overcautious.
- D) admirable.

(A) is the correct answer: In the final paragraph, the author states that "the mechanisms of contamination often remain poorly understood. Health, food, and agriculture authorities need to coordinate their efforts to monitor and limit the spread of the drug-resistant strains like *Salmonella* Kentucky." Thus, the author feels that more could be done to prevent such outbreaks, making (A) correct. (B) is incorrect as the author indicates current efforts are insufficient, not misguided or moving in the wrong direction. (C) is incorrect as it is the opposite of the author's position, which is that more needs to be done, not less (as "overcautious" would imply that more is being done than what is necessary). (D) is incorrect because the author's critique of current efforts and offering suggestions for improvements in coping with the outbreaks, indicates that the author does not think that the efforts being made are admirable, but rather found wanting.

Comprehension: Passage as a Whole

Questions on the passage as a whole test the student's ability to synthesize information from the entire passage to understand its framework and main ideas. A student's score in this subdomain indicates their mastery of skills including:

- Comprehending the main ideas of a passage
- Synthesizing information in a text
- Identifying an author's argument
- Understanding how a given paragraph or portion of a passage relates to the text as a whole
- Understanding how an author uses rhetorical devices in a passage
- Identifying the proper sequence of events in a passage

Particularly important topics in the Passage as a Whole subdomain are Main Ideas and Structure.

I. Main Ideas Questions. These questions test the student's ability to identify the main ideas of a text and to identify the author's main argument. Examples include:

Example #1: Main Ideas (The example question follows the passage. Read the passage carefully.)

Science

This passage is adapted from George Stanciu's "Wonder & Love: How Scientists Neglect God and Man," first published on November 26, 2017 in The Imaginative Conservative.

[1] In a classic study at Massachusetts Institute of Technology, Lettvin, Maturana, McCulloch, and Pitts inserted tiny electrodes into a living frog's optic nerve so that they could measure the electrical impulses traveling to the frog's brain. Using this technique, the researchers formed a good picture of what the frog sees. They found that when a small object is brought into the frog's field of vision and left immobile, the frog's eye sends electrical impulses to the brain for a few minutes, but then ceases to do so. After a short time, then, the object is no longer there, as far as the frog is concerned. The reason for this disappearance is that the frog's retina is designed to detect small moving objects. If a small object ceases to move in the frog's field of vision, the retina cancels it out of the frog's world. The researchers concluded that a frog cannot see a fly as such; it sees only small moving objects.

[2] Ethologist Jacob von Uexküll, among the first to document the remarkable specificity of animal perception, discovered that a jackdaw is unable to see a grasshopper that is not moving: "A jackdaw simply does not know the shape of a motionless grasshopper and is so constituted that it can only apprehend the moving form." This explains why so many insects feign death. The motionless form of an insect does not exist in the field of vision of frogs, birds, and snakes, so by shamming death, insects drop out of their prey's world and cannot be found even if searched for.

[3] Primates, considered the most intelligent of animals, also do not perceive what things are. Primatologist Wolfgang Kohler reports on the narrow perception of chimpanzees. He tested them with primitive stuffed toys on wooden frames padded with straw sewn inside cloth covers with black buttons for eyes. Kohler discovered that he could not get his normally docile chimpanzees near these small toys, which bore little resemblance to any kind of animal. The chimpanzees went into paroxysms of terror and threatened recklessly to bite his fingers when he tried to draw them towards the toys. The chimps perceived the shape, size, color, and design of the stuffed toys but could not see what they were—harmless cloth and wood. Psychologist Francine Patterson discovered the same thing while training her female gorilla Koko: "Although Koko has never seen a real alligator, she is petrified of toothy stuffed or rubber facsimiles. I have exploited Koko's irrational fear of this reptile by placing toy alligators in parts of the trailer I don't want her to touch."

[4] Because animals do not grasp what things are, their innate responses are keyed to a few external stimuli. To picture the impoverished perceptual life of an animal is extraordinarily difficult. We perceive the what and the why of things, substances and causes. Only extreme and rare pathology can cause the perceptual life of a human being to approximate that of an animal. From the scientific study of animal perception, Uexküll concludes that an animal's world is not the world we see at all, but more closely resembles "a small, poorly furnished room."

[5] Of all the natural creatures, only human beings can grasp a whole. A frog cannot see the iridescent, filigreed wing of a fly, nor can the fly see the glistening head and jet black eyes of the frog. A ten-year-old boy seated on the bank of the pond can take in the frog and the fly, can see the puffy white clouds racing across the blue sky, and can feel the warm spring breeze. Without the presence of a human being, the scene does not exist.

[6] We know from experience that human sensory perception is not limited to rigid categories of utility. Without human beings, the universe would be a drama played before an empty theater and thus would be pointless. Nature, without human beings, would be like a superb book with no reader. Remove humankind from nature and you erase the perception of all its wonder, its beauty, and its mystery—the world becomes meaningless.

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Which of the following is NOT a main topic of the passage?

- A) Perception of animals
- B) Perception of humans
- C) Limitations of perception
- D) Morality of perception

(D) is the correct answer: The passage is concerned with how the perception of humans and animals differ, and the various limits of perception in animals in particular. Thus, (A), (B), and (C)

can all be eliminated. (D) is the correct answer because the author nowhere discusses the morality of perception, only its differences and functions in different animals.

Example #2: Main Ideas (The example question follows the passage. Read the passage carefully.)

Science

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[1] Recent outbreaks of enterohaemorrhagic *Escherichia coli* (E. *coli*) in Germany and reports about the emergence of "new superbug" *Salmonella* Kentucky have re-focused public attention on foodborne diseases. However, both pathogens aren't new and have been recognized as bio-threats for decades. *Salmonella*, for instance, is a well-known bacterial cause of foodborne disease with a significant impact on the population and health-care systems. But improvements in sanitation have drastically reduced the frequency of people in the industrialized world getting ill from bacteria. It has essentially been reduced to a condition travelers contract in countries where it occurs naturally.

[2] In the last century, microbiologists have discovered hundreds of other types of *Salmonella* that can infect humans and some warm-blooded animals. They are especially widespread in areas with sub-standard farming practices and supplies of drinking water. These types of *Salmonella* have been responsible for acute gastroenteritis following the ingestion of contaminated water and undercooked meat or other animal products.

[3] *Salmonella enterica* serotype Kentucky represents one of the non-typhoidal types of *Salmonella* that microbiologists and public health professionals encounter from time to time. In Australia, more than 12,000 cases of *Salmonella* infection were reported in 2010, but only five of them were related to *Salmonella* Kentucky. What makes *Salmonella* Kentucky stand out and explains its rather hyperbolic label of "new superbug" is that this pathogen has managed to develop resistance to some antibiotics, making it more difficult to treat. Of particular concern is the fact that many strains of this serotype circulating in countries in the Middle East and Africa display high-level resistance to ciprofloxacin, which is one of the drugs used for treatment of *Salmonella* disease.

[4] The development and dissemination of its antimicrobial resistance is thought to be caused by the antibiotics used in animals farmed for food. Multinational surveillance has noted a recent increase in the frequency of the detection of drug-resistant *Salmonella enterica* serotype Kentucky and its potential for global spread. A recently published report suggested this drug-resistant type spread from Africa and the Middle East to Western Europe and Asia.

[5] The global human health impact of non typhoidal *Salmonella* infection could be as high as one hundred million illnesses and 155,000 deaths each year. Many cases of salmonellosis would be prevented if common outbreak sources could be rapidly identified to allow for

earlier public health interventions. But changes in foodborne disease epidemiology have complicated recognition and investigation of outbreaks.

[6] The most important contributors to the evolution of disease epidemiology are many. Socioeconomic changes because of booming Southeast Asian economies have affected food production, food supply, and food consumption habits, resulting in an increase in the number of foodborne diseases like *Salmonella*. Increased mobility of people and complex food production, processing, and distribution systems, as well as the plethora of retail fast-food outlets, mean we need more efficient and effective ways to identify sources of foodborne outbreaks.

[7] *Salmonella*-related outbreaks are increasingly linked to a diverse range of food sources, but the mechanisms of contamination often remain poorly understood. Health, food, and agriculture authorities need to coordinate their efforts to monitor and limit the spread of the drug-resistant strains like *Salmonella* Kentucky. Increasing the timeliness of case follow-up and linking laboratory results to public health actions are critical for reducing delays in the investigation of outbreaks.

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The primary purpose of the passage is most likely to

A) assure the public that *Salmonella* contamination is unlikely and overblown.

B) detail the risks of Salmonella exposure and the best ways to respond to outbreaks.

C) lay out a plan to reduce *Salmonella* outbreaks by a third in the next few decades.

D) suggest that researchers must turn their attention to superbugs instead of local *Salmonella* outbreaks.

(B) is the correct answer: The passage outlines the spread of *Salmonella* Kentucky and possible future tactics to combat the illness, so (B) is the best answer. (A) is incorrect; the author refers to the label "new superbug" as hyperbolic, but does not indicate that the threat from *Salmonella* Kentucky is overblown. (C) is incorrect as the passage is not focused on an action plan to reduce *Salmonella* outbreaks, but instead discusses the phenomenon more generally. (D) is incorrect because *Salmonella* Kentucky is considered a superbug; the two are not separate.

Example #3: Main Ideas (The example question follows the passage. Read the passage carefully.)

Philosophy/Religion

This passage is adapted from John Paul II's "On the Christian Meaning of Human Suffering," first published in 1984.

[1] Even though in its subjective dimension, suffering seems almost inexpressible and not

transferable, perhaps at the same time nothing else requires as much as does suffering, in its "objective reality," to be dealt with, meditated upon, and conceived as an explicit problem. Therefore, it also requires that basic questions be asked about it and the answers sought.

[2] Medicine, as the science and also the art of healing, discovers in the vast field of human sufferings the best-known area, the one identified with greater precision and relatively more counterbalanced by the methods of "reaction" (that is, the methods of therapy). Nonetheless, this is only one area. The field of human suffering is much wider, more varied, and multi-dimensional. Man suffers in different ways, ways not always considered by medicine, not even in its most advanced specializations.

[3] Suffering is something which is still wider than sickness, more complex, and at the same time still more deeply rooted in humanity itself. A certain idea of this problem comes to us from the distinction between physical suffering and moral suffering. This distinction is based upon the double dimension of the human being and indicates the bodily and spiritual element as the immediate or direct subject of suffering. Insofar as the words "suffering" and "pain" can, up to a certain degree, be used as synonyms, physical suffering is present when "the body is hurting" in some way, whereas moral suffering is "pain of the soul." In fact, it is a question of pain of a spiritual nature, and not only of the "psychological" dimension of pain, that accompanies both moral and physical suffering. The vastness and the many forms of moral suffering are certainly no less in number than the forms of physical suffering. But at the same time, moral suffering seems, as it were, less identified and less reachable by therapy.

[4] It can be said that man suffers whenever he experiences any kind of evil. Suffering expresses a situation in which man experiences evil and in doing so becomes the subject of suffering. Even when man brings suffering on himself, when he is its cause, this suffering remains something passive in its metaphysical essence.

[5] This does not, however, mean that suffering in the psychological sense is not marked by a specific "activity." This is, in fact, that multiple and subjectively differentiated "activity" of pain, sadness, disappointment, discouragement, or even despair, according to the intensity of the suffering subject and his or her specific sensitivity. In the midst of what constitutes the psychological form of suffering there is always an experience of evil, which causes the individual to suffer.

[6] Thus the reality of suffering prompts the question about the essence of evil: what is evil? This question seems, in a certain sense, inseparable from the theme of suffering. Man suffers on account of evil, which is a certain lack, limitation, or distortion of good. We could say that man suffers because of a good in which he does not share, from which in a certain sense he is cut off, or of which he has deprived himself. He particularly suffers when he ought—in the normal order of things—to have a share in this good and does not have it.

This passage has been excerpted and adapted from the original, including minor punctuation changes, spelling changes, and other modifications that have not substantially changed content or intent.

The passage addresses all of the following EXCEPT

- A) the different types of suffering.
- B) the causes of suffering.
- C) the complexity of suffering.
- D) the duration of suffering.

(D) is the correct answer: The author does not address the duration of suffering, so (D) is correct. (A) is incorrect because in the third paragraph the author discusses "the distinction between physical suffering and moral suffering." (B) is incorrect, for in the final paragraph the author explicitly states that "Man suffers on account of evil." (C) is incorrect because in the third paragraph the author compares suffering to sickness in terms of complexity.

II. Structure Questions. These questions test the student's ability to understand how different portions of a passage relate to one another, how an author structures his or her argument, and how events in a passage are ordered. Examples include:

Example #1: Structure (The example question follows the passage. Read the passage carefully.)

Historical/Founding Documents

This passage is adapted from Aristotle's "A Treatise on Government," written around the 4th century BC and translated here by William Ellis, A.M.

[1] Nature requires that we should not only be properly employed, but to be able to enjoy leisure honorably: for this (to repeat what I have already said) is of all things the principal.

[2] But, though both labor and rest are necessary, yet the latter is preferable to the first, and by all means we ought to learn what we should do when at rest. For we ought not to employ that time at play, for then play would be the necessary business of our lives. But if this cannot be, play is more necessary for those who labor than those who are at rest, for he who labors requires relaxation, which play will supply. For as labor is attended with pain and continued exertion, it is necessary that play should be introduced, under proper regulations, as a medicine.

[3] Now rest itself seems to partake of pleasure, of happiness, and an agreeable life: but this cannot be theirs who labor, but theirs who are at rest; for he who labors, labors for the sake of some end which he has not: but happiness is an end which all persons think is attended with pleasure and not with pain.

[4] It is evident that to live a life of rest there are some things which a man must learn and be instructed in, and that the object of this learning and this instruction centers in their acquisition. But the learning and instruction which is given for labor has for its object other things, for which reason the ancients made music a part of education, not as a thing necessary, for it is not of that nature, nor as a thing useful, as reading, in the common course

of life, or for managing of a family, or for learning anything as useful in public life.

This passage has been excerpted and adapted from the original, including minor punctuation changes, spelling changes, and other modifications that have not substantially changed content or intent.

The passage is primarily

- A) a lament.
- B) a retraction.
- C) an apology.
- D) an argument.

(D) is the correct answer: The passage discusses labor, rest, and play, and emphasizes the place and necessity of each. The author asserts his opinion about each of these three things, so (D) is correct. (A) is incorrect because the passage is not a mournful outcry about the topic. (B) is incorrect because the author is not taking back any previous statement or argument in the passage. (C) is incorrect because, similarly, the author is not apologizing for his position or anything else in the passage.

Example #2: Structure (The example question follows the passage. Read the passage carefully.)

Literature

This passage is adapted from Anne Brontë's Agnes Grey, first published in 1847.

[1] I spare my readers the account of my delight on coming home, my happiness while there, and my sorrow on being obliged to bid them, once more, a long adieu.

[2] I returned, however, with unabated vigor to my work—a more arduous task than anyone can imagine, who has not felt something like the misery of being charged with the care and direction of a set of mischievous, turbulent rebels, whom his utmost exertions cannot bind to their duty, while, at the same time, he is responsible for their conduct to a higher power, who exacts from him what cannot be achieved without the aid of the superior's more potent authority, which, either from indolence, or the fear of becoming unpopular with the said rebellious gang, the latter refuses to give. I can conceive few situations more harassing than that wherein, however you may long for success, however you may labor to fulfill your duty, your efforts are baffled and set at naught by those beneath you, and unjustly censured and misjudged by those above.

[3] I have not enumerated half the vexatious propensities of my pupils, or half the troubles resulting from my heavy responsibilities, for fear of trespassing too much upon the reader's patience.

[4] But I particularly remember one wild, snowy afternoon, soon after my return in January: the children had all come up from dinner, loudly declaring that they meant "to be naughty,"

and they had well kept their resolution, though I had talked myself hoarse, and wearied every muscle in my throat, in the vain attempt to reason them out of it. I had got Tom pinned up in a corner, whence, I told him, he should not escape till he had done his appointed task. Meantime, Fanny had possessed herself of my work-bag, and was rifling its contents—and spitting into it besides. I told her to let it alone, but to no purpose, of course.

[5] "Burn it, Fanny!" cried Tom: and this command she hastened to obey. I sprang to snatch it from the fire, and Tom darted to the door. "Mary Ann, throw her desk out of the window!" cried he: and my precious desk, containing my letters and papers, my small amount of cash, and all my valuables, was about to be precipitated from the three-story window. I flew to rescue it. Meanwhile Tom had left the room, and was rushing down the stairs, followed by Fanny. Having secured my desk, I ran to catch them, and Mary Ann came scampering after. All three escaped me, and ran out of the house into the garden, where they plunged about in the snow, shouting and screaming in exultant glee.

[6] What must I do? If I followed them, I should probably be unable to capture one, and only drive them farther away. If I did not, how was I to get them in? While I stood in this perplexity, I heard a voice behind me, in harshly piercing tones, exclaiming—

[7] "Miss Grey! Is it possible? What, in the devil's name, can you be thinking about?"

[8] "I can't get them in, sir," said I, turning round, and beholding Mr. Bloomfield, with his hair on end, and his pale blue eyes bolting from their sockets.

[9] "But I insist upon their being got in!" cried he.

[10] "Then, sir, you must call them yourself, if you please, for they won't listen to me," I replied, stepping back.

This passage has been excerpted and adapted from the original, including minor punctuation changes, spelling changes, and other modifications that have not substantially changed content or intent.

Which of the following best describes the movement of the passage?

- A) General reflections to a specific incident
- B) Personal musings to objective proof
- C) Firsthand narration to secondhand storytelling
- D) Calm rationalization to a vehement internal debate

(A) is the correct answer: The narrator begins the passage by speaking in general of her frustration with her job, and then describes events on one particular afternoon to show why she is so frustrated. (A) is therefore the correct answer. Both the style of the passage and the anecdotal content are more experiential than objective, so (B) is incorrect. (C) is incorrect because the second half of the passage is not secondhand, but rather the immediate experience of the narrator. Finally, while the narrator does express some internal

uncertainty in the sixth paragraph, it is not particularly intense debate, and the narrator does not calmly rationalize her plight anywhere in the passage; therefore, (D) is incorrect.

Example #3: Structure (The example question follows the passages. Read the passages carefully.)

Historical/Founding Documents

Passage 1 is adapted from H.R. James's translation of The Consolation of Philosophy by Boethius, originally written in the sixth century. In this passage, Philosophy appears to Boethius as a woman and talks to him about the state of current thought and the human mind.

Passage 2 is adapted from William Graham Sumner's "The Challenge of Facts," first published in 1914.

Passage 1

[1] Do you think that now, for the first time in an evil age, Wisdom has been assailed by peril? Did I not often in days of old, before my servant Plato lived, wage stern warfare with the rashness of folly? In his lifetime, too, Socrates, his master, won with my aid the victory of an unjust death. And when, one after the other, the Epicurean herd, the Stoic, and the rest, each of them as far as they could, went to seize the heritage he left, and were dragging me off protesting and resisting, as their booty, they tore in pieces the garment which I had woven with my own hands, and, clutching the torn pieces, went off, believing that the whole of me had passed into their possession.

[2] And some of them, because some traces of my vesture were seen upon them, were destroyed through the mistake of the lewd multitude, who falsely deemed them to be my disciples. It may be that you do not know of the banishment of Anaxagoras, of the poison draught of Socrates, nor of Zeno's torturing, because these things happened in a distant country. These men were brought to destruction for no other reason than that, settled as they were in my principles, their lives were a manifest contrast to the ways of the wicked.

[3] Though the host of the wicked is many in number, yet is it contemptible, since it is under no leadership, but is hurried here and there at the blind driving of mad error. But we from our vantage ground, safe from all this wild work, laugh to see them making prize of the most valueless of things, protected by a bulwark which aggressive folly may not aspire to reach.

Passage 2

[1] It is by strenuous exertion only that each one of us can sustain himself against the destructive forces and the ever recurring needs of life; and the higher the degree to which we seek to carry our development, the greater is the proportionate cost of every step.

[2] For help in the struggle we can only look back to those in the previous generation who are responsible for our existence. In the competition of life, the son of wise and prudent ancestors has immense advantages over the son of vicious and imprudent ones. The man who has capital possesses immeasurable advantages for the struggle of life over him who has none. The more we break down privileges of class, or industry, and establish liberty, the greater will be the inequalities and the more exclusively will the vicious bear the penalties. Poverty and misery will exist in society just so long as vice exists in human nature.

[3] Some, however, step over such facts with a genial platitude, a consoling commonplace, or a gratifying dogma. The effect is to spread an easy optimism, under the influence of which people spare themselves trouble and reflection, which are hard things, and to admit the necessity for which would be to admit that the world is not all made smooth and easy for us to pass through it surrounded by love, music, and flowers. Under this philosophy, "progress" has been represented as a steadily increasing and unmixed good; as if the good steadily encroached on the evil without involving any new and other forms of evil; and as if we could plan great steps in progress in our academies, and then realize them by resolution.

These passages have been excerpted and adapted from their originals, including minor punctuation changes, spelling changes, and other modifications that have not substantially changed content or intent.

Which of the following best describes the difference in how the two passages conclude?

A) Passage 1 concludes with a discussion of victory, while Passage 2 concludes with a hint of discouragement.

B) Passage 1 concludes with a personal anecdote, while Passage 2 concludes with a discussion of a historical trend.

C) Passage 1 concludes with a direct quote, while Passage 2 concludes with a paraphrase.

D) Passage 1 concludes with a definition, while Passage 2 concludes with a biography.

(A) is the correct answer: Passage 1 ends as Philosophy describes the lack of leadership among the wicked, and how she and her followers are "protected by a bulwark which aggressive folly may not aspire to reach." Passage 2 ends as the author discusses a group of people who "spread an easy optimism" and choose to falsely view progress as a "steadily increasing and unmixed good." Thus, Passage 1 ends on a note of optimism and hints at a potential victory, while Passage 2 ends on a note of discouragement. (A) is therefore correct. (B) is incorrect as Passage 1 does not conclude with a personal anecdote. (C) is incorrect because Passage 1 offers no direct quote at its conclusion. (D) is incorrect because Passage 1 does not define a term at its end, nor does Passage 2 offer a specific biography at its conclusion.

Comprehension: Passage Details

Questions on passage details test the student's ability to understand key facts and concepts discussed in a passage. A student's score in this subdomain indicates their mastery of skills including:

- Understanding essential details in a passage
- Locating crucial information in a passage
- Identifying important figures, dates, and concepts in a passage
- Understanding the meaning of words or phrases in a passage in context

Particularly important topics in the Passage Details subdomain are Finding Details and Vocabulary Use in Context.

I. Finding Details Questions. These questions test the student's ability to identify key facts and concepts within a passage. Examples include:

Example #1: Finding Details

(A truncated excerpt from the original passage from "Wonder & Love" by George Stanciu)

Psychologist Francine Patterson discovered the same thing while training her female gorilla Koko: "Although Koko has never seen a real alligator, she is petrified of toothy stuffed or rubber facsimiles. I have exploited Koko's irrational fear of this reptile by placing toy alligators in parts of the trailer I don't want her to touch."

According to this passage excerpt, when Francine Patterson discovered Koko was afraid of toy alligators, she

- A) tried to encourage Koko to overcome this irrational phobia.
- B) published a paper documenting her discovery.
- C) rid her entire house of any offending toys.
- D) used this fear to help her control Koko's behavior.

(D) is the correct answer: The last sentence of the passage excerpt describes how Patterson "exploited Koko's irrational fear of this reptile by placing toy alligators in parts of the trailer [she] don't want her to touch." Thus, (D) is the correct answer. (A) is incorrect because Patterson exploited Koko's fear, and did not try to encourage her to overcome it. (B) is incorrect because the excerpt mentions none of Patterson's papers. (C) is incorrect because Patterson placed toys in places she did not want Koko to go; she did not rid the home of any offending toys.

Example #2: Finding Details

(A truncated excerpt from the original passage from White Nights by Fyodor Dostoevsky, translated by Constance Garnett)

Leaning on the canal railing stood a woman with her elbows on the rail; she was apparently looking with great attention at the muddy water of the canal. She was wearing a very charming yellow hat and a jaunty little black mantle. "She's a girl, and I am sure she is dark," I thought. She did not seem to hear my footsteps, and did not even stir when I passed by with bated breath and loudly throbbing heart.

"Strange," I thought; "she must be deeply absorbed in something," and all at once I stopped as though petrified. I heard a muffled sob. Yes! I was not mistaken, the girl was crying, and a minute later I heard sob after sob. Good Heavens! My heart sank. And timid as I was with women, yet this was such a moment . . . I turned, took a step towards her, and should certainly have pronounced the word "Madam!" if I had not known that that exclamation has been uttered a thousand times in every Russian society novel.

When the narrator first notices the woman, he

- A) hears her crying.
- B) sees a suspicious figure across the street.
- C) grows curious about her appearance.
- D) believes that he knows her.

(C) is the correct answer: Upon first noticing the woman, the narrator wonders about her complexion, and whether "she is dark." Only later does he hear a "muffled sob." Thus, (C) is correct. (A) is incorrect because he notices the woman is crying only afterwards, when he passes by her. (B) is incorrect because the narrator only sees the older gentleman across from the woman after he has taken in her appearance. (D) is incorrect because the narrator never suggests that he knows the woman.
Example #3: Finding Details

(A truncated excerpt from the original passage from "A Farewell Sermon" by Jonathan Edwards)

Remember the day when you and I must meet again before the great Judge of quick and dead, when it will appear whether the things I have taught you were true, whether the counsels I have given you were good, whether I truly sought your welfare, and whether you have well improved my endeavors.

The author indicates that he believes that one day the "great Judge" will

- A) condemn him and all other sinners.
- B) confirm the wisdom of the author's counsel.
- C) pardon all souls who defied his laws.
- D) testify against the author's wayward congregation.

(B) is the correct answer: In this passage excerpt, the author notes that he and his people "must meet again before the great Judge of quick and dead, when it will appear whether the things I have taught you were true, whether the counsels I have given you were good, whether I truly sought your welfare, and whether you have well improved my endeavors." Thus, he and his people will meet again where the great Judge, God, will determine whether his advice was good, as stated in (B). The author does not suggest he will be condemned, so (A) is incorrect. (C) is incorrect because the author also does not suggest all will be pardoned. (D) is incorrect as it does not make sense that a great Judge would also testify at a case he is trying.

II. Vocabulary Use in Context Questions. These questions test the student's ability to understand the meaning of specific words or phrases in context. Examples include:

Example #1: Vocabulary Use in Context

(A truncated excerpt from the original passage from "A Farewell Sermon" by Jonathan Edwards)

Thus ministers and people, between whom there has been the greatest mutual regard and strictest union, may not only differ in their judgments and be alienated in affection, but one may rend from the other and all relation between them be dissolved.

In this excerpt, the word "strictest" most closely means

- A) sternest.
- B) harshest.
- C) closest.
- D) narrowest.

(C) is the correct answer: The word "strictest" here modifies "union," the union between ministers and people. Thus, "closest" would be most applicable to describe that union, as the author is trying to suggest that ministers and their people are at times strongly united. (A) and (B) are incorrect because they refer to the meaning of "strictest" that connotes rules, which is not applicable to this union. (D) is incorrect because "narrowest union" would imply a specific, thin union between the ministers and people, which is not what the author is trying to convey here.

Example #2: Vocabulary Use in Context

(A truncated excerpt from the original passage from "Salmonella Kentucky: stopping the spread of a new superbug" by Vitali Sintchenko)

What makes *Salmonella* Kentucky stand out and explains its rather hyperbolic label of "new superbug" is that this pathogen has managed to develop resistance to some antibiotics, making it more difficult to treat.

In this excerpt, the word "hyperbolic" most closely means

- A) exaggerated.
- B) false.
- C) spectacular.
- D) pretentious.

(A) is the correct answer: Here, "hyperbolic" is used to describe the label of "new superbug," a strong label for *Salmonella* Kentucky. In context, this most likely means "exaggerated," given the extremity of the label. (B) is incorrect as it implies not merely that the label is exaggerated, but that it is actually untrue, which the author does not imply. (C) is incorrect as the author is not praising the label. (D) is incorrect as "pretentious" indicates the author views the label as showy or conceited, which is not the case.

Example #3: Vocabulary Use in Context

(A truncated excerpt from the original passage from Beyond Good and Evil by Friedrich Nietzsche, translated by Helen Zimmern)

Having kept a sharp eye on philosophers and having read between their lines long enough, I now say to myself that the greater part of conscious thinking must be counted among the instinctive functions, and it is so even in the case of philosophical thinking; one has here to learn anew, as one learned anew about heredity and "innateness."

As used in this excerpt, "counted among" most closely means

- A) written down before.
- B) considered part of.
- C) regarded as encompassing.
- D) esteemed greater than.

(B) is the correct answer: The phrase "counted among" here is used to indicate that "conscious thinking" is one of the "instinctive functions." Thus, (B) is correct. (A) is incorrect as it is too literal. (C) is incorrect as it reverses the correct relationship. (D) is incorrect as it implies a value judgment of the two that is not intended in the paragraph.

Comprehension: Passage Relationships

Questions on passage relationships test the student's ability to recognize important connections between different parts of the passage. A student's score in this subdomain indicates their mastery of skills including:

- Understanding analogies
- Seeing how different parts of a passage relate to one another
- Identifying key relationships between elements of a passage
- Making high-level inferences about passage connections

The questions in the Passage Relationships subdomain are Analogy questions. The CLT explicitly covers this area.

Analogies Questions. These questions use analogies to test the student's ability to understand how different parts of a passage relate to one another. Examples include:

Example #1: Analogies

(A truncated excerpt from the original passage from "Inside the Mind of an Octopus" by Naureen Ghani)

In an octopus, the majority of neurons are in the arms themselves—nearly twice as many as in the central brain. Each arm has about 300 suckers and each sucker contains up to 10,000 neurons. Octopus suckers attach to surfaces as a local reflex. Since chemoreceptors line the sucker rim, the octopus can taste surfaces as it moves. There are also mechanoreceptors and proprioceptors present: the former provide information on touch and pressure, while the latter supply information about muscle activity.

mechanoreceptors : proprioceptors ::

- A) arms : suckers
- B) touch and pressure : muscle activity
- C) movement and sound : taste
- D) sensors : neurons

(B) is the correct answer: This excerpt states that mechanoreceptors "provide information on touch and pressure" and proprioceptors "supply information about muscle activity." (B) is therefore the correct answer. (A) is incorrect because an octopus's suckers are located on its arms, but proprioceptors are not located on mechanoreceptors. C) is incorrect because while both types of receptors do take in different types of sensory input, chemoreceptors, not proprioceptors, take in tastes. (D) is incorrect because both mechanoreceptors and proprioceptors are described as types of neurons, which serve to sense the external world.

Example #2: Analogies

(A truncated excerpt from the original passage from "On the Christian Meaning of Human Suffering" by John Paul II)

Suffering is something which is still wider than sickness, more complex, and at the same time still more deeply rooted in humanity itself. A certain idea of this problem comes to us from the distinction between physical suffering and moral suffering. This distinction is based upon the double dimension of the human being and indicates the bodily and spiritual element as the immediate or direct subject of suffering. Insofar as the words "suffering" and "pain" can, up to a certain degree, be used as synonyms, physical suffering is present when "the body is hurting" in some way, whereas moral suffering is "pain of the soul." In fact, it is a question of pain of a spiritual nature, and not only of the "psychological" dimension of pain, that accompanies both moral and physical suffering. The vastness and the many forms of moral suffering are certainly no less in number than the forms of physical suffering. But at the same time, moral suffering seems, as it were, less identified and less reachable by therapy.

physical suffering : body ::

- A) mental suffering : soul
- B) spiritual suffering : mind
- C) moral suffering : soul
- D) corporeal suffering : mind

(C) is the correct answer: The author notes in this paragraph that "physical suffering is present when 'the body is hurting ... whereas moral suffering is 'pain of the soul.'" Therefore, (C) is correct. (A) is incorrect because the author focuses on the distinction between physical and moral suffering and does not mention mental suffering directly. The author does state that the "psychological" dimension of pain (which could suggest mental suffering) accompanies both moral (soul) and physical (body) suffering, making the analogy in (A) at best, incomplete, and therefore incorrect. (B) is incorrect because spiritual suffering would afflict the spirit or soul, rather than the mind. (D) is incorrect because corporeal suffering would afflict the body, not the mind (the English word "corporeal" comes from the Latin word "corpus," meaning "body").

Example #3: Analogies

(A truncated excerpt from the original passage from Beyond Good and Evil by Friedrich Nietzsche, translated here by Helen Zimmern)

[1] The Will to Truth, which is to tempt us to many a hazardous enterprise, what questions has this Will to Truth not laid before us! What strange, perplexing, questionable questions! Is it any wonder if we at last grow distrustful, lose patience, and turn impatiently away? That this Sphinx teaches us at last to ask questions ourselves? Who is it really that puts questions to us here? What really is this "Will to Truth" in us? Granted that we want the truth: Why not rather untruth? And uncertainty? Even ignorance?

[2] Having kept a sharp eye on philosophers and having read between their lines long enough, I now say to myself that the greater part of conscious thinking must be counted among the instinctive functions, and it is so even in the case of philosophical thinking; one has here to learn anew, as one learned anew about heredity and "innateness." The greater part of the conscious thinking of a philosopher is secretly influenced by his instincts, and forced into definite channels. And behind all logic and its seeming sovereignty of movement, there are valuations, or to speak more plainly, physiological demands, for the maintenance of a definite mode of life.

[3] The falseness of an opinion is not for us any objection to it: it is here, perhaps, that our new language sounds most strangely. The question is, how far an opinion is life-furthering, life-preserving, species-preserving, perhaps species-rearing, and we are fundamentally inclined to maintain that the falsest opinions (to which the synthetic judgments *a priori* belong), are the most indispensable to us. Without a recognition of logical fictions, without a comparison of reality with the purely imagined world of the absolute and immutable, without a constant counterfeiting of the world by means of numbers, man could not live—the renunciation of false opinions would be a renunciation of life, a negation of life. To recognize untruth as a condition of life; that is certainly to impugn the traditional ideas of value in a dangerous manner, and a philosophy which ventures to do so, has thereby alone placed itself beyond good and evil.

logic : instinct ::

- A) hatred : fear
- B) happiness : joy
- C) truth : myth
- D) error : philosophy

(A) is the correct answer: The author of this excerpt writes that: "The greater part of the conscious thinking of a philosopher is secretly influenced by his instincts, and forced into definite channels." Thus, logic is influenced by instinct, and in a similar way, hatred is influenced by fear; (A) is therefore correct. (B) is incorrect because "happiness" and "joy" do not share this influenced-influencer relationship. (C) is incorrect because myth does not directly influence truth, based on the information in the passage. (D) is incorrect because "error" is a specific mistake and "philosophy" is a wider system of thinking, which does not fit with the influenced-influencer relationship of the original pair of terms.

Section 2: Grammar/Writing

Grammar: Agreement

Questions on agreement test the student's ability to recognize how individual elements of a sentence link up or agree with one another. A student's score in this subdomain indicates their mastery of skills including:

- Understanding the relationship between a pronoun and its antecedent
- Identifying subject-verb agreement
- Distinguishing between the proper use of adjectives and adverbs
- Choosing the proper verb tense in a given sentence
- Understanding parts of speech
- Identifying the proper spelling of common homophones

Particularly important topics in the Agreement subdomain are Spelling and Parts of Speech.

I. Spelling Questions. These questions test the student's ability to identify the correct spelling of a word. Examples include:

Example #1: Spelling

(A truncated excerpt from the original passage from William Harvey's "Of the blood as prime element in the body" in Animal Generation, first published in 1651)

Blood is both the author and preserver of the body. It is the [61] <u>principle</u> element, moreover, and that in which the vital principle (*anima*) has its dwelling place.

61. principle

- A) NO CHANGE
- B) principal
- C) principial
- D) princified

(B) is the correct answer: (B) is correct because "principal" means "first," "foremost," or "primary in importance;" since the first two sentences of the passage say that blood is the "author and preserver of the body" and "that in which the vital principle (anima) has its dwelling place," the author is clearly trying to express its great importance. (A) is incorrect because a "principle" is a "truth," "proposition," "theorem," or "law;" most importantly, these words are nouns, so they cannot modify the word "element." (C) is incorrect because "principial" means "original" or "elementary," which does not make sense in the context of the sentence. (D) is incorrect because "princified" is a rare, colloquial adjective primarily used in British English to mean "like a prince."

Example #2: Spelling

(A truncated excerpt from the original passage from a speech given by Robert Mueller at the RSA Cyber Security Conference on March 1, 2012, in San Francisco, CA)

[74] Stately-sponsored hackers are patient and calculating.

74. Stately-sponsored

- A) NO CHANGE
- B) Stately sponsored
- C) State-sponsored
- D) State sponsored

(C) is the correct answer: The underlined phrase modifies "hackers" and so should be some form of an adjective. (A) and (B) are incorrect as "stately" is an adjective with a meaning that does not fit the context. (D) is incorrect as it is missing a hyphen. A hyphen is required in between "state" and "sponsored" in order to form the phrasal adjective modifying "hackers," so (C) is correct.

Example 3: Spelling

(A truncated excerpt from the original passage from G.K. Chesterton's "The Fallacy of Success," first published in 1915)

It is perfectly obvious that in any decent occupation (such as bricklaying or writing books) there are only two ways of succeeding. One is by doing very good work; the other is by cheating. Both are much [48] to simple to require any literary explanation.

48. to simple

- A) NO CHANGE
- B) too simple
- C) to simply
- D) too simply

(B) is the correct answer: The adjective "simple" is modifying "both [ways of succeeding]," and "too" is an adverb that can be used to intensify adjectives & adverbs, which is appropriate for the meaning the author is attempting to convey here. (A) is incorrect because "to" is a preposition, not an adverb, so it cannot modify "simple." (C) is incorrect because "simply" is an adverb, so it cannot modify "both [ways of succeeding]." Furthermore, "to" is a preposition, not an adverb, so it cannot modify "simply." (D) is incorrect because "simply" is an adverb, not an adjective, so it cannot modify "both [ways of succeeding]."

II. Parts of Speech Questions. These questions test the student's ability to understand how different individual elements of a sentence link together, including verb tense, pronoun-subject agreement, subject-verb agreement, modifier-modified agreement and understanding parts of speech (including prepositions) in context. Examples include:

Example #1: Parts of Speech: Verb Tense

(A truncated excerpt from the original passage from An Inquiry into the Nature and Causes of the Wealth of Nations by Adam Smith)

But when the division of labor first began to take place, this power of exchanging must frequently [43] <u>has been</u> very much clogged and embarrassed in its operations. One man, we shall suppose, has more of a certain commodity than he himself has occasion for, while another has less.

43. has been

- A) NO CHANGE
- B) have been
- C) have gone
- D) have to be

(B) is the correct answer: This sentence uses "must" to express certainty about the past. In English, we always use "must have" and a past participle in this type of construction. "Been" is the past participle of the verb "to be" that here links "this power of exchanging" to its modifiers, "clogged" and "embarrassed in its operations," so (B) is correct. (A) is incorrect because "has" is a present tense form of the verb. (C) is incorrect because "gone" is not a linking verb that connects "this power of exchanging" to its modifiers. (D) is incorrect because "to be" is an infinitive, and the sentence needs an independently conjugated verb.

Example #2: Parts of Speech: Pronoun-Subject Agreement

(A truncated excerpt from the original passage from Susan Rice's "Human Rights First Annual Summit Address," delivered on December 4, 2013, in Washington, D.C.)

We must constantly question and challenge [74] <u>themselves</u> to be on the right side of history—to do our part so that more and more of our fellow human beings can enjoy the rights and freedoms that are the birthright of all mankind.

74. themselves

- A) NO CHANGE
- B) itself
- C) himself
- D) ourselves

(D) is the correct answer: The pronoun needed here must be consistent with the author's use of "we" and "our" in the sentence, so the first-person plural pronoun is required. Since the author is exhorting her readers to direct their efforts inward, the reflexive pronoun "ourselves" is needed, making (D) correct. (A) is incorrect because it is third-person. (B) and (B) are incorrect because they are third-person and singular.

Example #3: Parts of Speech: Subject-Verb Agreement

(A truncated excerpt from the original passage from The Book of Healing by Avicenna, translated here by Jon McGinnis)

In the latter case, it would not be unlikely that the science of physics establishes the existence of those principles and at the same time [67] <u>identify</u> their essence.

67. identify

- A) NO CHANGE
- B) identifies
- C) identifying
- D) identified

(B) is the correct answer: The subject of the second clause in this sentence is "the science of physics," which is singular (despite the -s) and therefore requires a verb in singular form. The clause is making a statement about a general truth, so the present tense is required (in parallel with "establishes"). "Identifies" is both singular and present tense, making (B) correct. (A) is incorrect because it is in the plural form. (C) is incorrect because it is a gerund rather than a conjugated verb, which is required for a clause. (D) is incorrect because it is in the past tense.

Example #4: Parts of Speech: Modifier-Modified Agreement

(A truncated excerpt from the original passage from The Book of Healing by Avicenna, translated here by Jon McGinnis)

The natural philosopher must simply accept their existence as a postulate and conceptualize their essence as [68] <u>full</u> real.

68. full

- A) NO CHANGE
- B) fulsome
- C) fuller
- D) fully

(D) is the correct answer: Adjectives are modified by adverbs, which qualify the degree of the adjective. "Real" is an adjective and therefore requires an adverb like "fully" to modify it, making (D) correct. (A), (B), and (C) are incorrect because they are all adjectives, which cannot modify other adjectives.

Example #5: Parts of Speech: Parts of Speech in Context

(A truncated excerpt from the original passage from An Inquiry into the Nature and Causes of the Wealth of Nations by Adam Smith)

But if this latter should chance to have nothing that the former stands in need [45] <u>of</u>, no exchange can be made between them.

45. of

A) NO CHANGEB) forC) byD) to

(A) is the correct answer: The verbal phrase "stand in need" requires the preposition "of," so (A) is correct. However, that phrase cannot take the prepositions "for," "by," or "to," which eliminates (B), (C), and (D).

Grammar: Punctuation and Sentence Structure

Questions on punctuation and sentence structure test the student's ability to understand how different elements of a sentence are linked together through punctuation and how to properly construct a sentence. A student's score in this subdomain indicates their mastery of skills including:

- Identifying how dependent and independent clauses are punctuated
- Recognizing when parallelism is needed in a given sentence
- Understanding the punctuation of parenthetical elements in a sentence
- Understanding how to link two independent clauses
- Recognizing complex sentences and how their individual clauses are linked
- Identifying how to properly use apostrophes to indicate possession and contractions
- Understanding the use of the serial comma

Particularly important topics in the Punctuation and Sentence Structure subdomain are Sentence Parts and Punctuation.

I. Sentence Parts Questions. These questions test the student's ability to link clauses in a sentence, recognize where parallelism is required in a sentence, and punctuate complex sentences. Examples include:

Example #1: Sentence Parts

(A truncated excerpt from the original passage from The Case of Wagner by Friedrich Nietzsche, translated here by Anthony M. Ludovici)

Bizet's music seems to me perfect. It comes forward lightly, gracefully, stylishly. [55] <u>It is</u> <u>lovable</u>; and it does not sweat.

55. It is lovable; and it does not sweat.

- A) NO CHANGE
- B) It is lovable, it does not sweat.
- C) It is lovable; and does not sweat.
- D) It is lovable; it does not sweat.

(D) is the correct answer: "It is lovable" is an independent clause (with the subject "It" and conjugated verb "is"). "It does not sweat" is also an independent clause (with the subject "It" and conjugated verb "does"). (D) is correct because it combines these clauses with a semicolon, which separates two independent clauses. (A) is incorrect because coordinating conjunctions (for, and, nor, but, or, yet, so) are not used after a semicolon to combine clauses, but after a comma. (B) is incorrect because a comma alone cannot combine independent clauses; it requires a coordinating conjunction as well. (C) is incorrect because separating the two phrases with a

semicolon results in "and does not sweat," which lacks a subject, and therefore is not an independent clause.

Example #2: Sentence Parts

(A truncated excerpt from the original passage from "The Feelings of Things, the Contemplation of Beauty" by Joseph Ratzinger)

To move from here [73] to disdain or to reject the impact produced by the response of the heart in the encounter with beauty as a true form of knowledge would impoverish us.

73. to disdain or to reject

- A) NO CHANGE
- B) to disdain or rejecting
- C) disdaining or to reject
- D) disdain or reject

(A) is the correct answer: "To disdain" and "to reject" should be parallel to "to move," as in (A). (B), (C), and (D) are all incorrect because they lack this parallelism in one or both verbs.

Example #3: Sentence Parts

(A truncated excerpt from the original passage from An Inquiry into the Nature and Causes of the Wealth of Nations by Adam Smith)

In order to avoid the inconvenience of such situations, every prudent man in every period of [47] <u>society</u>, <u>after the first establishment of the division of labor must</u> naturally have endeavored to have at all times by him, besides the peculiar produce of his own industry, a certain quantity of some one commodity or other, such as he imagined few people would be likely to refuse in exchange for the produce of their industry.

47. society, after the first establishment of the division of labor must

- A) NO CHANGE
- B) society, after the first establishment of the division of labor, must
- C) society-after the first establishment of the division of labor must
- D) society, after the first establishment of the division of labor; must

(B) is the correct answer: "After the first establishment of the division of labor" is a dependent clause that should be set off from the rest of the sentence by commas, as in (B). (A) is incorrect as it lacks a closing comma. (C) uses a single dash and no other punctuation to separate the dependent clause from the rest of the sentence, making it incorrect. (D) uses a comma and a

semicolon; as the portion of the sentence prior to the semicolon does not form an independent clause, (D) cannot be correct.

II. Punctuation Questions. These questions test the student's ability to use punctuation appropriately within individual words and elements in a sentence, including how best to use the serial comma and how to appropriately use apostrophes to indicate possession and contractions. Examples include:

Example #1: Punctuation

(A truncated excerpt from the original passage from "Beauty or Bloodshed" by Dwight Longenecker)

When we apprehend beauty, we become aware of all that is [41] <u>bigger better, and more</u> <u>beautiful</u> than we are.

41. bigger better, and more beautiful

- A) NO CHANGE
- B) bigger, better, and more beautiful
- C) bigger better and more beautiful
- D) bigger, better and, more beautiful

(B) is the correct answer: To punctuate this list consistently, a comma is needed in between each description, so (B) is correct. (A) is incorrect as the original is missing a comma after "bigger." (C) is incorrect as it is missing commas entirely. (D) is incorrect as it places the final comma incorrectly after "and" instead of after "better."

Example #2: Punctuation

(A truncated excerpt from the original passage from "Beauty or Bloodshed" by Dwight Longenecker)

If a tree, for example, is not revered for [46] <u>its</u> innate beauty and the sacredness of the life of the forest, and if it is merely viewed as an obstacle to a commercial development or as a commodity to be exploited, then we will quite happily, in Saruman-esque fashion, fell the forests.

46. its'

- A) NO CHANGE
- B) it's
- C) its
- D) their

(C) is the correct answer: Because the antecedent of the pronoun, "tree," requires the possessive case to relate it to its modifier, "innate beauty," "its" is the appropriate spelling of the possessive singular pronoun here, requiring no apostrophe; therefore, (C) is correct. (A) is incorrect as the word "its" with an apostrophe after the 's' is never correct spelling. (B) is incorrect because "it's" is a contraction of "it is", the third person singular conjugation of the verb "to be"; however, the possessive case of a pronoun is required in this context. (D) is incorrect because "their" is plural and "its" refers to its antecedent, "tree"; therefore, the singular form of the possessive case of the pronoun is required.

Example #3: Punctuation

(A truncated excerpt from the original passage from The Varieties of Religious Experience by William James)

The whole force of the Christian religion, therefore, so far as belief in the divine personages determines the prevalent attitude of the believer, is in general exerted by the instrumentality of pure ideas, of which nothing in the [46] <u>individuals</u> past experience directly serves as a model.

46. individuals'

- A) NO CHANGE
- B) individually
- C) individuals
- D) individual's

(D) is the correct answer: Here, the underlined word should be both singular and possessive, as in (D). (A) and (C) are the possessive plural and plural form of the noun, respectively, and can be eliminated. (B) changes the noun to an adverb, which would not make sense here.

Writing: Structure

Questions on structure test the student's ability to recognize how different parts of a passage, paragraph, and sentence relate to one another. A student's score in this subdomain indicates their mastery of skills including:

- Logically grasping an argument or statement
- Recognizing irrelevant information
- Revising work for clarity
- Understanding key relationships between parts of a passage
- Building evidence in an argument

Particularly important topics in the Structure subdomain are Insertions/Evidence and Transitions.

I. Insertions/Evidence Questions. These questions test the student's ability to revise and strengthen a work, choose appropriate examples for its thesis, and identify how a sentence relates to other parts of the passage. Examples include:

Example #1: Insertions/Evidence

(A truncated excerpt from the original passage from "How Sharks Could Help Us Regrow Our Own Human Teeth" by Gareth J. Fraser)

My colleagues and I recently studied the key genes involved in tooth regeneration in a small species of shark known as the catshark (*Scyliorhinus canicula*). Its eggs can easily be collected, and the embryos inside can be raised to show us the precise set of developmental stages that tooth formation and regeneration goes through. We found that within the epithelial cells that line sharks' mouths, there are special compartments of stem cells that are key to their continuous tooth regrowth.

[64] We analyzed the stem cell compartments in sharks' mouths and deciphered all the active genes involved in shark tooth development and regeneration.

64. The writer is considering inserting the following sentence here:

Without these stem cells, the sharks would suffer like humans with only a restricted set of teeth, which would in turn affect their success as hunters at the top of the food chain.

Should the writer make this insertion?

- A) Yes, because it shows why humans have fewer teeth than sharks.
- B) Yes, because it explains the importance of these stem cells in sharks.
- C) No, because it provides unnecessary details about tooth development.
- D) No, because it puts too much focus on stem cells rather than teeth.

(B) is the correct answer: The given sentence builds on the previous one by explaining the importance of the scientists' discovery of the stem cells. (A) is incorrect because the sentence provides crucial information about the importance of stem cells; the author is not concerned here with comparing humans and sharks. (C) and (D) are incorrect because the sentence should be included—it is not irrelevant, and the topic of stem cells is related to teeth.

Example #2: Insertions/Evidence

(A truncated excerpt from the original passage from The Case of Wagner by Friedrich Nietzsche, translated here by Anthony M. Ludovici)

I become a better man, a better musician, a better listener when Bizet speaks to me. Is it in any way possible to listen better? I even burrow behind this music with my ears. I hear its very cause. I seem to assist at its birth. I tremble before the dangers which this daring music runs. [58] <u>I attend the orchestra show once a week, regardless of the featured</u> <u>composer.</u>

58. I attend the orchestra show once a week, regardless of the featured composer.

A) NO CHANGE

B) Bizet grew up in the countryside, and his mother played a large part in his musical inspiration.

C) I am enraptured over those happy accidents for which even Bizet himself may not be responsible.

D) Delighted as I am with Bizet, Wagner also has many songs that are at least equal in perfect execution.

(C) is the correct answer: Of all the answer choices, (C) supports the paragraph's main idea: the author's experience of Bizet's music. (A) and (B) are incorrect because they are irrelevant to the author's opinion of Bizet. (D) is incorrect because it contradicts the author's prior statements.

Example #3: Insertions/Evidence

(A truncated excerpt from the original passage from The Book of Healing by Avicenna, translated here by Jon McGinnis)

From the part of *The Healing* where we concisely presented the science of demonstration, you have already learned that some sciences are universal and some are [61] <u>earthly</u>, and that some are related to others. So now what you need to learn is that the science we are engaged in explaining is physics, which is a particular science in relation to what comes later.

61. earthly

- A) NO CHANGEB) particular
- C) narrow-minded
- D) expansive

(B) is the correct answer: "Particular" (meaning "limited in scope") provides the correct contrast to the word "universal" to describe the two types of sciences the author is referring to, making (B) correct. (A) is incorrect because "universal" does not refer to the physical universe, so "earthly" is not the correct contrast. (C) is incorrect because "narrow-minded" is a specifically negative term for limited thinking, which does not fit the context here. (D) is incorrect because "expansive" is a synonym for "universal," rather than an antonym.

II. Transition Questions. These test the student's ability to link parts of a sentence and paragraph, recognize relationships between different parts of a passage, and organize arguments. Examples include:

Example #1: Transitions

(A truncated excerpt from the original passage from An Inquiry into the Nature and Causes of the Wealth of Nations by Adam Smith)

In the rude ages of society, cattle are said to have been the common instrument of commerce; and, though they must have been a most inconvenient one, yet, in old times, we find things were frequently valued according to the number of cattle which had been given in exchange for them. The armor of Diomedes, says Homer, cost only nine oxen, but that of Glaucus cost a hundred oxen.

In all countries, [49] <u>therefore</u>, men seem at last to have been determined by irresistible reasons to give the preference, for this employment, to metals above every other commodity. Metals cannot only be kept with as little loss as any other commodity, scarcely any thing being less perishable than they are, but they can likewise, without any loss, be divided into any number of parts, as by fusion those parts can easily be reunited again.

49. therefore

- A) NO CHANGE
- B) notwithstanding
- C) however
- D) as a result

(C) is the correct answer: The given sentence marks a transition from one idea to the next. "However" indicates an exception or contrast, which matches the contrast between valuing in cattle and valuing in metal. Thus, (C) is correct. (A) is incorrect because the adverb "therefore" indicates some sort of conclusion, but this sentence describes an exception to the prior paragraph. (B) is incorrect because "notwithstanding" generally requires an object, which is not present. (D) is incorrect because "as a result" is generally used to draw a direct causal line, which is not the purpose of the sentence here.

Example #2: Transitions

(A truncated excerpt from the original passage from "The Feelings of Things, the Contemplation of Beauty" by Joseph Ratzinger)

[79] <u>Today another objection has even greater weight</u>. The message of beauty is thrown into complete doubt by the power of falsehood, seduction, violence, and evil. Can the beautiful be genuine, or, in the end, is it only an illusion? Isn't reality perhaps basically evil? The fear that, in the end, it is not the arrow of the beautiful that leads us to the truth, but that falsehood, all that is ugly and vulgar, may constitute the true "reality" has at all times caused people anguish.

79. The writer wants to combine the two underlined sentences. Which choice best accomplishes this?

A) Today another objection has even greater weight: the message of beauty is thrown into complete doubt by the power of falsehood, seduction, violence, and evil.
B) Today another objection has even greater weight, the message of beauty is thrown into complete doubt by the power of falsehood, seduction, violence, and evil.
C) Today another objection has even greater weight, so the message of beauty is thrown into complete doubt by the power of falsehood, seduction, violence, and evil.
D) Today another objection has even greater weight; thus the message of beauty is thrown into complete doubt by the power of falsehood, seduction, violence, and evil.
D) Today another objection has even greater weight; thus the message of beauty is thrown into complete doubt by the power of falsehood, seduction, violence, and evil.

(A) is the correct answer: The second part of the sentence represents the objection to which the first part of the sentence refers. Thus, a colon would connect the two ideas well, making (A)

correct. (B) is incorrect as a comma doesn't suggest that the second part of the sentence is the objection referenced in the first and represents a comma splice. (C) is incorrect because it falsely links the two ideas; the second part is not an effect of the first. (D) is incorrect because it also falsely links the two ideas by suggesting that the second is a consequence of the first, rather than the objection referenced by it.

Example #3: Transitions

(A truncated excerpt from the original passage from Catharine of Aragon by Patrick Fraser Tytler)

Henry VII, divided between his policy and his conscience, first agreed to have Catharine marry his son Henry. But when the latter reached his fourteenth year, Henry VII became alarmed and insisted on formally ending the engagement.

[53] <u>In light of this, neither</u> Catharine's father nor Catharine herself was told of the end of the engagement. The marriage between Catharine and Henry had required a papal dispensation, which had already been secured. In the meantime, Henry's heart became touched by the good-natured qualities of Catharine, who showed no reluctance or displeasure about the match.

53. In light of this, neither

- A) NO CHANGE
- B) Naturally, it was crucial that neither
- C) Broadly received, neither
- D) Yet, strange as it may appear, neither

(D) is the correct answer: In the previous paragraph, the author explains how Henry VII formally ended the engagement between Catharine and his son, Henry. After the underlined transition, the author says that neither Catharine's father nor she herself were told of the end of the engagement; withholding this information is surprising, considering that it directly impacts her. The transition must acknowledge this tension. (D) is correct because it recognizes that it is "strange" but true that Catharine and her father were not told; it makes the transition between the two ideas easier for the reader to follow. (A) is incorrect because "in light of this" implies that some reason or explanation has been given that would explain why Catharine and her father were not told; this is not so. (B) is incorrect for a similar reason, because "naturally" implies that it logically follows that Henry VII would not tell Catharine and her father of the broken engagement. (C) is incorrect because it does not acknowledge the tension between the two ideas.

Writing: Style

Questions on style test the student's ability to understand a writer's tone and intent. A student's score in this subdomain indicates their mastery of skills including:

- Understanding language that matches the tone of a passage
- Recognizing the character of an author's argument
- Identifying irrelevant or repetitive information
- Determining where modifiers in sentences belong
- Recognizing the difference between extraneous and relevant information

Particularly important topics in the Style subdomain are Matching the Tone of a Passage and Conciseness/Clarity questions.

I. Matching the Tone of a Passage Questions. These questions test the student's ability to identify key passage excerpts that match the tone and language of the overall passage. Examples include:

Example #1: Matching the Tone of a Passage

(A truncated excerpt from the original passage from "Beauty or Bloodshed" by Dwight Longenecker)

If we extrapolate our attitude about the trees, and apply it to our neighbor, we will soon see that if we do not regard each person as sacred and transcendent in his beauty, then we might as well annihilate our neighbor in the same way that we [47] <u>knock some stuff down.</u>

47. Which of the following choices best matches the tone of the passage?

knock some stuff down.

- A) NO CHANGE
- B) get rid of some growing things.
- C) destroy arborous plant-filled land.
- D) tear down the trees.

(D) is the correct answer: (D) best matches the tone of the passage and so is correct. (A) and (B) are incorrect because they are too casual and vague. (C) is incorrect because it is wordy and confusing, as "arborous plant-filled" is redundant and circuitous.

Example #2: Matching the Tone of a Passage

(A truncated excerpt from the original passage from "The tale of a spymaster and his avian double" by Helen Macdonald)

In *A Cuckoo in the House* (1955), Knight tells the story of how he [59] jumped at the chance to rescue a cuckoo. He'd wanted to hand-rear a cuckoo for years. Why? Because they are interesting, he explained, and because they are familiar, but not well-known. Though everyone knows the cuckoo's call, he continued, the bird itself was "not thoroughly understood." It is "mysterious," he explained, with evident relish.

59. Which of the following choices best matches the tone of the passage?

jumped at the chance

- A) NO CHANGE
- B) was so excited
- C) felt so much joy
- D) experienced a cerebral thrill

(A) is the correct answer: (A) is correct as it best matches the tone of the passage, which is formal without being scholarly. (B) and (C) are both too casual and thus incorrect. (D) is incorrect as it is overly formal and unnecessarily wordy.

Example #3: Matching the Tone of a Passage

(A truncated excerpt from the original passage from The Case of Wagner by Friedrich Nietzsche, translated here by Anthony M. Ludovici)

To sit for five hours: the first step to holiness! [53] <u>Can I just say that Bizet's tunes are the only ones I can basically stand?</u> That other orchestration which is all the rage at present—the Wagnerian—is brutally artificial and "unsophisticated." Hence its appeal to all the three senses of the modern soul at once. How terribly Wagnerian orchestration affects me! I call it the Sirocco. A disagreeable sweat breaks out all over me. All my fine weather vanishes.

61. Which of the following choices best matches the tone of the passage?

Can I just say that Bizet's tunes are the only ones I can basically stand?

A) NO CHANGE

B) May I confess to you all Bizet's orchestration is really the only one I can deal with right now?

C) May I perchance pontificate that Bizet's orchestration is the sole instrumentation I can brave in modern times?

D) May I be allowed to say that Bizet's orchestration is the only one that I can endure now?

(D) is the correct answer: Of all the answer choices, (D) is the one that maintains the author's style while being clear and concise. (A) and (B) are incorrect because "tunes," "basically stand," and "deal with right now" are modern slang expressions. (C) is incorrect because it uses unnecessarily sophisticated vocabulary in awkward phrasing.

II. Concision/Clarity Questions. These questions test the student's ability to understand how different parts of the sentence link up, to identify extraneous or repetitive information, and to convey essential information in a succinct and clear manner. Examples include:

Example #1: Concision/Clarity

(A truncated excerpt from the original passage from "The Feelings of Things, the Contemplation of Beauty" by Joseph Ratzinger)

I was sitting next to the Lutheran Bishop Hanselmann. When the last note of one of the great Thomas-Kantor-Cantatas triumphantly faded away, [76] <u>we looked spontaneously</u>, <u>each at the other</u>, and said: "Anyone who has heard this knows that the faith is true."

76. Which of the following choices represents the clearest and most concise way to convey all of the information in the sentence?

we looked spontaneously, each at the other

- A) NO CHANGE
- B) we, out of spontaneity, turned to look at one another
- C) we in a spontaneous moment looked at each other
- D) we looked at each other spontaneously

(D) is the correct answer: The most concise, clear way to state this idea is in (D). (A) is incorrect because "each at the other" is awkward and clunky. (B) is incorrect because it is circuitous and introduces unnecessary disruptions in the flow of the sentence. (C) is incorrect because it is overly wordy.

Example #2: Concision/Clarity

(A truncated excerpt from the original passage from An Inquiry into the Nature and Causes of the Wealth of Nations by Adam Smith)

But they have nothing to offer in exchange, except the different productions of their respective trades, and the butcher is already provided with all the bread and beer which he has immediate occasion for. No exchange can, in this case, be made between them. [46] <u>He cannot be their merchant, and they cannot be his customers, and so as a result they are, altogether, less serviceable from one to the other, considered as a whole.</u>

46. Which of the following choices represents the clearest and most concise way to convey all of the information in the sentence?

He cannot be their merchant, and they cannot be his customers, and so as a result they are, altogether, less serviceable from one to the other, considered as a whole.

A) NO CHANGE

B) They cannot be each other's customers.

C) He cannot be their merchant, nor they his customers, and they are all of them thus mutually less serviceable to one another.

D) He cannot be their merchant or vice versa, and thus he is less serviceable to them and they are less serviceable to him, as a result.

(C) is the correct answer: The answer (C) concisely summarizes the relationship between the merchant and his customers—along with its consequences—and is therefore the best answer. (A) is incorrect because it includes redundant clauses ("altogether," "from one to the other," "as a whole"). (B) is incorrect because it is too simplistic; it loses meaning. (D) is incorrect because it is needlessly repetitive ("he is less serviceable to them and they are less serviceable to him").

Example #3: Concision/Clarity

(A truncated excerpt from the original passage from The Physics of Healing by Avicenna, translated here by Jon McGinnis)

Now, these bodies are considered natural things and so, too, is whatever is accidental to them. All of them are called natural in relation to that power called nature. [64] <u>Some of them are subjects</u>, for it, some of them effects, motions, and dispositions proceeding from <u>nature</u>. If, as was explained in the science of demonstration, natural things have principles, reasons, and causes without which the science of physics could not be attained, then the only way to acquire genuine knowledge of those things possessing principles is simple. First, one must know their principles and, from their principles, to know them, for this is the way to teach and learn that gives us access to the authentic knowledge of things that possess principles.

64. Which of the following choices represents the clearest and most concise way to convey all of the information in the sentence?

Some of them are subjects, for it, some of them effects, motions, and dispositions proceeding from nature.

A) NO CHANGE

B) Some of them, for nature, proceeding from it, are subjects, effects, motions, and dispositions.

C) Some of them are subjects, and some are effects, for the nature of motions and dispositions proceeding from it.

D) Some of them are subjects for nature and some are effects, motions, and dispositions proceeding from it.

(D) is the correct answer: Commas should not be placed indiscriminately in a sentence. (D) correctly limits the use of commas to separating items in a list. (A) incorrectly places commas around the prepositional phrase "for it" and creates a run-on sentence. (B) and (C) both awkwardly misplace the modifier "proceeding from it" and lack the clear distinction between "subjects" and "effects, motions, and dispositions," and so are incorrect as well.

Writing: Word Choice

Questions on word choice test the student's ability to recognize how different words fit into different contexts. A student's score in this subdomain indicates their mastery of skills including:

- Understanding complex vocabulary
- Understanding the nuances of word choice in a sentence
- Comprehending the various meanings that a particular word may have
- Distinguishing between similar vocabulary words to choose the best word in context

Word Choice questions are an independent category on the CLT, with no further subdomains. Examples include:

Example #1: Word Choice

(A truncated excerpt from the original passage from An Inquiry into the Nature and Causes of the Wealth of Nations by Adam Smith)

One man, we shall suppose, has more of a certain commodity than he himself has occasion for, while another has less. The former, consequently, would be glad to dispose of, and the latter to purchase, a part of this [44] <u>supposition</u>.

44. supposition

- A) NO CHANGE
- B) supplication
- C) supremacy
- D) superfluity

(D) is the correct answer: The previous sentence discusses a man who produces too much of a thing for his own use, so (D) is the correct answer. (A) is incorrect since a "supposition" is an uncertain belief. (B) is incorrect because "supplication" is a humble request. (C) is incorrect because "suppremacy" refers to a status of superiority.

Example #2: Word Choice

(A truncated excerpt from the original passage from The Case of Wagner by Friedrich Nietzsche, translated here by Anthony M. Ludovici)

And, strange to say, at bottom I do not give it a thought, or am not aware how much thought I really do give it. For quite other ideas are running through my head the while. Has anyone ever observed that music [59] <u>eradicates</u> the spirit? It gives wings to thought, and the more one becomes a musician the more one is also a philosopher.

59. eradicates

- A) NO CHANGE
- B) emancipates
- C) eviscerates
- D) extrapolates

(B) is the correct answer: The verb here should express a parallel thought to what the author says in the next sentence: "It gives wings to thought . . . " (B) is correct because "to emancipate" means to set free, which in context is synonymous to "gives wings." (A) is incorrect because "to eradicate" means to destroy. (C) is incorrect because "to eviscerate" means to kill violently (literally, to gut). (D) is incorrect because "to extrapolate" means to draw a predictive conclusion from other data.

Example #3: Word Choice

(A truncated excerpt from the original from The Physics of Healing by Avicenna, translated here by Jon McGinnis)

If, as was explained in the science of demonstration, natural things have principles, reasons, and causes without which the science of physics could not be attained, then the only way to acquire [65] generic knowledge of those things possessing principles is simple.

65. generic

- A) NO CHANGE
- B) genuine
- C) gentile
- D) genteel

(B) is the correct answer: The sentence following this one refers to "authentic knowledge," so the reference to "knowledge" at this point should be modified by a similar adjective; "genuine" is a synonym for "authentic," making (B) correct. (A) is incorrect because "generic" means not specific. (C) is incorrect because "gentile" is a term for someone who is not Jewish. (D) is incorrect because "genteel" means refined and well-mannered.

Section 3: Quantitative Reasoning

Algebra: Algebraic Expressions and Equations

Questions on algebraic expressions and equations test the student's ability to simplify algebraic expressions, solve equations and inequalities, and substitute variables into algebraic expressions. A student's score in this subdomain indicates their mastery of skills including:

- Simplifying algebraic expressions
- Combining like terms in algebraic expressions
- Simplifying quadratic equations
- Solving simple equations
- Solving simple inequalities
- Solving systems of equations
- Solving systems of inequalities
- Finding the roots and solutions of quadratic equations
- Substituting variables
- Understanding input and output of a function

Particularly important topics in the Algebraic Expressions and Equations subdomain are Simplifying Algebraic Expressions, Systems and Solutions of Equations and Inequalities, Special Symbols and Substitution, and Quadratic Equations with Real Roots. The CLT explicitly covers these three areas.

I. Simplifying Algebraic Expressions Questions. These questions test the student's ability to simplify algebraic expressions and combine like terms. Examples include:

Example #1: Simplifying Algebraic Expressions

If $x = y^2$, then which of the following is equivalent to $x^4 y^5$?

A) y^{9} B) y^{13} C) y^{20} D) y^{40}

(B) is the correct answer: Substitute y for x: $x^4y^5 = (y^2)^4y^5 = y^8y^5 = y^{13}$.

Example #2: Simplifying Algebraic Expressions

If $y^{z} = x$ and 8z = xy, then what is x + y + z if y is half the value of z?

- A) 13
- B) 22
- C) 71
- D) There is not enough information to answer the question.

(B) is the correct answer: If *y* is half the value of *x*, then we can write three equations that relate the three variables:

 $y^2 = x$ 8z - xy 2y = zSubstitute and solve: 8(2y) = xy 16 = xSubstituting x into the other equations gives us three relationships total between y and z: $y^z = 16$ 8z = 16y 2y = zThe bottom two equations are equivalent. But, if we know that z is double the value of y, we can rewrite the first equation as: $y^{2y} = 16$

One value that could work is 2, $2^4 = 16$. In this case, y = 2, and z = 4. Check this by substituting all values into all three equations.

 $y^{z} = x : 2^{4} = 16$ 8z = xy : 8(4) = 2(16) 2y = z : 2(2) = 4 All are correct. Thus, x + y + x = 2 + 4 + 16, which is equal to 22.

Example #3: Simplifying Algebraic Expressions

Which of the following is equivalent to x(x + 3)?

A) 5xB) 2x + 3C) $x^{2} + 3$ D) $x^{2} + 3x$

(D) is the correct answer: Distribute the *x* through the binomial to find an equivalent expression. x(x + 3)

x(x) + 3(x) $x^{2} + 3x$ Thus, (D) is correct.

II. Systems and Solutions of Equations and Inequalities Questions. These questions test the student's ability to solve both simple equations and inequalities and systems of equations and inequalities, including systems of equations involving three inequalities and/or equations. Questions also test a student's ability to simplify and solve quadratic equations. Examples include:

Example #1: Systems and Solutions of Equations and Inequalities

Which of the following are solutions to $36x^2 + 30x - 24 = 0$?

A)
$$-\frac{4}{3}$$
 and $\frac{1}{2}$
B) $-\frac{1}{2}$ and $\frac{4}{3}$
C) 6 and 4
D) 6 and 5

(A) is the correct answer: Factoring out 6 leaves $6(6x^2 + 5x - 4) = 0$, or (3x + 4)(2x - 1) = 0. The two solutions are therefore 3x + 4 = 0, or 3x = -4; $x = -\frac{4}{3}$ and 2x - 1 = 0, or 2x = 1; $x = \frac{1}{2}$.

Example #2: Systems and Solutions of Equations and Inequalities

If *a* is an integer and a < -3, which of the following MUST be a solution (*x*, *y*) of the following system of inequalities?

$$y^2 > x + 15$$
$$y < \sqrt{6x} - 5$$

A) $\left(-\frac{2}{3}, a\right)$ B) $\left(-6, a\right)$ C) $\left(\frac{2}{3}, a\right)$ D) $\left(6, a\right)$

(C) is the correct answer: (A) and (B) can be eliminated because a negative value cannot be beneath a radical, as would occur in the second inequality. (D) can be eliminated because 6 + 15 = 21, which is not always less than a^2 (for instance, if a was - 4, then 16 is not greater than 21). (C) is correct because $\sqrt{6(\frac{2}{3})} - 5 = 2 - 5 = -3$, and a is always less than -3, as stated in the stem of the question. In addition, $a^2 > \frac{2}{3} + 15$, as the smallest possible value of a^2 would be 16 (when a = -4).

Example #3: Systems and Solutions of Equations and Inequalities

Which of the following is a solution (x, y, z) to the system of equations below?

$$3x + 2 = y$$

$$2y - z = 12x$$

$$x + y + z = 4$$

A) (- 1, - 1, 10)
B) (0, 2, - 4)
C) (0, 2, 2)
D) (1, 5, - 2)

(D) is the correct answer: If 3x + 2 = y, then 2(3x + 2) - z = 12x, or 6x + 4 - z = 12x; 4 - z = 6x. In addition, x + 3x + 2 + z = 4, or 4x + z = 2. Combining those two equations, 4 + 4x = 2 + 6x, or 2 = 2x, and x = 1. This means that 3(1) + 2 = y, so y = 5. Plugging these values into the third equation, 1 + 5 + z = 4, or z = -2. **III. Special Symbols and Substitution Questions.** These questions require students to analyze a given expression specifying a given input and output, and then extrapolate that to a new input. Examples include:

Example #1: Special Symbols and Substitution

If the operation \otimes is defined such that $a \otimes b = a^2 - 2b$, then which of the following yields the largest value?

 $\begin{array}{cccc} A) & 3 & \otimes & 4 \\ B) & 3 & \otimes & 5 \\ C) & 4 & \otimes & 2 \\ D) & 5 & \otimes & 7 \end{array}$

(C) is the correct answer: The symbol, \otimes , represents the given function, $a \otimes b = a^2 - 2b$, where a and b are variables. In each answer choice, the number on the left side of the symbol is equal to a and the number on the right side of the operator is equal to b. Therefore, the solution can be found by substituting each number (a and b) into the function and then evaluating: $3^2 - 2(4) = 9 - 8 = 1$, so (A) is $1.3^2 - 2(5) = 9 - 10 = -1$, so (B) is $-1.4^2 - 2(2) = 16 - 4 = 12$, so

(C) is $12.5^2 - 2(7) = 25 - 14 = 11$, so (D) is 11. (C) therefore yields the largest value.

Example #2: Special Symbols and Substitution

If the operation φ is defined such that $p\varphi q = \frac{p^3}{q} + 5$, then which of the following is equivalent to $3\varphi 9$?

A) 3
B) 6
C) 8
D) 248

(C) is the correct answer: Plug 3 in for p and 9 in for $q: \frac{3^3}{9} + 5 = \frac{27}{9} + 5 = 3 + 5 = 8$.

Example #3: Special Symbols and Substitution

If the operation Ψ is defined such that $a\Psi b = 2a^2 + 3b$, then which of the following is equivalent to $4\Psi 2$?

A) 10B) 18C) 20D) 38

(D) is the correct answer: Substitute 4 for *a* and 2 for *b* and solve: 2(4)²+ 3(2)
2 (16) + 3(2)
32 + 6
38

IV. Quadratic Equations with Real Roots Questions. These questions test the student's ability to factor or otherwise understand/manipulate quadratic equations. Examples include:

Example #1: Quadratic Equations with Real Roots

Which of the following is NOT an x-intercept of $y = x^3 - 4x^2 - 21x$?

A) (- 7, 0) B) (- 3, 0) C) (0,0) D) (7,0)

(A) is the correct answer: Set *y* equal to 0 and solve for every solution of *x*.

 $0 = x^{3} - 4x^{2} - 21x$ $0 = x(x^{2} - 4x - 21)$ 0 = x(x - 7)(x + 3)Thus, x = 0, 7, -3, and the *x*-intercepts are (0, 0), (7, 0), and (-3, 0). (A) is not one of these x-intercepts, and so is the correct answer.

Example #2: Quadratic Equations with Real Roots

What are the solutions of the equation $0 = 3x^2 + 2x - 8$?

A) $-\frac{4}{3}$ and 2 B) $\frac{4}{3}$ and -2C) $-\frac{1}{3}$ and 8 D) $\frac{1}{3}$ and -8

(B) is the correct answer: $0 = 3x^2 + 2x - 8 = (3x - 4)(x + 2)$, so the solutions are given by: 3x - 4 = 0, or $x = \frac{4}{3}$ x + 2 = 0, or x = -2

Example #3: Quadratic Equations with Real Roots

Which of the following has a double root?

A)
$$y = x^{2} - 9$$

B) $y = x^{2} + 6x + 9$
C) $y = x^{2} + 9x + 18$
D) $y = 3x^{2} + 12x + 3$

(B) is the correct answer: A double root of a quadratic equation occurs when the quadratic equation $(ax^2 + bx + c = 0)$ has only one real root (or solution). The quickest way to determine how many solutions a quadratic has is to check the discriminant $(b^2 - 4ac)$: if the discriminant is equal to zero, then the quadratic has only one real root. (The discriminant is the portion of the quadratic formula, $x = -b \pm \frac{\sqrt{b^2-4ac}}{2a}$, that lies under the radical sign. It is easy to see that a discriminant of 0 leads to only one solution.) In choice (B), we see a = 1, b = 6, and c = 9. So we can evaluate the discriminant $b^2 - 4ac = 6^2 - 4(1)(9) = 36 - 36 = 0$. This tells us that the equation in (B) has only one real root, and is therefore the correct answer. Alternatively, we can find double roots by setting the quadratic function equal to 0, factoring, and identifying equivalent factors. By factoring answer choice (B), we see that $0 = x^2 + 6x + 9 = (x + 3)(x + 3)$. Setting the first factor equal to 0, we get (x + 3) = 0 and x = -3. The same is true for the second factor. Therefore, $y = x^2 + 6x + 9$ has a double root at x = -3. Answer choice (B) is the correct answer.

Algebra: Arithmetic and Operations

Questions on arithmetic and operations test the student's ability to utilize basic rules of arithmetic to simplify expressions and solve equations, as well as their ability to recognize patterns. A student's score in this subdomain indicates their mastery of skills including:

- Understanding the properties of prime numbers
- Understanding the properties of even and odd integers
- Understanding the properties of negative and positive integers
- Recognizing patterns and identifying terms in a sequence
- Simplifying absolute value expressions
- Solving absolute value equations and inequalities
- Using the properties of exponents to simplify expressions
- Understanding probability

Particularly important topics in the Arithmetic and Operations subdomain are Properties of Integers and Exponents, Patterns, Probability, and Absolute Value.

I. Properties of Integers and Exponents Questions. These questions test the student's ability to simplify expressions and draw conclusions based on their knowledge of integers and rules of exponents. Examples include:

Example #1: Properties of Integers

If integer *a* is a factor of 14, which of the following must always also be true?

- A) *a* is a multiple of 7.
- B) *a* is a factor of 7.
- C) *a* is a multiple of 28.
- D) *a* is a factor of 28.

(D) is correct: A factor of a number is an integer that can be multiplied with another integer to obtain the original number. 14 is a factor of 28. Thus, any integer that is a factor of 14 will also be a factor of 28, so (D) is correct.

Example #2: Properties of Exponents

Which of the following is equivalent to $(e^5)^3 + e^2$?

A)
$$e^{8} + e^{2}$$

B) e^{10}
C) $e^{15} + e^{2}$
D) e^{17}

(C) is the correct answer: Multiply the exponents in the first term; the terms then cannot be combined further. $(e^5)^3 + e^2 = e^{15} + e^2$.

Example #3: Properties of Exponents

What is the value of $\frac{3^2}{2^2}$?

A) 0 B) 1 C) $\frac{3}{2}$ D) $\frac{9}{4}$

(D) is the correct answer: Simplify the numerator and denominator: $\frac{3^2}{2^2} = \frac{(3)(3)}{(2)(2)} = \frac{9}{4}$

II. Patterns and Probability Questions. These questions test the student's ability to identify patterns and terms in a sequence (both arithmetic and geometric) and to calculate probability. Examples include:

Example #1: Pattern

Which of the following could be the missing term in the sequence below?

$$\frac{1}{4}$$
, ?, $\frac{1}{16}$, $\frac{1}{25}$, $\frac{1}{36}$...

A) $\frac{1}{6}$ B) $\frac{1}{8}$ C) $\frac{1}{9}$ D) $\frac{1}{10}$
(C) is the correct answer: Choose the term that fits the pattern given in the sequence. The first term has a numerator of 1 and a denominator that is the square of 2. The third term has a numerator of 1 and a denominator that is the square of 4. The fourth term has a numerator of 1 and a denominator that is the square of 5. The fifth term has a numerator of 1 and a denominator that is the square of 5. The fifth term has a numerator of 1 and a denominator that is the square of 6. The pattern is that the numerator is 1 and the denominator of each term is the square of successive integers. Thus, the second term should have a numerator of 1 and a denominator that is the square of 3, which is the integer that is after 2 and before 4. The second term would be $\frac{1}{9}$. Thus, (C) is correct.

Example #2: Probability

A flower shop always chooses a different type of flower at random to feature in its front display case. The shop sells 22 different types of flowers in total, including roses. If the store is out of stock of sunflowers and orchids, then what is the chance that the shop will feature roses in the display case that day?

A) 1.0%
B) 4.5%
C) 5.0%
D) 20.0%

(C) is the correct answer: The store is out of two types of flowers, so it has only 22 - 2 = 20 flowers to choose from for the display. The chance that roses will be picked is thus 1 in 20, which comes out to $\frac{1}{20} = 0.05 = 5.0\%$.

Example #3: Pattern

What is the next term in the following sequence?

$$\frac{1}{5}, \frac{1}{15}, \frac{1}{45}, \dots$$

A) $\frac{1}{55}$ B) $\frac{1}{65}$ C) $\frac{1}{90}$ D) $\frac{1}{135}$

(D) is the correct answer: The denominators of each term in the sequence is three times the denominator of the previous term in the sequence (15 is three times 5, 45 is three times 15). So, the next term will have a numerator of 1 and a denominator that is three times 45, or $\frac{1}{135}$.

III. Absolute Value Questions. These questions test the student's ability to simplify absolute value expressions, solve absolute value equations and inequalities, and understand related properties of absolute value. Examples include:

Example #1: Absolute Value

Which of the following is the solution to the following inequality?

|x - 5| > 7

A) -2 < x < 12B) 2 < x < 12C) x > 2 or x < 12D) x < -2 or x > 12

(D) is the correct answer: If |x - 5| > 7, then x - 5 > 7 or x - 5 < -7. Simplified, that means that x > 12 or x < -2.

Example #2: Absolute Value

Which of the following is equivalent to |3x - 5| > 7?

A)
$$x = 4$$

B) $x > 4$ or $x < -\frac{2}{3}$
C) $x > \frac{2}{3}$ or $x < -4$
D) $-\frac{2}{3} < x < 4$

(B) is the correct answer: For this absolute value inequality, either 3x - 5 > 7 or 3x - 5 < -7. Solve for *x* in each inequality.

3x - 5 > 7 3x > 12 x > 4and 3x - 5 < -7 3x < -2 $x < -\frac{2}{3}$

Example #3: Absolute Value

What is (are) the solution(s) to the equation below?

$$5|x - 2| + 1 = 16$$

A) x = 3 only B) x = 5 only C) x = 5 and x = 3D) x = 5 and x = -1

(D) is the correct answer: Simplify and solve:

5|x - 2| + 1 = 16 5|x - 2| = 15 |x - 2| = 3Since the absolute value of x - 2 is equal to 3, then x is either 5 (since |5-2|=3) or -1(since |-1-2|=3).

Geometry: Plane Geometry

Questions on plane geometry test the student's ability to analyze two-dimensional shapes and to understand points, lines, figures, and functions in the (x, y)-coordinate plane. A student's score in this subdomain indicates their mastery of skills including:

- Finding the slope of a line, given an equation
- Finding the slope of a line, given two points
- Finding the *x* and *y*-intercepts of a line
- Understanding the slope of parallel and perpendicular lines
- Transforming points, lines, and figures in the (x, y)-coordinate plane, including reflections over the x-axis, y-axis, the line y = x, and more
- Understanding the slope of vertical and horizontal lines
- Finding missing angle measures or chord lengths in two-dimensional shapes

Particularly important topics in the Plane Geometry subdomain are Coordinate Geometry, Transformations, Properties of Parallel and Perpendicular Lines, and Finding Missing Measures of Angles and Segments.

I. Coordinate Geometry/Slope Questions. These questions test the student's ability to determine the slope of a line and to identify the slope of parallel and perpendicular lines. Examples include:

Example #1: Coordinate Geometry/Slope

Which of the following is the slope of the line 3x + 4y = 1 in the (x, y)-coordinate plane?

A)
$$-\frac{4}{3}$$

B) $-\frac{3}{4}$
C) 1
D) $\frac{3}{4}$

(B) is the correct answer: In slope-intercept form, 3x + 4y = 1 is equivalent to 4y = -3x + 1 $y = -\frac{3}{4}x + \frac{1}{4}$, so the slope is $-\frac{3}{4}$.

Example #2: Coordinate Geometry/Slope

Which of the following lines in the (x, y)-coordinate plane is perpendicular to a line whose slope is undefined?

A) y = xB) y = 5C) x = -7D) x = 0

(B) is the correct answer: A line with an undefined slope is a vertical line. A line perpendicular to a vertical line is horizontal, with a slope of 0. Here, the only equation given that is a horizontal line is y = 5, thus (B) is correct.

Example #3: Coordinate Geometry/Slope

A right triangle in the (x, y)-coordinate plane has one leg that passes through (2, 1) and (3, 2) and a hypotenuse that passes through (0 - 2) and (6, 0). Which of the following is perpendicular to the line that contains the remaining leg of the triangle?

A)
$$y = 2$$

B) $x = 2$
C) $y = x + 2$
D) $y = -x + 2$

(C) is the correct answer: The remaining leg must be perpendicular to the leg that passes through (2, 1) and (3, 2). The slope of the given leg is $\frac{2-1}{3-2} = \frac{1}{1} = 1$, so the remaining leg has a slope of -1, the negative reciprocal. A line parallel to that leg would have a slope of 1.

II. Transformations Questions. These questions test the student's ability to analyze how points, lines, and other figures are transformed in the (x, y)-coordinate plane. Examples include:

Example #1: Transformations

A line in the (x, y)-coordinate plane has a slope of 3. If the line is rotated 180°, what is the slope of the new line?

A) -3B) $-\frac{1}{3}$ C) $\frac{1}{3}$ D) 3 **(D) is the correct answer:** If a line is rotated 180°, it will have the same slope; thus, the slope will still be 3.

Example #2: Transformations

A point (4, -3) in the (x, y)-coordinate plane is reflected across the *x*-axis. The resulting point is on a line with the point (2, 1). What is the equation of this line?

A)
$$y = \frac{2}{3}x - \frac{1}{3}$$

B) $y = -\frac{1}{3}x + \frac{5}{3}$
C) $y = x - 1$
D) $y = -2x + 5$

(C) is the correct answer: Flipping point (4, -3) across the *x*-axis will reverse the sign of the *y*-value, yielding the point (4, 3). A line passing through both (4, 3) and (2, 1) will have the slope $\frac{3-1}{4-2} = \frac{2}{2} = 1$. Plugging the points and slope into a slope-intercept equation will yield a *y*-intercept of -1, but the only choice with a slope of 1 is (C).

Example #3: Transformations

The line y = x + 5 in the (x, y)-coordinate plane undergoes a transformation such that every point (a, b) is now (a, b + 3) on the resulting line. What is the equation of the line after the transformation?

A)
$$y = 3x + 5$$

B) $y = 3x + 8$
C) $y = x + 2$
D) $y = x + 8$

(D) is the correct answer: On the new line, every *x*-coordinate is the same, but every *y*-coordinate is increased by 3. Thus, add 3 to the right side of the equation:

y = x + 5 + 3y = x + 8

III. Properties of Parallel and Perpendicular Lines Questions. These questions test a student's ability to know the relationship between the slope of parallel lines and the slope of perpendicular lines or use properties of parallel lines and transversals to answer the question. Examples include:

Example #1: Properties of Parallel and Perpendicular Lines

Line Z passes through the points (2, 7) and (4, 5) in the (x, y)-coordinate plane. Which of the following is true about the line that is perpendicular to Line Z at its y-intercept?

- A) It has a slope of 4.
- B) It passes through the point (4, -2).
- C) Its *x*-intercept is $(\frac{1}{4}, 0)$.
- D) It has a greater *y*-value than Line Z at x = 4.

(B) is the correct answer: To determine anything about the line that is perpendicular to Line *Z* at its *y*-intercept, we will first need to known the slope and *y*-intercept of Line *Z*. Using the two coordinates we are given, we can calculate the slope of Line *Z*:

slope =
$$\frac{rise}{run} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{15 - 7}{4 - 2} = \frac{8}{2} = 4$$

Thus, the slope of Line Z is 4. Since perpendicular lines have negative reciprocal slopes, the slope of any line perpendicular to Line Z will have a slope that is the negative reciprocal of 4, which is $-\frac{1}{4}$. (At this point, we can already tell that (A) is incorrect.) Next we need to know the y -intercept of Line Z. Using the slope we already found and obtaining values for x and y from either one of the given points, we can calculate the y-intercept by solving for b in the y = mx + b equation. We show the work for using both of the given points here:

$$y = mx + b$$

7 = 4 \cdot 2 + b = 15 = 4 \cdot 4 + b
7 = 8 + b = 15 = 16 + b
- 1 = b

Thus, the *y*-intercept of Line *Z* is -1, and the complete equation of Line *Z* is y = 4x - 1. Since we already found the slope of any line perpendicular to Line *Z*, we can also determine the equation of the line that is perpendicular to Line *Z* with the same *y*-intercept: $y = -\frac{1}{4}x - 1$. From here, we can determine that (B) is correct by substituting the values from the point (4, - 2) for *x* and *y* and evaluating to check for equivalence:

$$y = -\frac{1}{4}x - 1$$

- 2 = -\frac{1}{4} \cdot 4 - 1
- 2 = -1 - 1 - 1
- 2 = -2

This confirms that (B) is correct; the point (4, -2) lies on the line perpendicular to Line Z at its y -intercept. We can also confirm that (C) and (D) are incorrect. For (C), confirm that the point ($\frac{1}{4}$, 0) does not lie on the line, using a similar substitution method:

$$y = -\frac{1}{4}x - 1$$

$$0 = -\frac{1}{4} \cdot \frac{1}{4} - 1$$

$$0 = -\frac{1}{16} - 1$$

$$0 \neq -1\frac{1}{16}$$

For (D), we can rely on work we have already done. We are given that Line Z passes through the point (4, 15); in other words, its y-value at x = 4 is 15. The line perpendicular to Line Z through (0, - 1) passes through the point (4, - 2), as we determined in (B); in other words, its y-value at x = 4 is -2. x = 4 is not greater than 15, so (D) is false.

Example #2: Properties of Parallel and Perpendicular Lines

Line *Q* is perpendicular to y = -2x + 6 and contains the point (4, 3). What is the *x*-intercept of Line *Q*?

A) (- 2,0)
B) (1,0)
C) (2,0)
D) (3,0)

(A) is the correct answer: Solve for Line *Q*. Since *Q* is perpendicular to a line with a slope of -2, the slope of Line *Q* is the negative reciprocal of -2, or $\frac{1}{2}$. Since Line *Q* contains the point (4, 3), the slope-intercept equation for line can be used to solve for the *y*-intercept: y = mx + b, or $3 = 4(\frac{1}{2}) + b$, so b = 1. Line *Q* is thus represented by the equation $y = \frac{1}{2}x + 1$. The *x*-intercept occurs when y = 0. Let y = 0 and solve for $x: 0 = \frac{1}{2}x + 1$, or $-1 = \frac{1}{2}x$, so x = -2, and the *x*-intercept is (-2, 0).

Example #3: Properties of Parallel and Perpendicular Lines

The amount of Pollutant A in a lake increases linearly over time, at a rate given by the equation below, where x is time in days and y is units of Pollutant A. Pollutant B, on the other hand, is decreasing at the same rate that Pollutant A is increasing. If on Day 0, there were 108 units of Pollutant B in the lake, which equation represents the amount of Pollutant B in the lake after x days?

$$y = \frac{3}{2}x + 221$$

A) $y = \frac{2}{3}x + 221$ B) $y = -\frac{2}{3}x + 108$ C) $y = -\frac{3}{2}x + 108$ D) $y = -\frac{3}{2}x + 113$

(C) is the correct answer: If on Day 0, there were 108 units of Pollutant B in the lake, then the y -intercept of the equation is (0, 108), where the y is units of Pollutant B and x is time in days. The slope of the line will be the same as the slope of the equation that represents the amount of Pollutant A in the lake-but negative, since the amount of Pollutant B is decreasing at the same rate that Pollutant A is increasing. Thus, the equation that represents the amount of Pollutant B in the lake is given by $y = -\frac{3}{2}x + 108$.

IV. Finding Missing Measures of Angles and Segments Questions. These questions test a student's ability to determine the measure of a missing angle or length of a segment given information about various figures. Examples include:

Example 1: Finding Missing Measures of Angles and Segments

In the figure below, Lines \overrightarrow{AD} and \overrightarrow{EF} intersect at *C*. Two line segments intersect at *B*, and $\angle BCA$ is a right angle. If $\overrightarrow{AB} | \overleftarrow{EF}$ and $\angle ECD = 125^\circ$, what is $m \angle ABC$? (Note: The figure is not necessarily drawn to scale.)



A) 35°B) 45°C) 55°

D) 60°

(A) is the correct answer: Since we know that $\angle ECD = 125^\circ$, we can find the measure of its supplementary angle, $\angle DCF$, by subtracting 125° from 180°. Thus, $m \angle DCF = 55^\circ$. Because \overline{AB} and \overleftarrow{EF} are parallel, corresponding angles (angles in the same relative position along the transversal). $\angle BAC$ and $\angle DCF$ have equal measures and so $m \angle BAC = m \angle DCF = 55^\circ$. $m \angle ABC = 180^\circ - (90^\circ + 55^\circ) = 35^\circ$, because the sum of the angles of a triangle equals 180°.

Example 2: Finding Missing Measures of Angles and Segments

In the circle below, *C* is the center and points *A* and *B* lie on the circle. AC = 3cm and $AB = 3\sqrt{2}$ cm. What is the length of the minor arc *AB*?



A) π cm

- B) $\frac{3\pi}{2}$ cm
- C) 2π cm
- D) It is impossible to determine from the given information.

(B) is the correct answer: If AC = 3 cm, $AB = 3\sqrt{2} \text{ cm}$, and AC = BC, then the ratio of the lengths of the sides of $\triangle ABC$ is $1:1:\sqrt{2}$. We can then recognize this as a $45^{\circ} - 45^{\circ} - 90^{\circ}$ right triangle, where $m \angle ACB = 90^{\circ}$. Thus, AB is equal to one-quarter the circumference of the entire circle (since a circle is 360° , and 90° is one-quarter of 360°). As both \overline{AC} and \overline{BC} are radii of the circle that measure 3 cm, the circumference of the circle is equal to $2\pi r = 2\pi(3) = 6\pi$ cm. Thus, AB is equal to $\frac{6\pi}{4} = \frac{3\pi}{2}$ cm.

Example 3: Finding Missing Measures of Angles and Segments

In the circle below, *P* is the center of the circle and \overline{PQ} and \overline{PR} are radii. *PQ* and *PR* are both 6cm. If $m \angle QPR = 60^\circ$, then what is the measure of QR?

A) 1 cm

B) π cm

C) 2π cm

D) 4π cm

(C) is the correct answer: We are given the radius (r = 6 cm) of the circle, as well as the angle (60°) which forms QR. We can find the length of QR by first recognizing that 60° is $\frac{1}{6}$ of 360°. This means that the length of QR will be $\frac{1}{6}$ the circumference of the circle: $QR = \frac{1}{6} \cdot 2\pi r = \frac{1}{6} \cdot 2\pi (6 \text{ cm}) = 2\pi \text{ cm}$

 $QR = 2\pi$ cm; therefore, choice (C) is correct.



Geometry: Properties of Shapes

Questions on properties of shapes test the student's ability to analyze triangles, circles, and polygons and determine additional information about those shapes. A student's score in this subdomain indicates their mastery of skills including:

- Determining triangle congruence
- Using properties of similar shapes to find missing angle measures, side lengths, perimeters, areas, etc.
- Finding the area and circumference of a circle
- Using the Pythagorean theorem
- Understanding special properties of isosceles, right, and equilateral triangles
- Determining the area and perimeter of polygons
- Determining the surface area and volume of three-dimensional shapes

Particularly important topics in the Properties of Shapes subdomain are Properties of Triangles, 2-D Shapes, and 3-D Shapes.

I. Properties of Triangles. These questions test the student's ability to use special properties of isosceles, right, and equilateral triangles in order to find missing information. These questions also test the student's ability to identify congruent triangles. Examples include:

Example #1: Properties of Triangles

An isosceles triangle has one angle that measures 112°. What are the measures of its other angles?

- A) 34° and 34°
- B) 68° and 68°
- C) 12° and 90°
- D) It is impossible to determine from the given information.

(A) is the correct answer: The two other angles in the triangle must be equal, and all angles must add up to 180° . $180^{\circ} - 112^{\circ} = 68^{\circ}$, and $68^{\circ} \div 2 = 34^{\circ}$ each.

Example #2: Properties of Triangles

A right triangle has a hypotenuse measuring 26 units and one leg measuring 10 units. What is the length of its other leg?

A) 10 unitsB) 16 unitsC) 20 unitsD) 24 units

(D) is the correct answer: Using the Pythagorean theorem $(a^2 + b^2 = c^2)$, where *a* and *b* are the length of the legs of the triangle and *c* is the length of the hypotenuse), you can solve for the missing leg. However, you can also recognize that this is a multiple of the Pythagorean triple 5-12-13, as 10 is double 5 and 26 is double 13. The length of the missing leg should be double 12 units or 24 units.

Example #3: Properties of Triangles

 ΔDFG below is equilateral. \overline{JG} is 6 inches. \overline{DH} is one third the length of \overline{DF} . What is the length of \overline{FH} ?



- A) 2 inches
- B) 8 inches
- C) 12 inches
- D) It is impossible to determine from the information given.

(B) is the correct answer: First, find line *DF*. Since the triangle is equilateral, FG = DF. \overline{DJ} bisects \overline{FG} , so FJ = JG. Thus, FJ = 6 inches as well and FG = 6 + 6 = 12 inches. *DF* must also then be 12 inches. If *HD* is one third of the length of *DF*, then *HF* is two thirds of the length of *DF*, or $\frac{2}{3}(12) = 8$ inches.

Example #4: Properties of Triangles

Given the figure shown, which of the following facts would be enough information to prove $\triangle AGE$ and $\triangle BHC$ are congruent?



- A) Quadrilateral *DHFG* is a rhombus.
- B) $m \angle EGA \cong m \angle BHC$ and $m \angle AEG \cong m \angle GDB$
- C) $DE \cong FC$ and Quadrilateral *DHFG* is a Parallelogram.
- D) None of the above.

(D) is the correct answer: Answer choice (A) establishes that $m \angle EGA \cong m \angle BHC$, but does not establish another necessary condition for congruence. The second condition of answer choice (B), $m \angle AEG \cong m \angle GDB$, does not represent corresponding angles (i.e., $\angle GDB$ is not one of the interior angles of $\triangle BHC$). And finally, answer choice (C) would result in the condition of two pairs of congruent sides which do not share congruent included angles and is therefore not a sufficient condition for congruence. Thus, answer choice (D) is the only correct choice.

II. 2-D Shapes Questions. These questions test the student's ability to analyze circles, quadrilaterals, and various polygons and find missing information including interior angles, side lengths, perimeter/circumference, area, similarity, etc. Examples include:

Example #1: 2-D Shapes

A group of biologists is studying the growth of a species of mold on trees. The particular species they are studying only grows in exact circles. They find one tree with two patches of mold, one with a radius of 5 inches and one with a diameter of 6 inches. What is the difference in area between the two circles of mold?

- A) 2π square inches
- B) 4π square inches
- C) 9π square inches
- D) 16π square inches

(D) is the correct answer: The first patch of mold has an area of $\pi r^2 = \pi (5^2) = 25\pi$. The second patch of mold has a radius of $\frac{6}{2} = 3$ and an area of $\pi r^2 = \pi (3^2) = 9\pi$. The difference in area is thus $25\pi - 9\pi = 16\pi$.

Example #2: 2-D Shapes

A regular hexagon has one side measuring 2 inches. What is the area of the hexagon?

- A) 3 square inches
- B) $6\sqrt{3}$ square inches
- C) 12 square inches
- D) $12\sqrt{3}$ square inches

(B) is the correct answer: A hexagon is made up of 6 equilateral triangles. If one side measures 2 inches, then an equilateral triangle with one side of 2 has an area of $\frac{1}{2}$ (2)(*h*), where *h* is the height. The height forms a right triangle with half of the base and one of the sides, which have lengths of 1 and 2 respectively, making the height $\sqrt{3}$. Multiplied by 6, this means the area of the hexagon is $6\sqrt{3}$.

Example #3: 2-D Shapes

2

What is the circumference of the circle that is represented by the equation below?

$$36 = \left(x - \frac{1}{2}\right)^{2} + \left(y + \frac{5}{2}\right)^{2}$$
A) $\frac{5}{2}\pi$
B) 6π
C) 12π
D) 78π

(C) is the correct answer: The equation is in the center-radius form, or $(x - h)^2 + (y - k)^2 = r^2$, where *r* is the radius of the circle. First, solve for the radius of the circle, and then use that radius to calculate the circumference:

 $36 = r^{2}$ 6 = r *Circumference* = $2\pi r$ *Circumference* = $2\pi(6)$ *Circumference* = 12π Thus, (C) is correct.

Example #4: 2-D Shapes

Triangle *A* is similar to Triangle *B*. If Triangle *B* has angles measuring 60° and 70°, which of the following is the measure of one of the angles of Triangle *A*?

- A) 20°
- B) 40°
- C) 60°
- D) 80°

(D) is the correct answer: Similar triangles have angles with equal measures. The interior angles of a triangle add up to 180°. Thus, the remaining angle in Triangle *B* is $180^\circ - 30^\circ - 70^\circ = 80^\circ$. The measures of the angles of both Triangle *A* and Triangle *B* are thus 30° , 70° , and 80° . Of the answer choices only (D) matches these angle measures, so (D) is correct.

III. 3-D Shapes Questions. These questions test the student's ability to analyze three-dimensional shapes including cylinders, spheres, boxes, cones, and more in order to solve for surface area, volume, or other missing information. Examples include:

Example #1: 3-D Shapes

What is the surface area of a sphere whose largest cross section has an area that measures 16π units squared?

- A) 64π units squared
- B) $\frac{256\pi}{3}$ units squared
- C) 1024 π units squared
- D) It is impossible to determine from the given information.

(A) is the correct answer: The largest cross-section of a sphere is a circle that intersects the sphere's center. The circle will thus share a radius with the sphere. If the cross-section has an area of 16π units², its radius is given by:

```
16\pi = \pi r^{2}
16 = r^{2}
4 = r
Then, solve for the surface area of the sphere with a radius of 2 units:
4\pi r^{2}
4\pi (4^{2})
4(16)\pi
64\pi \text{ units}^{2}
```

Example #2: 3-D Shapes

A cylinder that is 5 feet high, with a circumference of 2π feet, is about 40% full of water. The company that owns the cylinder plans to dye the water blue. If they need 4 ounces of dye for every 12 cubic inches of water, how much dye will they need to dye all of the water currently in the cylinder?

- A) $\frac{2\pi}{3}$ ounces of dye
- B) 1152π ounces of dye
- C) 2880π ounces of dye
- D) 3456π ounces of dye

(B) is the correct answer: The volume of the cylinder is given by the area of the base times its height. Given a circumference of 2π feet, or 24π inches, you can solve for the radius: $24\pi = 2\pi r$, or 12 inches. The area is thus $\pi r^2 = 144\pi$ square inches. Multiplied by the height, the volume is thus (60)144 π cubic inches. Since the cylinder is only 40%, or $\frac{2}{5}$, full, there is only

 $\left(\frac{60(2)}{5}144\pi\right) = (24)(144)\pi = 3456\pi$ cubic inches of water in the cylinder. The company needs 4 ounces of dye for every 12 cubic inches of water, or 1 ounce of dye for every 3 cubic inches of water. Divide 3456π by 3 to determine how many ounces of dye are needed: 1152π .

Example #3: 3-D Shapes

A chemical company uses the cylindrical storage containers shown below to store a viscous fluid with a density of $1.5 \frac{g}{cm^3}$. If the radius of the container is 4 m, and the height of the container is 10 m, what is the weight of the cylinder when it is full, in grams? (1 m = 100 cm)



A) 450g B) 1,600,000 π g C) 2,400,000 π g

D) 240,000,000 $\pi~g$

(D) is the correct answer: First, calculate the volume of the cylinder by multiplying the area of the base by the height. Because the density is measured in grams (g) per centimeter (cm) cubed, we begin by converting meters (m) into centimeters (cm): 10 m is 1000 cm, and 4 m is 400 cm. $Volume = \pi r^2 h$

 $Volume = \pi (400 \text{ cm})^{2} (1000 \text{ cm})$ $Volume = \pi (160,000 \text{ cm}^{2}) (1000 \text{ cm})$ $Volume = 160,000,000\pi \text{ cm}^{3}$

Since the density of the fluid is $1.5 g/cm^3$, we can calculate the weight.

 $\frac{1.5 g}{1 cm^3} = \frac{x}{160,000,000 \pi cm^3}$ 240,000,000 $\pi g \cdot cm^3 = x cm^3$

240, 000, $000\pi g = x$ and (D) is the answer.

Geometry: Trigonometry

Questions on trigonometry test the student's ability to use a right triangle's angle measures and the ratios between its side lengths in order to deduce additional information. Advanced questions also look at students' ability to understand and manipulate trigonometric identities and the graphing of trigonometric functions. A student's score in this subdomain indicates their mastery of skills including:

- Solving simple trigonometric equations
- Using trigonometric identities to simplify an expression
- Recognizing equivalent trigonometric expressions
- Identifying graphs of trigonometric equations in the (x, y)-coordinate plane
- Understanding the unit circle
- Identifying trigonometric functions of any angles on the unit circle

Trigonometric identities and ratios are always provided in a list of formulas at the start of each Quantitative Reasoning section.

Particularly important topics in the Trigonometry subdomain are Sine, Cosine, Tangent Ratios; Cosecant, Secant, and Cotangent Ratios; Graphing of Trigonometric Functions; and the Unit Circle. The CLT explicitly covers these areas.

I. Sine, Cosine, Tangent Ratio Questions. These questions test the student's ability to understand the relationships between side lengths in right triangles. Examples include:

Example #1: Sine, Cosine, Tangent Ratios

In the triangle below, the hypotenuse is 17 inches long, and \overline{BC} is 8 inches long. What is $\cos (A)$?



A) $\frac{8}{12}$ B) $\frac{8}{17}$ C) $\frac{15}{17}$ D) $\frac{13}{8}$ (C) is the correct answer: The cos(A) is equal to the length of the adjacent side over the length of the hypotenuse, 17. Since this is a Pythagorean triple, the missing side length is 15 inches. The cos(A) is thus equal to $\frac{15}{17}$.

Example #2: Sine, Cosine, Tangent Ratios

A triangle has side lengths that measure 6 inches, 8 inches, and 10 inches. What is the cosine of the smallest angle of the triangle?

A) $\frac{3}{5}$ B) $\frac{3}{4}$ C) $\frac{4}{5}$ D) $\frac{5}{3}$

(C) is the correct answer: The shortest angle of the triangle will be opposite the shortest leg, which here is 6 inches. Thus, the adjacent side measures 8 inches and the hypotenuse of the triangle is 10 inches (you can double-check this is a right triangle using the Pythagorean theorem: $10^2 = 8^2 + 6^2$). The cosine is the length of the adjacent side over the length of the hypotenuse, or $\frac{8}{10} = \frac{4}{5}$.

Example #3: Sine, Cosine, Tangent Ratios

Which of the following is equivalent to $1 + \sin^2 x + \cos^2 x$?

A) 2 B) $\tan^2 x$ C) $-\sin^2 x - \cos^2 x$ D) $1 - \sin^2 x + \cos^2 x$

(A) is the correct answer: From the formulas provided in the Quantitative Reasoning section, we can see that $\sin^2 \theta + \cos^2 \theta = 1$. Here θ is represented by x, so $\sin^2 x + \cos^2 x = 1$. Therefore, substituting this into the original expression, $1 + (\sin^2 x + \cos^2 x) = 1 + 1 = 2$ and the correct answer is (A).

II. Cosecant, Secant, and Cotangent Ratio Questions. These questions test the student's ability to understand the inverse of the three main trigonometric functions. Examples include:

Example #1: Cosecant, Secant, and Cotangent Ratios

Which of the following is equivalent to the expression below?

$$\frac{4}{\cot\theta}\sqrt{rac{1}{\csc^2 heta}+rac{\sin^2 heta}{\tan^2 heta}}$$

A) 0B) 1C) 4

D) 4tan0

(D) is the correct answer: Use the formulas provided within the test to put the expression into another form:



Tour o

4tan θ

Example #2: Cosecant, Secant, and Cotangent Ratios

Which of the following is NOT equivalent to $\frac{\csc \theta}{\cot \theta \sec \theta}$?

A) 1
B)
$$\csc \theta \tan \theta \cos \theta$$

C) $\frac{1}{\csc^2 \theta} + \frac{1}{\sec^2 \theta}$
D) $\frac{\csc^2 \theta}{\cot^2 \theta - 1}$

(D) is the correct answer: Using trigonometry identities, we can simplify $\frac{\csc \theta}{\cot \theta \sec \theta}$. By their basic definitions, $\csc \theta = \frac{1}{\sin \theta}$, $\sec \theta = \frac{1}{\cos \theta}$, and $\cot \theta = \frac{\cos \theta}{\sin \theta}$. Substituting these expressions into the original expression, we can obtain the following:

A) $\frac{\csc \theta}{\cot \theta \sec \theta} = \frac{1}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta} \cdot \cos \theta = 1$ B) $\frac{\csc \theta}{\cot \theta \sec \theta} = \csc \theta \cdot \frac{\sin \theta}{\cos \theta} \cdot \cos \theta = \csc \theta \tan \theta \cos \theta$

For (C), we can use our result from part (A) along with the identity $\sin^2 \theta + \cos^2 \theta = 1$ to show that the expression in (C) is equivalent to the original expression:

(C)
$$\frac{\csc\theta}{\cot\theta\sec\theta} = 1 = \sin^2\theta + \cos^2\theta = \frac{1}{\csc^2\theta} + \frac{1}{\sec^2\theta}$$

The only expression that cannot be shown to be equal to $\frac{\csc \theta}{\cot \theta \sec \theta}$ is the expression in (D), making (D) the correct answer.

Example #3: Cosecant, Secant, and Cotangent Ratios

Which of the following is equivalent to $\frac{\cot 30^\circ}{\sec 30^\circ}$?

A) $\frac{\sqrt{3}}{2}$ B) $\frac{1}{2}$ C) $\frac{3}{2}$ D) 2

(C) is the correct answer: The answer could be found simply in the following manner:

 $\frac{\cot 30^{\circ}}{\sec 30^{\circ}} = \frac{\text{side length adjacent the } 30^{\circ}}{\text{side length opposite the } 30^{\circ}} \div \frac{\text{hypotenuse length}}{\text{side length adjacent the } 30^{\circ}} = \frac{\sqrt{3}}{1} \div \frac{2}{\sqrt{3}} = \frac{\sqrt{3}}{1} \cdot \frac{\sqrt{3}}{2} = \frac{3}{2}$

Alternatively, the answer can be found as follows: In a right triangle with one angle measuring 30°, the remaining angle measure must equal 60° ($180^\circ - 90^\circ - 30^\circ = 60^\circ$). We are given, at the start of the section, the side length ratios for a 30° - 60° - 90° triangle: $1:\sqrt{3}: 2$. In the figure below, the side length ratios are shown relative to their opposite angles.



From here, using the trigonometric ratios for such a triangle, as shown in the figure above, we find that $\cot 30^\circ = \frac{\text{side length adjacent the } 30^\circ \text{ angle}}{\text{side length opposite the } 30^\circ \text{ angle}} = \frac{\sqrt{3}}{1}$ and $\sec 30^\circ = \frac{\text{hypotenuse length}}{\text{side length adjacent the } 30^\circ \text{ angle}} = \frac{2}{\sqrt{3}}$ (Note: because these are trigonometric ratios, we can use the side length ratios as though there were actual side lengths). Dividing these two fractions, we get the result of $\frac{3}{2}$.

III. Graphing of Trigonometric Functions Questions. These questions test the student's ability to recognize graphs of trigonometric functions in a Cartesian coordinate system. Examples include:

Example #1: Graphing of Trigonometric Functions

If $y = 6 \cos(x - 2) + 9$ is graphed in the (x, y)-coordinate plane, which of the following is the maximum value of y?

A) 2

B) 6C) 9

D) 15

(D) is the correct answer: The equation for a trigonometric function graphed in the (x, y)-coordinate plane is given by y = Atrig(Bx - C) + D, where *A* is the amplitude of the function, *trig* is a placeholder for a specific trigonometric function, such as cosine, $\frac{2\pi}{B}$ is the period, *C* is the phase shift, and *D* is the vertical shift. The amplitude of y = 6cos(x - 2) + 9 is equal to 6 and so its maximum *y*-value is 6. However, the vertical shift, *D*, in this case is equal to 9. Since this number is positive, this represents a vertical shift of the entire function upward by 9 units. Therefore, the maximum value for *y* would be 6 + 9 = 15.

Example #2: Graphing of Trigonometric Functions

Which transformation of the function $y = sin \theta$ yields a function with an amplitude of 3 and a period of π radians?

A) $y = 2 \sin \theta + 3$ B) $y = \frac{1}{2} \sin 3\theta$ C) $y = 3 \sin \frac{1}{2} \theta$ D) $y = 3 \sin 2\theta$

(D) is the correct answer: Multiplying the value of the *y* coordinates by $(sin \theta)$ by 3 will give a function with an amplitude of 3 instead of the amplitude of 1 that the original function has. In order to shorten the period from 2π to π radians, we remember that if our equation is in the form y = A sin Bx, then the period of the function is equal to $\frac{2\pi}{B}$. In this case, this means that we have *Period* = $\frac{2\pi}{B}$, which means that B = 2, and so the equation that we're looking for is $y = 3 sin 2\theta$, making (D) the correct response.

IV. Unit Circle Questions. These questions test the student's ability to relate side lengths and angles to trigonometric ratios in the Cartesian coordinate system. Examples include:

Example #1: The Unit Circle

Circle *C* has its center at the origin. Point *A*, whose coordinates are $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, lies on Circle *C*. Point *B*, whose coordinates are $\left(\frac{\sqrt{2}}{2}, 0\right)$, is the third vertex in $\triangle ABC$. AC = 1. What is $m \angle ACB$ in $\triangle ABC$?

- A) 30°
- **B**) 45°
- **C)** 60°
- D) 90°

(B) is the correct answer: Since the radius of Circle *C* is 1 and its center is the origin, we know that Circle *C* is the unit circle. From the coordinates of points *A* and *B*, we know that \overline{AB} , a side of $\triangle ABC$, is a vertical line segment and that its length is $\frac{\sqrt{2}}{2}$. From the coordinates of points *B* and *C* (remembering that *C* is at the origin), we know that \overline{BC} is a horizontal line segment and that its length is also $\frac{\sqrt{2}}{2}$. We now know that $\angle ABC$ is a right angle and that $\triangle ABC$ is a right triangle. The side lengths of the right triangle have a ratio of $1: 1: \sqrt{2}$, meaning that this is a $45^{\circ}-45^{\circ}-90^{\circ}$ triangle, and $m \angle ACB = 45^{\circ}$.

Example #2: The Unit Circle

A unit circle centered at the origin in the (x, y)-coordinate plane has a point that lies on its edge with coordinates (x, y). An angle, θ , is in standard position and its terminal side goes through that point. If $\cot \theta = -\sqrt{3}$, which of the following could be the coordinates of the point?



A) $A(-\frac{1}{2}, \frac{\sqrt{3}}{2})$ B) $B(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$ C) $C(\frac{\sqrt{3}}{2}, -\frac{1}{2})$ D) $D(\frac{1}{2}, -\frac{\sqrt{3}}{2})$

(C) is the correct answer: The simplest first step is to recognize that the cotangent function is negative only in Quadrants II and IV. That quickly eliminates point B as a possible answer. That leaves us with points *A*, *B*, and *D*. Each of the triangles corresponding to those answer choices can be constructed along the unit circle as in the figure shown below. The arrows indicate the direction of angle θ from standard position (along the positive *x*-axis) to the terminal side. From here, we can use the given value for *cot* θ to form a type of reference triangle for θ . The "adjacent" side has length 1, the "opposite" has a length of $\sqrt{3}$, therefore the hypotenuse has length 2. We see now that the absolute value of the *x*-coordinate of our point will be *cos* θ or $\frac{1}{2}$. We now look through the answer choices to see which option meets these criteria and also puts our point in the second or fourth quadrant. Thus, answer choice C is correct.



Example #3: The Unit Circle

A unit circle is centered at the origin of the (x, y)-coordinate plane. The point $(\frac{\sqrt{3}}{2}, \frac{1}{2})$ lies along the circumference of the circle. If a diameter is drawn from point $(\frac{\sqrt{3}}{2}, \frac{1}{2})$ to point *P* on the opposite side of the circle, then what is tangent of the angle formed between the radius of the circle from the origin to point *P* and the *x*-axis?



(B) is the correct answer: We are given $(\frac{\sqrt{3}}{2}, \frac{1}{2})$ on the unit circle. We consider point *P* also on the unit circle and directly opposite the given point. This means that point *P* has the coordinates

 $\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$, as shown in the figure below. Let us call the angle formed between the radius of the circle from the origin to point *P* and the negative *x*-axis, β , and let us call the angle formed between the positive *x*-axis and our given point $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$, θ . On the unit circle, $tan \theta$ is the slope of the radius from the origin to $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$. We can see from the figure below that the slope will be the same between our given point $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$ and point $P\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$. Therefore $tan \theta = tan \beta$. We know that $tan \theta = \frac{\sin \theta}{\cos \theta}$ and that on the unit circle $x = \cos \theta$ and $y = \sin \theta$. For our given point, $x = \frac{\sqrt{3}}{2}$ and $y = \frac{1}{2}$. Substituting *x* and *y*, we get $tan \theta = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$. Given that we have shown that $tan \beta = tan \theta$, then $tan \beta = \frac{\sqrt{3}}{3}$. Therefore, choice (B) is correct.

Mathematical Reasoning: Logic

Questions on logic test the student's ability to use given information to arrive at a new conclusion. A student's score in this subdomain indicates their mastery of skills including:

- Analyzing a set of numbers based on a pair of conditions
- Identifying counterexamples
- Identifying false and true statements based on a given information
- Drawing conclusions about an unknown integer from given information

Particularly important topics in the Logic subdomain are Basic Logic Problems and Integer Problems.

I. Basic Logic Problems. These questions test the student's ability to identify counterexamples as well as true and false statements based on given information. Many of these question types are rooted in geometry. Some questions within this category also ask students to analyze a set of numbers based on a pair of given conditions. Examples include:

Example #1: Basic Logic Problems: "Conditions" Logic Problem

How many integers between 20 and 70 (inclusive) meet both conditions below?

- 1. The integer is prime.
- 2. The product of the digits of the integer is six.
- A) 2
- B) 3
- C) 6
- D) 11

(A) is the correct answer: Prime numbers between 20 and 70 include 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, and 67. Of those, only two have digits whose product is six: 23 and 61. An alternate method that yields the same result is to recognize that between 20 and 70, numbers 23, 32, and 61 have a product of six—but only 23 and 61 are prime

Example #2: Basic Logic Problems: Which of the following is a counterexample?

A student studying triangles comes up with the following rule:

The hypotenuse of a right triangle is always exactly two units longer than at least one leg of the right triangle.

Which of the following is a counterexample that disproves the above statement?

- A) A right triangle with side lengths of 3 units, 4 units, and 5 units.
- B) A right triangle with side lengths of 6 units, 8 units, and 10 units.
- C) A right triangle with side lengths of 8 units, 15 units, and 17 units.
- D) A right triangle with side lengths of 7 units, 24 units, and 25 units.

(D) is the correct answer: To find the counterexample that disproves the statement, find the example which demonstrates that the hypotenuse of a right triangle is NOT necessarily two units longer than the leg of a right triangle. (D) is correct because a right triangle with side lengths of 7, 24, and 25 units has a hypotenuse that is, respectively, 1 unit and 18 units longer than its legs, not 2 units longer than the leg of 3 units. (A) cannot be correct because the hypotenuse is 2 units longer than the leg of 3 units. (B) cannot be correct because the hypotenuse is 2 units longer than the leg of 8 units. (C) cannot be correct because the hypotenuse is 2 units longer than the leg of the 15 units.

Example #3: Basic Logic Problems: Which of the following is false?

For rectangle *ABCD*, AB = BC = CD. Which of the following is false?

A) $m \angle A = 90^{\circ}$ B) AD = BCC) $m \angle A > m \angle B$ D) $m \angle A + m \angle D = 180^{\circ}$

(C) is the correct answer: If three sides of a rectangle are congruent, the fourth side must also be equal in length, as opposite sides of a rectangle are equal in length. Every interior angle of a rectangle measures 90°, so (A) is true. All sides of this rectangle are congruent, so (B) is true. (D) is true because, as each interior angle of the rectangle measures 90°, the sum of the measures of two of the interior angles is equal to 180°. (C) is not true because each interior angle of a rectangle is congruent, so (C) is false and thus the correct answer.

Example #4: Basic Logic Problems: Which of the following is true?

A right triangle's hypotenuse is twice as many inches long as one of its legs. Which of the following must be true?

A) The remaining leg of the triangle measures $\sqrt{3}$ inches.

B) The triangle is similar to all right triangles with one angle measuring 60°.

C) The two legs of the right triangle are equal in length.

D) The triangle is congruent to a right triangle which has a hypotenuse measuring 8 inches and one leg measuring $4\sqrt{3}$ inches.

(B) is the correct answer: If the ratio of the length of the leg of a right triangle to the hypotenuse of a right triangle is 1: 2, it is a $30^{\circ} - 60^{\circ} - 90^{\circ}$ triangle, whose legs have a ratio of $1:\sqrt{3}: 2$, as noted in the formulas provided in the Quantitative Reasoning section. (B) is true because right triangles with one angle measuring 60° are always $30^{\circ} - 60^{\circ} - 90^{\circ}$ triangles, since their remaining angles must be $180^{\circ} - 90^{\circ} - 60^{\circ} = 30^{\circ}$. Since the triangles share their angle measures, they are similar. (A) is incorrect because while the ratio of the measure of the final leg of the triangle to the hypotenuse of the triangle is $\sqrt{3}: 2$, we do not have any information about actual measures of the triangle—the measures could be $2\sqrt{3}$ inches and 4 inches, for example. (C) is incorrect because the ratio of the sides of the triangle is $1: \sqrt{3}: 2$; no side lengths are equal. (D) is incorrect because to be congruent, the measures of the sides of the triangle must be equal, and we have no information about the concrete measures of the triangle sides in the question—only a ratio.

Example #5: Basic Logic Problems: Given xxx, Find xxx

An online candle business open five days a week (Monday through Friday) sells fifteen candles a day its first week. If each week, it sells three more candles a day, how many candles IN TOTAL does that business sell in Week 5?

- A) 24 candlesB) 27 candles
- C) 120 candles
- D) 135 candles

(D) is the correct answer: If each week, the candle business sells 3 more candles than the first week, the number of candles it sells EACH DAY in Week 5 is given by 15 + 3 + 3 + 3 + 3 = 27. Multiplied by 5 days of the week, the total number of candles sold in Week 5 is given by 5(27)=135 candles.

II. Integer Problems. These questions test the student's ability to assess and comprehend given information about an integer and to draw logical conclusions based on that information. Examples include:

Example #1: Integer Problem

Which of the following is always true for the integers w and z, if w > z and z is negative?

A) $w^{2} > z^{2}$ B) $w + 1 > z^{3}$ C) -w < zD) $w^{2} < z^{3}$

(B) is the correct answer: To investigate the answer choices with some possible numbers, let w = 1 and z = -1

(A) 1 is equal to, not greater than, 1, so eliminate (A).

(B) 2 is greater than -1, so keep option (B).

(C) -1 is not less than -1, so eliminate (C).

(D) 1 is not less than -1, so eliminate (D).

But is (B) always true? Since z is an integer, and z < 0, it follows that $z^3 < 0$ and $z^3 \le z$. But z < w, so $z^3 \le z < w < w + 1$, and $w + 1 > z^3$.

Example #2: Integer Problem

If w is an even integer, then which of the following is odd?

A) 25wB) w^{3} C) $w^{5} - 1$ D) $3w^{2} + 8$

(C) is the correct answer: Let *w* be 2, and test each choice:

(A) 25(2) = 50, which is even.

(B) $2^3 = 8$, which is even.

- (C) $2^5 1 = 32 1 = 31$, which is odd.
- (D) $3(2)^2 + 8 = 12 + 8 = 20$, which is even.

Example #3: Integer Problem

If *a* is an even positive integer, which of the following must be true?

A) 3a is odd. B) a^2 is even. C) a + 2 is odd. D) a - 3 is even.

(B) is the correct answer: Let *a* be an even, positive integer such as 2, and test each of the answer choices with this integer. (A) is not true because 3a = 3(2) = 6, which is not odd. (B) is true because $a^2 = 2^2 = 4$, which is even. (C) is not true because a + 2 = 2 + 2 = 4, which is not odd. (D) is not true because a - 3 = 2 - 3 = -1, which is not even. Thus, (A), (C), and (D) can all be eliminated. (B) is true because any even positive integer multiplied by itself will also be even.

Mathematical Reasoning: Word Problems

Word problems test the student's ability to use reasoning and logic to draw conclusions about real-life scenarios. A student's score in this subdomain indicates their mastery of skills including:

- Drawing logical conclusions
- Identifying logical fallacies
- Reasoning about geometric shapes in real-life scenarios
- Using ratios to identify additional information
- Understanding percent increase and decrease
- Solving quadratic word problems
- Solving work-rate word problems

Particularly important topics in the Word Problems subdomain are Reasoning and Logic Word Problems, Geometric Word Problems, and Arithmetic Word Problems.

I. Reasoning and Logic Word Problems. These questions test the student's ability to draw logical conclusions about real-life scenarios given a set of limited conditions and facts. Students will have to identify relevant information, discard irrelevant information, and draw a conclusion based on given evidence. Examples include:

Example #1: Reasoning and Logic Word Problems

An explorer discovers a new type of cat with purple and white stripes. Every cat with purple and white stripes also has a brown nose. If a cat does not have a brown nose, then which of the following is true?

- A) It only has white fur.
- B) It has no stripes.
- C) It does not have purple and white stripes.
- D) It has a black nose.

(C) is the correct answer: The question states that every cat with purple and white stripes has a brown nose. Thus, no purple and white cat will have a different colored nose. If a cat does NOT have a brown nose, it can be inferred that it does not have purple and white stripes, making (C) correct. The question says nothing, however, about whether there may be cats with brown noses and different colored fur, or about any other color combinations. (A) and (B) are incorrect because the question says nothing about the color or pattern of fur paired with any other colored noses. (D) is incorrect because it cannot be inferred from the question that the cat has a black nose--it could have a red, pink, purple, etc. nose.

Example #2: Reasoning and Logic Word Problems

Diego is a member of the football team and the basketball team at his school, but not a member of any other teams. Brian is a member of three teams at the same school, only one of which he shares with Diego. Which of the following must be true?

- A) Brian and Diego are both members of the football team.
- B) Brian and Diego are both members of the basketball team.
- C) Brian does not play both football and basketball.
- D) Brian plays both football and basketball.

(C) is the correct answer: Brian plays on three teams, only one of which he shares with Diego, who plays on the football team and basketball team. Thus, Brian plays on either the football team or the basketball team, but not both. Thus, (C) is correct. (A) and (B) are incorrect because we do not have enough information to know whether Brian plays on the basketball or football team. (D) is incorrect because given the information in the question, Brian cannot play on both the basketball and football team, as he shares only one team with Diego.

Example #3: Reasoning and Logic Word Problems

A new type of multiplayer online game allows up to five players to choose avatars to represent them: a rabbit, an evergreen, a flower, a star, and a fish. Lydia, Clark, Marko, Priya, and Mike all decide to play together. Priya does not pick the evergreen or the star. Both Clark and Marko end up with animals as their avatars. When you combine the names of Mike and Marko's avatars, you get a new type of animal. Which avatars did Priya and Lydia end up with?

- A) Lydia chose the evergreen, and Priya chose the flower.
- B) Lydia chose the star, and Priya chose the flower.
- C) Lydia chose the star, and Priya chose the rabbit.
- D) Lydia chose the flower, and Priya chose the fish.

(A) is the correct answer: Create a visual representation to help determine which avatars Priya and Lydia end up with, as shown below. Note that combining "star" and "fish" creates a new type of animal—starfish.

	Rabbit	Evergreen	Flower	Star	Fish
Lydia	Х	0	Х	Х	Х
Clark	0	Х	Х	Х	Х
Marko	Х	Х	Х	Х	0
Priya	Х	Х	0	Х	Х
Mike	X	X	X	0	Х

Thus, Lydia ends up with the evergreen, and Priya ends up with the flower.
Example #4: Reasoning and Logic Word Problems

A chemist is studying the density of four different gases. If Gas X and Gas Y are each less dense than Gas Z, and Gas Y is more dense than Gas W, then which of the following must be true?

- I. Gas X is more dense than Gas W.
- II. Gas W is less dense than Gas Z.
- III. Gas X is less dense than Gas Y.
- A) I only
- B) II only
- C) I and II only
- D) I, II, and III

(B) is the correct answer: Based on the information in the question the densities of the gases are as follows: density of Gas W < density of Gas Y < density of Gas Z. Gas X is also less dense than Gas Z, but it is not certain how dense Gas X is in comparison to Gases W and Y. (I) and (III) therefore cannot be determined with any certainty. However, Gas W is less dense than Gas Z (by the transitive property of inequality), so (II) is true.

II. Geometric Word Problems. These questions test the student's ability to reason about directions, shapes, and figures in real-life scenarios. Examples include:

Example #1: Geometric Word Problems

Javier's parents buy him a gridded mat, shown below. When anyone steps on any one of the boxes in the grid, that box and all adjacent boxes light up. If Javier steps on A3 and A5, and at the same time his younger sister Sofia steps on C6 and C7, how many boxes are left unlit?



(C) is the correct answer: If Javier steps on A3 and A5, then A2, B2, B3, B4, A3, A4, B5, A5, B6, and A6 will light up. If Sofia steps on C6 and C7, then B7, C5, D5, C6, C7, D6, and D7 will light up. That means 17 boxes will light up, leaving $7 \times 4 = 28 - 17 = 11$ boxes unlit.

Example #2: Geometric Word Problems

A sports equipment company needs to package baseballs into boxes for shipments to outlet stores. If each baseball has a radius of 1.5 inches, and the baseballs are perfect spheres, then how many baseballs would be able to fit into a cubic box with one side measuring 24 inches? Assume that each layer of baseballs has the same number of balls as the layer beneath it.

- A) 48
- B) 64
- C) 128
- D) 512

(D) is the correct answer: Each baseball has a radius of 1.5 inches, or a diameter of 3 inches. Thus, 8 baseballs could fit along each side of the box, so the first layer would have $8 \times 8 = 64$ baseballs. As the box is also 24 inches tall, 8 layers of 64 baseballs would be able to fit inside, or $64 \times 8 = 512$.

Example #3: Geometric Word Problems

Six circular game chips can fit inside each holding cylinder in a board game. If the game comes with seven cylinders, each filled halfway with chips, then how many game chips does it come with?

A) 18
B) 21
C) 28
D) 36

(B) is the correct answer: Each holding cylinder can fit six game chips, but comes filled only halfway, or $6\left(\frac{1}{2}\right) = 3$. Thus, if seven holding cylinders each contain three game chips, the total number of game chips that come in the board game is 7(3) = 21.

III. Arithmetic Word Problems. These questions test the student's ability to solve relevant logic-based word problems including percent increase and decrease, ratios, proportion and rate word problems, quadratic word problems, and work-rate word problems. Examples include:

Example #1: Arithmetic Word Problems

A scientist arrives at her lab only to find that 24 out of 250 samples have been improperly stored. If she is able to salvage 12 of the improperly stored samples, what percent of the total number of samples was she not able to salvage?

A) 4.8%
B) 5.3%
C) 9.6%

D) 10.6%

(A) Is the correct answer: If the scientists saves 12 of the 24 improperly stored samples, then 12 out of 250 were not salvaged, or $\frac{12}{250} = 0.048 = 4.8\%$.

Example #2: Arithmetic Word Problems

Candace can plant six trees in 40 minutes. Working together, Candace and Janet can plant 12 trees in 20 minutes. How many trees could Janet plant in one hour?

- A) 6 trees
- B) 24 trees
- C) 27 trees
- D) 54 trees

(C) is the correct answer: To solve this problem, we can use a general expression for rate: $rate(r) = \frac{amount \ of \ some \ work \ done}{time}$. Candace can plant 6 trees in 40 minutes, which means that she can plant trees at a rate of $r_{Candace} = \frac{6}{40}$ trees/minute. Working together, Candace and Janet can plant trees at a rate of $r_{Both} = \frac{12}{20}$ trees/minute. To calculate how many trees Janet can plant in one hour (or 60 minutes), we need to recognize that the rate at which Candance and Janet plant trees is equivalent to the sum of their individual tree planting rates ($r_{Both} = r_{Candace} + r_{Janet}$). We can let x represent the number of trees that Janet plants. Using our rates, we can set up the following equation: $\frac{12}{20} = \frac{6}{40} + \frac{x}{60}$. Solving this equation for x, we get x = 27 trees.

Example #3: Arithmetic Word Problems

A soccer player kicks a ball straight up in the air. The equation $h(t) = -16t^2 + 8t$ approximates the height in feet, h(t), of the ball t seconds after it was kicked. How many seconds after being kicked will the ball reach its greatest height, according to this equation?

- A) 0.00 seconds
- B) 0.25 seconds
- C) 0.50 seconds
- D) 1.00 seconds

(B) is the correct answer: This question can be solved using the formula for finding the vertex (in this case, maximum) of a parabola. Given the general form of the equation for a parabola, $f(x) = ax^2 + bx + c$, the parabola's vertex is the point $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$. Utilizing this formula for our equation, $h(t) = -16t^2 + 8t$, we are solving for time, t, when the ball will reach its maximum height. We can find this time by setting $t = \frac{-b}{2a} = \frac{-8}{2(-16)}$. Simplifying this expression, we get t = 0.25 seconds. The answer can also be found by choosing some values for t and plotting the function h(t) to find the maximum value or finding the midpoint between the two solutions of the equation.

Example #4: Arithmetic Word Problems

A mother buys two types of shirts for her son: red shirts and blue shirts. Before she goes shopping, the ratio of red to blue shirts in the son's closet is 11 : 14. She buys 2 new red shirts and 4 new blue shirts. When she adds these to her son's closet, what is the new ratio of red to blue shirts?

- A) 3:4
- B) 11:14
- C) 13:18
- D) It is impossible to determine from the given information.

(D) is the correct answer: You are given the initial ratio of red to blue shirts in the closet. The son could have 11 red shirts and 14 blue shirts or any multiple of these numbers (such as 22 red shirts and 28 blue shirts, etc.). Adding 2 red shirts and 4 blue shirts would change the ratio in different ways depending on how many shirts the son has to begin with. Without that information, it is impossible to determine the new ratio of red to blue shirts.



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